NUMB3RS Activity: No Fly Zone
Episode: “Take Out”

Topic: Geometry, Probability
Grade Level: 9 - 10
Objective: Explore geometric probability
Time: 20 - 30 minutes

Introduction

In the episode “Take Out,” a series of restaurant robberies prompts the FBI to ask Charlie to help predict the next target. When his analysis is not successful, Charlie explains that his assumptions, not his math, were wrong. “Think of it like looking for a spider or its prey by studying a web. We can use tension values of the silk web, vibrations—mixed with weather conditions, surrounding terrain, other variables—to reliably predict both where the spider's location is and where the next insect will get tangled.”

Even a very well constructed web may not catch all insects. There exist regions where it will be safe for an insect to fly through. The purpose of this activity is to explore these “safe regions” by examining the structure of the web using geometric probability.

Discuss with Students

In the beginning of this activity, students calculate geometric probabilities by determining the areas of circles and rectangles. You may wish to review the area formulas with your students.

In the model of a fly traveling through a web, some assumptions were made; for example, the danger zone will be half of the width of the fly. Another point is the boundary of the cinder block and box for questions 3, 4, and 5. The boundary is there but was excluded from the picture so as to not be confused with the strands of web. Students should not consider the boundary of the block and box when calculating the probabilities. You may wish to discuss these assumptions with the students if questions arise.

In question 1, ensure that the students are using 0.5 cm for the amount that the radius decreases. Even though π is used in the calculation, you can ask the students if it has any effect on the probability. Many students will believe it does and will be surprised to learn that it does not. This also illustrates the usefulness of expressing your answers in exact terms.

In question 2, a common mistake is to subtract 0.5 cm from each of the dimensions. Point out to the students that they must subtract 0.5 cm from each side length of the rectangle – this means to subtract 1 cm from each dimension. You may find it useful to practice this by working on examples with situations such as sidewalks around pools, flowerbeds around plots of ground, and borders around pictures.

In question 4, there will be an optimal arrangement of the strands. If the student has all seven of the strands parallel either horizontal or vertical and spaced two units apart, this would minimize the safe area and result in a probability of 44%.

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The answer to question 5 is the complement of the answer to question 4. Students can count the shaded squares and divide that by the total; then later, as a class, you can verify that the result is equal to 1 minus their answer from question 4.

Student Page Answers:

1. *$12.25\pi/16\pi = 49/64 = 76.6\%$* **2.** *$6/12 = 50\%$* **3.** *$120/256 = 46.9\%$* **4.** *Answers will vary, but if all seven of the strands are parallel either horizontally or vertically and spaced at least two units apart, this would minimize the safe area and result in a probability of 44%.* **5.** *Answers will vary, however it will be the complement of the answer to question 4. The minimum probability is 56%.*

Extension Page Answer:

1. $\frac{39}{36} - \frac{\sqrt{3}}{3} \approx 50.5\%$ **2.** $P(safe) = 1 - \frac{2f\sqrt{3}}{s} + \frac{3f^2}{s^2}$

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Name: ________________________________ Date: _______________

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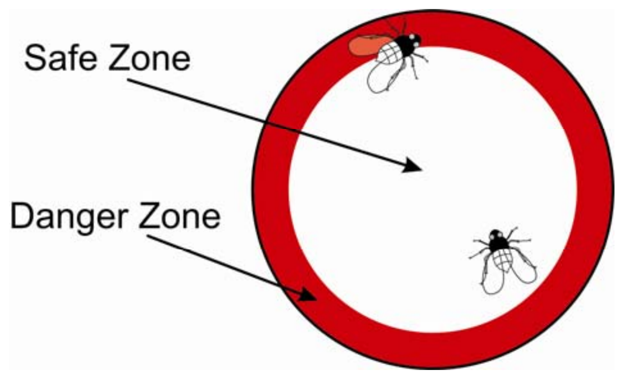
Even a very well constructed web may not catch all insects. There exist regions where it will be safe for an insect to fly through. The purpose of this activity is to explore these “safe regions” by examining the structure of the web using geometric probability.

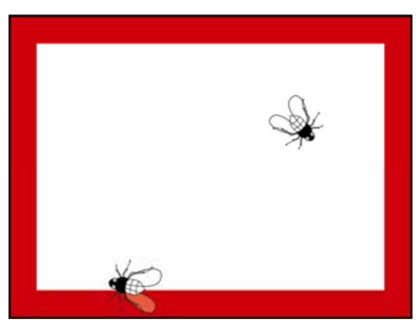
Begin with a spider web in the shape of a circle. Depending on the size of the insect, there will be an area on either side of the web that will still be able to trap the fly. We can mark this area just inside the web as the danger zone and the middle region as the safe zone. In the picture at the right, we are only concerned with the inside of the circular web and have marked the danger zone accordingly.



The probability that a fly, flying into the web at random, will be able to pass safely through the web is the area of the safe region divided by the total area.

1. Given a circular web with a radius of 4 centimeters and a nearly spherical fly with a width of 1 centimeter, what is the probability that the fly will pass safely through the web? Remember that the distance from the web to the safe zone will be half the width of the fly.

Looking at a rectangular web, how does this shape change the probabilities? As with the circle, the probability that the fly will travel safely through the web can be found using geometric probability.



2. Given a rectangular web of 4 centimeters by 3 centimeters and a fly with a width of 1 centimeter, what is the probability that the fly will travel safely through the web?

