How Many?

ID: 8982

Time required
30 minutes

Activity Overview

In this activity, students will explore Bernoulli probabilities. They will use them to calculate the probabilities of various single and cumulative events. They will also explore the Bernoulli probability distribution.

Topic: Discrete Random Variables

- *Derive the formula for the probability of exactly r success in n Bernoulli trials.*
- *Use the binomial probability function to calculate the probability of k successes in n trials.*
- *Use the binomial cumulative distribution function to calculate the probability that at least k successes will occur in n trials.*

Teacher Preparation and Notes

- *If someone is shooting free-throws in a basketball, what is the probability of making three out of ten? If an unfair coin is flipped 30 times, what is the probability of getting tails seven times?* ***Bernoulli probabilities*** *are useful for finding these probabilities and others that are binomial, independent events.*
- *Pages 1 and 2 of this teacher guide walk students through the process of deriving the Bernoulli Probability formula (Note: it is not part of the Student pages.)*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TINspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8982" in the keyword search box.***

Associated Materials

- *HowMany_Student.doc*
- *HowMany.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

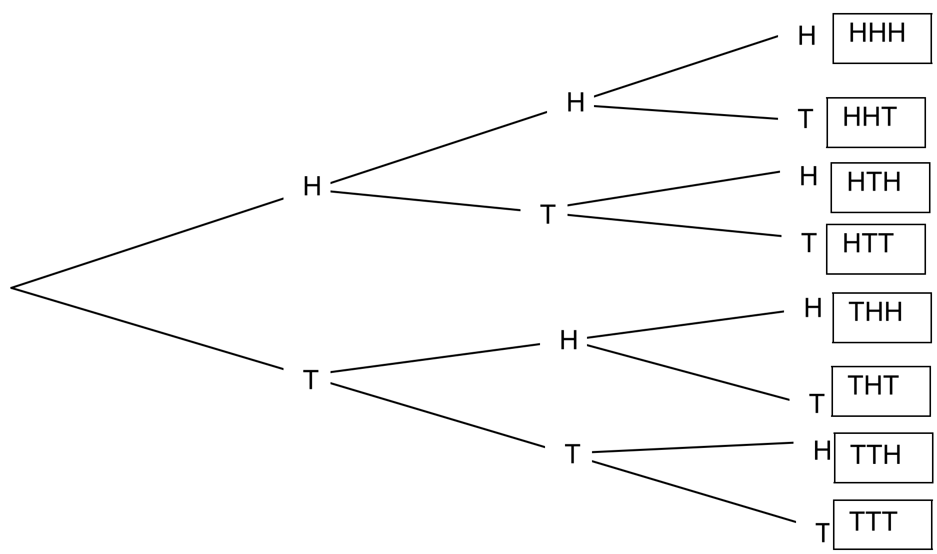
- *Sums of Powers (TI-89 Titanium) — 3484*
- *Plinko! (TI-73 Explorer) —6701*
- *How Likely is it? Exploring Probabilities (TI-Nspire technology) — 9236*
- *Binomial Probabilities (TI-Nspire technology) — 9836*

Before beginning the activity, use the example below to walk the students through the process of deriving the formula for the Bernoulli probability. The process is based on the tree diagram—something that the students might have already learned from other probability experiments.

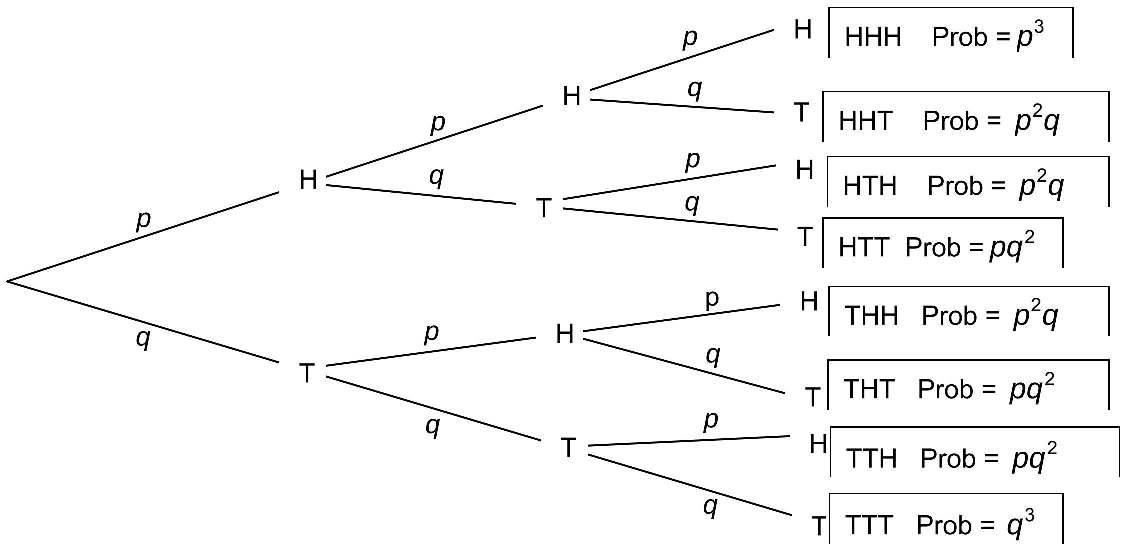
Remind the students that the probabilities on the tree are multiplied as the path is traced out. Since the probabilities are not known (the coin could be biased), variables are assigned instead.

