AC Circuits – ID: 9525

Time required
45 minutes

Activity Overview

In this activity, students explore a model of alternating electric current. They observe the effects of varying voltage, angular velocity, frequency, and phase shift on the shape of the waveform. They also calculate the relative phase shift between two waveforms. Finally, they create a model of a three-phase alternating current.

Concepts

- *Alternating electric current*
- *Mathematical models of alternating electric current*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes

Teacher Preparation

This activity will give students an introduction to the use of sine curves to model alternating current. Go over how parameters used in a sine function relate to wave properties such as amplitude, frequency, period, and phase.

- *If time allows, you may wish to have students experiment with oscilloscopes and relate their observations to the simulations provided in this activity.*
- *The screenshots on pages 2–8 demonstrate expected student results. Refer to the screenshots on pages 9 and 10 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9525" in the search box.***

Classroom Management

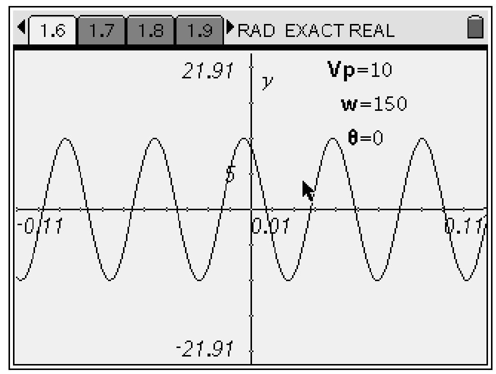
- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

- What waveform best models alternating current (AC) electricity?
- How do peak voltage, angular velocity and frequency of the generator, and phase shift affect the shape of the curve?
- What equations model these relationships?

In the first part of the activity, students explore the equation for a waveform describing the voltage of AC electricity over time. In the second part of the activity, students derive an equation for the phase shift of one waveform relative to another. In the final part of the activity, students use their acquired knowledge to solve a problem involving AC electricity generated by an electrical generator.

Problem 1 – Manipulation of a basic sine curve



Step 1: Students should open the file **PhyAct26_ACcircuits_EN.tns** and read the first five pages. Page 1.6 contains a sine curve representing the voltage in an alternating current with time. Students should vary the values of **Vp**, **w**, and **θ** and observe the effects on the waveform. Then, they should answer questions 1–3.

Q1. What characteristic of the curve does the variable **Vp** control?

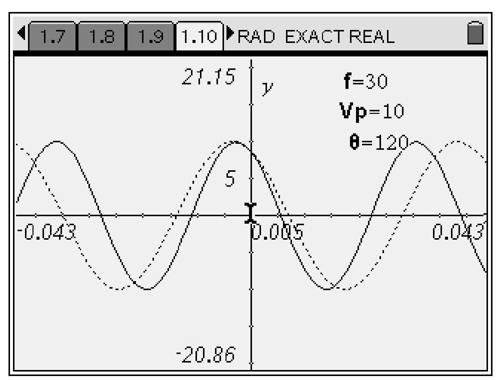
A. *the amplitude (height) of the curve*

Q2. What characteristic does the variable **w** control?

A. *the distance between successive peaks*

Q3. What characteristic does the variable **θ** control?

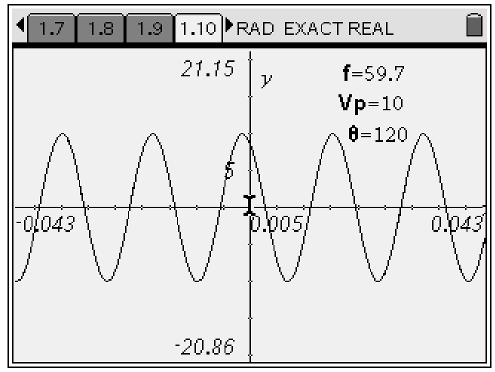
A. *the phase shift of the curve (the locations of the maxima and minima along the* x*-axis)*



Step 2: Next, students should read the information on pages 1.8 and 1.9. Page 1.10 shows two sine curves. The dotted curve is identical to the curve students manipulated on page 1.6. Students should vary the value of **f** and observe the effects on the waveform. Then, they should answer questions 4–7.