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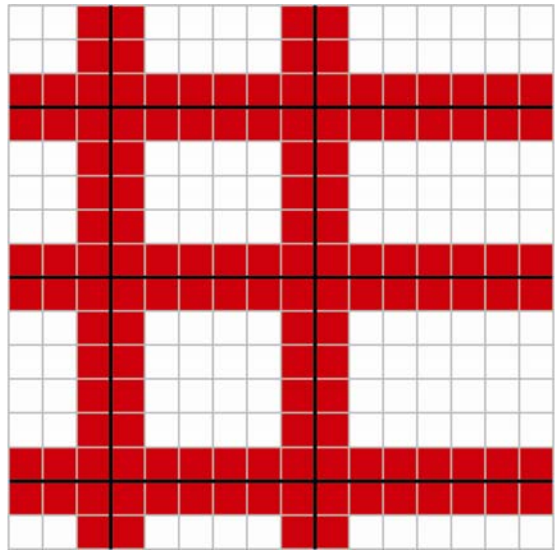
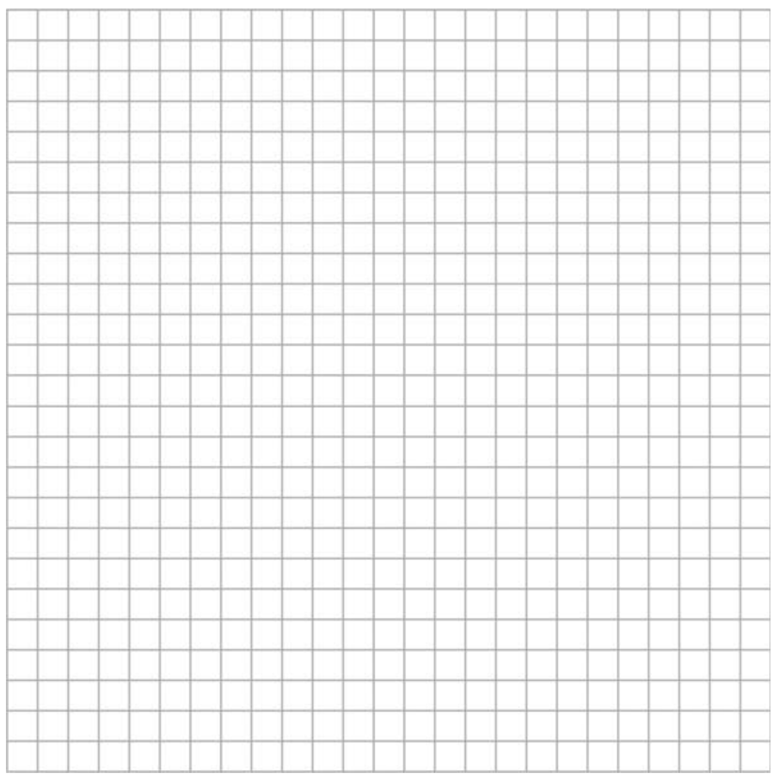
A spider web, however, is not composed of a single region. To examine the overall probability of a fly passing safely through the web, you must find the sum of the individual safe areas and divide by the total area.



3. A cinder block has a square hole with five strands of web stretched across the opening represented by the picture at the right. What is the probability that the fly will pass safely through the web?

Now **you** get to be the spider.



4. A cardboard box has an opening represented by the area at the right. As the spider, choose 7 strands to place either horizontally or vertically across the opening to compose your web. On each side of the strand, shade in the danger zone represented by one square width, then calculate the probability of a fly passing through your web.

5. Now calculate the probability that your web catches the fly.

Compare your web with the web of your neighbor to see whose web has a greater likelihood of catching a fly.

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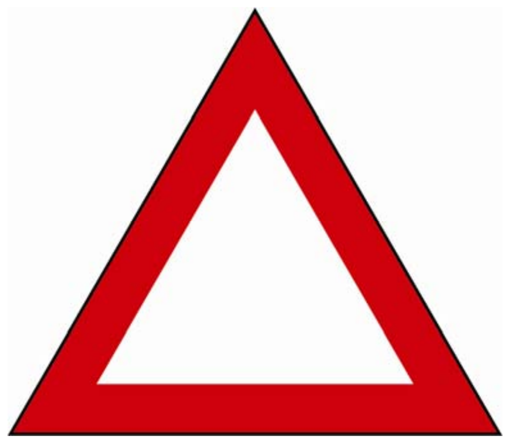
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

In an actual web you discover many beautiful and interesting shapes besides circles and rectangles we studied in the activity. One shape that is common is a triangle. In this extension, we will examine a web in the shape of an equilateral triangle.

1. Given a web in the shape of an equilateral triangle with a side of 6 centimeters and a fly with a width of 1 centimeter, what is the probability that the fly will fly safely through the web? Remember that distance from the web to the safe zone will be half the width of the fly. (Hint: Breaking the shapes up gives you rectangles, kites, and/or triangles can help you solve this problem. Keep in mind the 30-60-90 triangle relationships and/or trigonometric ratios.)

2. Generalize a formula for calculating the probability of the fly passing through the web. Use *s* for the side of the original triangle and *f* for the width of the fly.



Additional Resources

- For more information about geometric probability, go to: **http://home.wlu.edu/~mcraea/GeometricProbabilityFolder/Introduction/Problem0/problem0.html**

- For an interactive applet that uses throwing darts at a target to illustrate geometric probability, go to: **http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=59**

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