Before finishing with the Bernoulli probability formula, emphasize to the students that it can only be used with binomial events that are independent from each other.

Suppose we are flipping coins and wish to determine the probability of getting two tails after three flips. This is determined by finding all the possibilities and counting. To find all the possibilities, a tree diagram is employed.



By counting, we determine $P(2\text{ tails}) = \frac{3}{8}$. While this seems feasible to do, what would happen if we needed to do this for 30 coin flips or 120? What if we wanted to know how many times heads happened three, four, or five times in those trials? Better still, what if the coin was bias towards heads? To help us accomplish this, a formula is needed. Let's look at the coin flipping again, but attach probabilities *p* and *q* (heads and tails) to each edge.



Notice the following probabilities:

0 heads	1 head	2 heads	3 heads	Total probability
q^3	$3pq^2$	$3p^2q$	p^3	$q^3+3pq^2+3p^2q+p^3=(q+p)^3$

For these probabilities, there is a pattern:

- The power of *p* is always the number of heads
- The power of *q* is always the number of tails.

The number of ways of getting r successes in *n* trials is the number of ways of choosing the *r* branches which have a *p* on them. ${}_nC_r$ calculates this value.

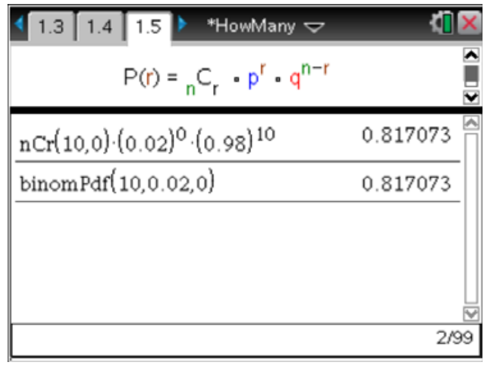
To calculate the probability of exactly *r* successes in *n* Bernoulli trials is ${}_nC_r \cdot p^r q^{n-r}$.

Note, that this only works in trials where there is a binomial distribution and the events are independent of each other (this constitutes a Bernoulli trial.)

In our example, *p* was heads and *q* was tails. This is more generalized to *p* being successes and *q* being failures.

Problem 1 – Exact Probabilities

In this problem students are to calculate the probability of no defective chips in a packet of 10 with an average of 2% defective. First, they are to use the formula for Bernoulli probability function and then use the **Binomial Pdf** command.



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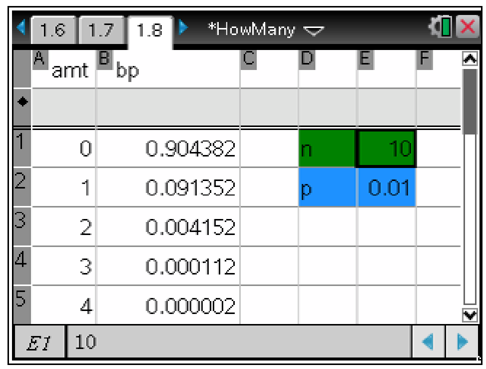
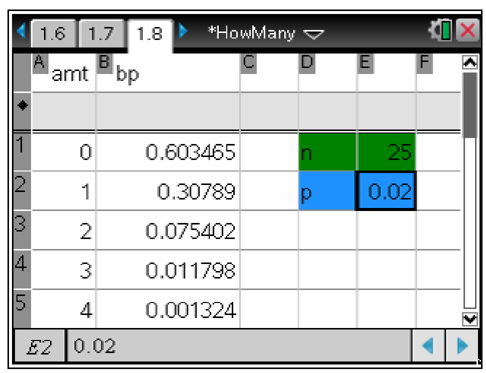
See Note 1 at the end of this lesson.