Q4. What characteristic of the curve does the variable **f** control?

A. *the frequency of the curve (the distance between successive peaks)*



Q5. Predict the approximate value of **f** required to produce a sine curve equivalent to the one produced by a generator with an angular velocity of 375.

A. *Students should use the relationship $w = 2\pi f$ to determine the required value of* **f** *(approximately 59.7 Hz).*

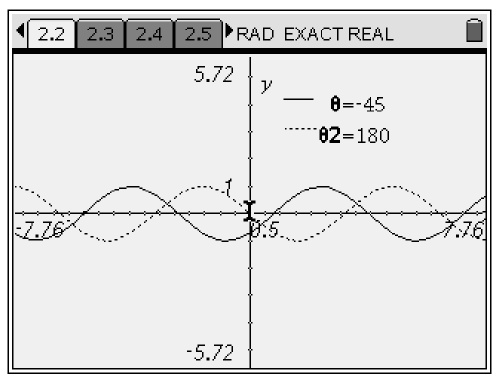
Q6. Predict the value of **w** required to produce a sine curve equivalent to one with a frequency of 45 Hz.

A. *Students should again use the relationship $w = 2\pi f$ to determine the required value of* **w** *(approximately 283).*

Q7. Use the graphs on pages 1.6 and 1.10 to test your predictions in questions 5 and 6. Were you correct? If not, explain any errors in your reasoning.

A. *Students should enter their predicted values of* **w** *and* **f** *into the appropriate simulations and observe how closely the two curves on page 1.10 match. Encourage students to discuss their results.*

Problem 2 – Phase shifts



Step 1: Students should read the information on page 2.1 and then move on to page 2.2, which shows waveforms for two different alternating currents. Each of the waveforms is phase shifted relative to the origin. The variable **θ** represents the phase shift of the solid curve, and the variable **θ2** represents the phase shift of the dotted curve. Students should vary the phase shifts of the two curves and observe the results. Then, they should answer questions 8–10.

Q8. Describe the relative positions of the two waves when the phase factors (**θ** and **θ2**) differ by 180º.

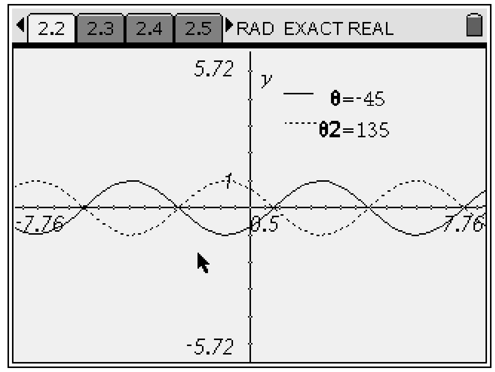
A. *When the phase factors are 180º apart, the crest of one curve is exactly aligned with the trough of the other curve.*

Q9. What positive phase shift is equivalent to a phase shift of –45°?

A. *315º*

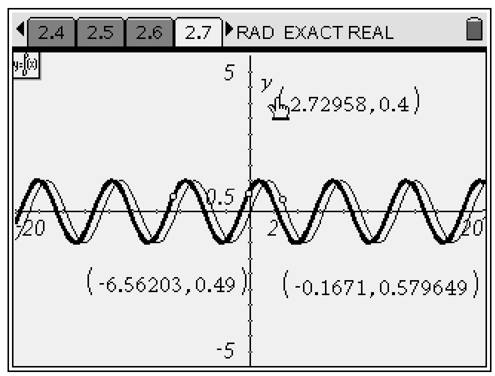
Q10. What difference between **θ** and **θ2** is required to produce two curves that would sum to zero?

A. *180º; if students struggle with this concept, remind them of the rules for adding waves, and encourage them to experiment with various combinations of* ***θ*** *and* ***θ2****.*

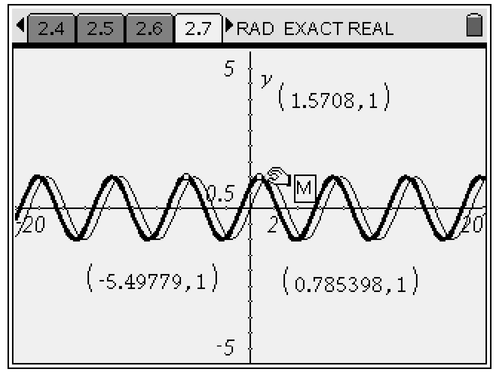


Step 2: Next, students should read pages 2.5 and 2.6. Then, they should move to page 2.7, which shows two waveforms that are phase shifted relative to each other. Students will use this simulation to calculate the phase shift of the two curves relative to each other.

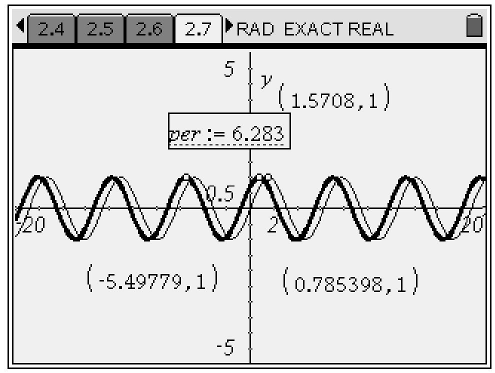
Step 3: To calculate the relative phase shift, students should first label the coordinates of the three points on page 2.7. They will drag these points along the curves and record their coordinates to calculate the phase shift. To label the coordinates of a point, students should select the **Coordinates and Equations** tool (**Menu > Actions > Coordinates and Equations**) and then click once on the point they wish to label. They can then drag the label to wherever they would like on the screen. Encourage them to keep the labels in logical places so that they know which label goes with which point. After they have labeled the coordinates of all three points, students should press (esc) to exit the **Coordinates and Equations** tool.



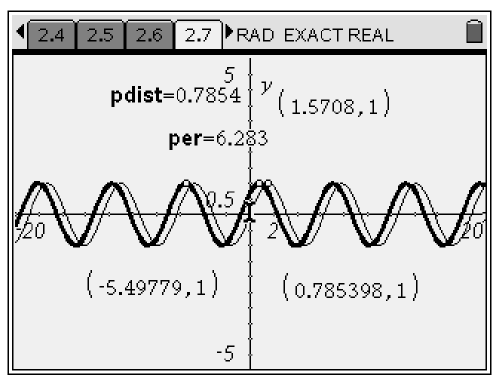
Step 4: After labeling the three points, students should drag one of the points on the thick curve to a peak of that curve. They should then drag the other point on the thick curve to an adjacent peak on the curve. Finally, they should move the point on the thin curve to the peak closest to the second labeled peak on the thick curve, as shown to the right.



Step 5: Next, students should calculate and record the horizontal (*x*) distance between the two points on the thick curve. They can calculate the distance by subtracting the *x*-values of the two points. This distance is the period of the waves. (Students should use the absolute value of the difference between the *x*-values.) Students should use the **Text** tool (**Menu > Actions > Text**) to create a text box somewhere on the screen, type the value of the period in the text box, and press [enter]. After exiting the **Text** tool, they should click once on the value they just entered and press [sto→ var]. They should choose **Store Var,** type the variable name **per,** and press [enter].



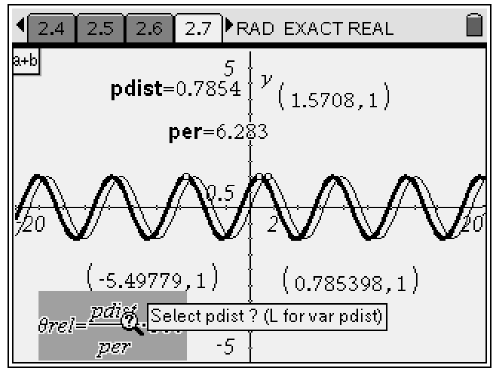
Step 6: Next, students should calculate and record the horizontal (*x*) distance between a peak on the thick curve and the nearest peak on the thin curve. They can calculate this distance by subtracting the *x*-value of the point on the thin curve from the *x*-value of the rightmost point on the thick curve. Students should again use the **Text** tool and the **Store Var** command to record this value in the variable **pdist**.



Step 7: Next, students should use the equation below to calculate the phase shift between the two curves:

$$\boldsymbol{\theta rel} = \frac{\textbf{pdist}}{\textbf{per}} \cdot 360$$

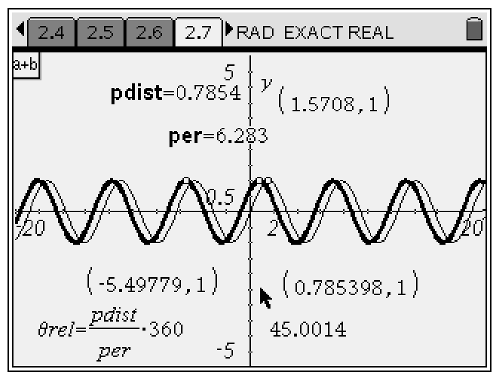
They should type this equation into a text box somewhere on the screen. They should then use the **Calculate** tool (**Menu > Actions > Calculate**) to determine the value of the equation. To use the **Calculate** tool, students should click once on the equation they entered. They will be prompted to select the values for **pdist** and **per**. They should click on each variable to select it. The value of **θrel** will then be displayed. Then, students should answer questions 11 and 12.



Q11. Write the equation describing the relative phase shift of two sine curves.

A. *The general equation is shown below:*

$$\textit{relative phase shift} = \frac{\textit{distance between peaks}}{\textit{period of one wave}} \cdot 360$$



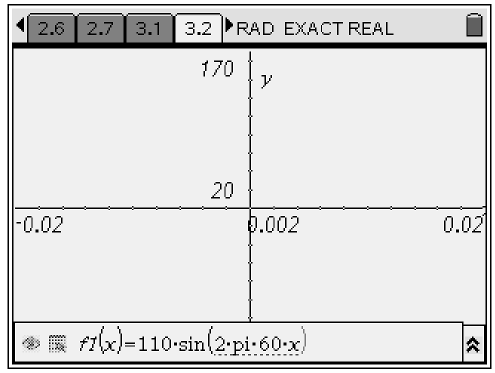
Q12. What is the relative phase shift of the two curves on page 2.7?

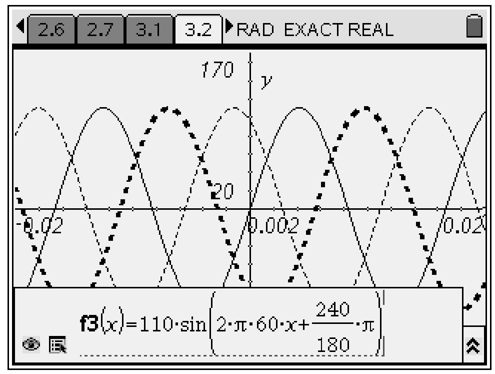
A. *approximately 45º; if you wish, you may have students use the simulation on page 2.2 to confirm their results.*

Problem 3 – Construction of a three-phase waveform

Step 1: In this problem, students use what they have learned about AC waveforms to construct phase-shifted waveforms with specific characteristics. Students should answer questions 13–15. Encourage student discussion and interaction.

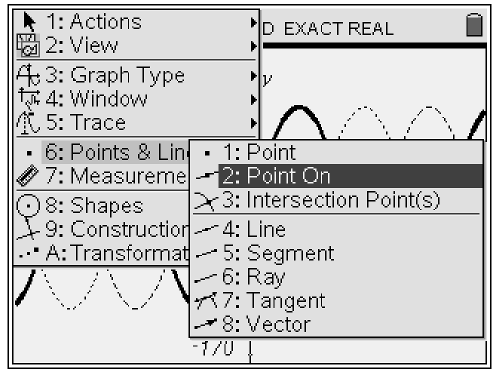
Q13. A three-phase AC generator produces three signals that are phase shifted by 120° relative to one another. The frequency of the generator is 60 Hz, and the peak voltage is 110 V. Graph the signals from this generator on the next page.





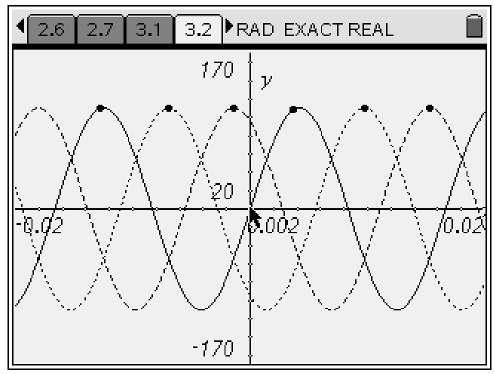
A. *Students should enter three equations in the function line on the* Graphs & Geometry *application on page 3.2. Each equation should represent one of the AC signals. They should use the equation relating peak voltage, frequency, and phase shift that they derived in problem 2. If necessary, remind students that the phase shift in this equation is the phase shift of the individual waveform relative to the origin. The first equation they enter should have a phase shift of 0º, the second should have a phase shift of 120º, and the third should have a phase shift of 240º. Remind students that they must convert these angles to radians when they enter them into the equation for* v(t)*. Students will also need to adjust the window settings of the graph in order to see the waveforms clearly. An* x*-range of –0.02 to 0.02 and a* y*-range of –170 to 170 will produce a reasonably scaled graph.*

Q14. Use the equation you derived in question 11 to verify that the three curves you graphed are phase shifted by 120° relative to one another. Show your work.



A. *Students should use the equation shown below for each pair of waveforms to verify the phase shift:*

$$\text{relative phase shift} = \frac{\text{distance between peaks}}{\text{period of one wave}} \cdot 360$$



*Students should place two points on each waveform using the **Point On** tool and then measure the* x*-distance between successive peaks, as they did in problem 2, steps 4 and 5. When students place the points on the curves, the coordinates of the points will automatically be displayed. Students may find it easier to hide or delete these coordinates and use the **Coordinates and Equations** tool to label only the coordinates of the points they are working with at each moment. They should obtain values close to 120º for the relative phase shift between each pair of waveforms.*

Q15. For a three-phase AC generator, what is the relationship between the separations between peak maxima and the period of the generator?

A. *The separations between the peak maxima are equally spaced and are equal to one-third the period of the generator.*

AC Circuits – ID: 9525

(Student)TI-Nspire File: *PhyAct26_ACcircuits_EN.tns*

AC CIRCUITS

Physics

Applications of Electricity & Magnetism

In an AC generator, a coil of wire rotates within a magnetic field. This generates an electric field within the wire, which produces current. Because the coil of wire is rotating within the magnetic field, the voltage produced by the generator alternates between positive and negative polarity.

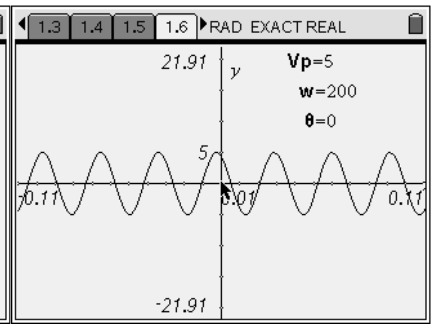
The alternating polarity produces current that periodically reverses direction–hence the name "alternating current." The speed at which the wire coil turns determines the frequency of the current.

Alternating current can be modeled by a sine curve like the one below:

$v(t) = Vp\cdot\sin(wt+\theta)$

In this equation, $v(t)$ is the voltage, Vp is the peak voltage, w is the angular velocity of the generator, t is time, and θ is the phase shift. In this equation, angles are in radians and voltages are in volts.

The next page shows a sine wave. Vary the values of **Vp**, **w**, and **θ**, and observe the effects on the shape of the curve. (**θ** is expressed in degrees in this simulation.)



1. What characteristic of the curve does the variable **Vp** control?

2. What characteristic does the variable **w** control?

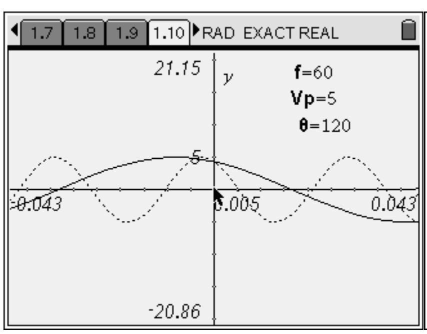
3. What characteristic does the variable **θ** control?

The angular velocity of the generator is related to frequency (f) by the equation $w = 2\pi f$. Therefore, we can use substitution to find the equation below, which relates voltage to frequency:

$v(t) = Vp\cdot\sin(2\pi f + \theta)$

In this equation, frequency is in hertz.

On the next page, you can vary the frequency (**f**) of the current. The dotted curve shows the sine curve you manipulated on page 1.6. Vary the value of **f** and observe the effects on the curve. (**θ** is expressed in degrees in this simulation.)



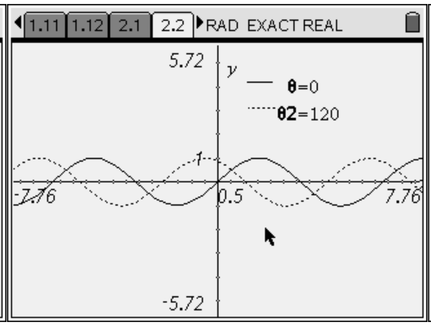
4. What characteristic of the curve does the variable **f** control?

5. Predict the approximate value of **f** required to produce a sine curve equivalent to the one produced by a generator with an angular velocity of 375.

6. Predict the value of **w** required to produce a sine curve equivalent to one with a frequency of 45 Hz.

7. Use the graphs on pages 1.6 and 1.10 to test your predictions in questions 5 and 6. Were you correct? If not, explain any errors in your reasoning.

The electricity that comes into a residential building typically consists of two signals that are phase shifted relative to each other. The next page shows two waveforms that are phase shifted by different amounts. Vary the values of **θ** and **θ2**, and observe the effects on the two waveforms. (**θ** and **θ2** are expressed in degrees.)



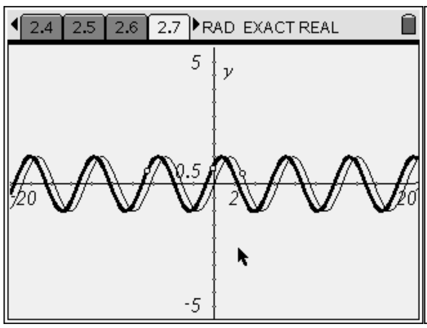
8. Describe the relative positions of the two waves when the phase factors (**θ** and **θ2**) differ by 180°.

9. What positive phase shift is equivalent to a phase shift of −45°?

10. What difference between **θ** and **θ2** is required to produce two curves that would sum to zero?

The phase shifts you explored on page 2.2 were phase shifts of the curves relative to the *y*-axis. However, it is often more useful to calculate the phase shift of one curve relative to another.

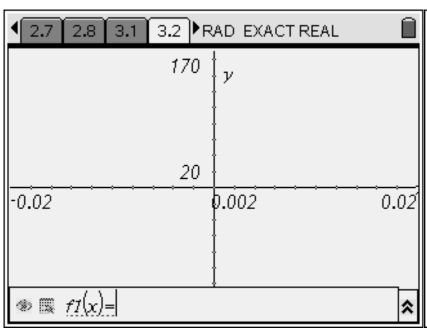
If you know the values of **θ** and **θ2**, you can calculate the relative phase shift through subtraction. However, if you do not know these values, you can calculate the relative phase shift from information on the period and the distance between the peaks. Use the simulation on the next page to calculate the phase shift of two curves.



11. Write the equation describing the relative phase shift of two sine curves.

12. What is the relative phase shift of the two curves on page 2.7?

13. A three-phase AC generator produces three signals that are phase shifted by 120° relative to one another. The frequency of the generator is 60 Hz, and the peak voltage is 110 V. Graph the signals from this generator on the next page.



14. Use the equation you derived in question 11 to verify that the three curves you graphed are phase shifted by 120° relative to one another. Show your work.

15. For a three-phase AC generator, what is the relationship between the separations between peak maxima and the period of the generator?