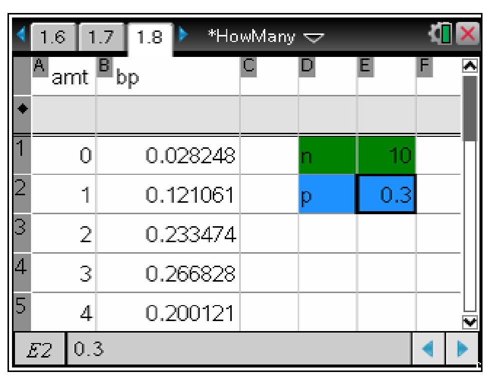
Then students need to use the spreadsheet on page 1.8 to calculate the probability of 1, 2, 3, 4, and 5 defective chips in a packet of 10 with an average of 2% defective. When they change the value of *p* in cell E2, Column B will automatically update.



Students will also use the spreadsheet to calculate the probability of 0, 1, 2, 3, 4, and 5 defective chips in a packet of 25 with an average of 2% defective and a packet of 10 with an average of 30% defective.

They should see that the probabilities of 30% defective are more evenly distributed than the probabilities of 2% defective.





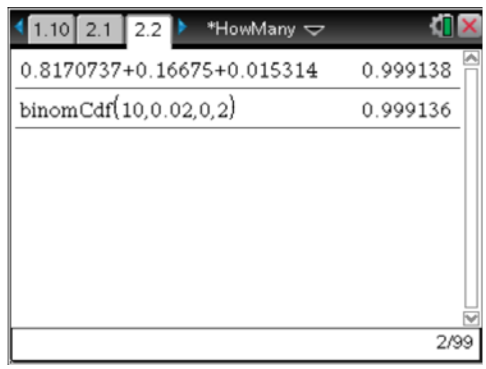
Problem 2 – Cumulative Probabilities

Use the following example to explain to students how to use the complement of an event to find the cumulative probability.

For any sample space, the sum of the probabilities of the outcomes is 1. So, for a given value of *m*, $P(0) + P(1) + P(2) + ... = 1$.

From this, you can use the **complement** of an event to help you find a probability. For example, given *m*, to find the probability that an event will happen more than 2 times ($k > 2$), subtract the probability that the event will happen 2 or fewer times ($k \le 2$) from 1.

$$P(\text{2 or fewer times}) + P(\text{more than 2 times}) = 1$$
$$P(\text{more than 2 times}) = 1 - P(\text{2 or fewer times})$$
$$P(\text{more than 2 times}) = 1 - P(\text{0 times}) - P(\text{1 time}) - P(\text{2 times})$$
$$P(\text{more than 2 times}) = 1 - \left(P(\text{0 times}) + \text{P}(\text{1time}) + P(\text{2 times})\right)$$

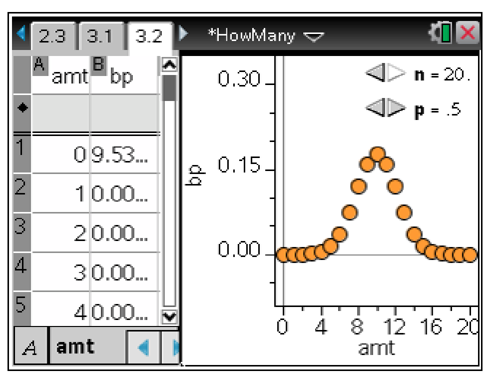


Students will use the numbers from the first example, where a packet of 10 memory chips had an average of 2% defective, to calculate the probability that there are less than 3 defective memory chips.

First students will add the probabilities of 0, 1, and 2 defects found in Problem 1 to find the probability and then check their answer using the **Binomial Cdf** command from the **Probability** menu.

Explain to students that the command provides a more accurate answer since the probabilities in the spreadsheet were rounded.

Problem 3 – Bernoulli Probability Distributions



Students are to change the value of *p*, using the slider, from 0.1 to 1 in increments of 0.1. They are to observe the changes of the scatter plot as they do so. The value of *n* will remain at 20.

They should see that low *p*-values skew the graph right, while high *p*-values skew the graph left. When the *p*-value is equal to 0.5, the distribution looks like a normal graph because a success and failure are equally likely.

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See Note 2 at the end of this lesson.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter*

On page 1.5, it may be a good idea to demonstrate to students how to select the **BinomPdf(** Command from the Menu as well as how to enter the values into the wizard.

Note 2
Problem 3, *Live Presenter*

Use this opportunity on page 3.2 to facilitate the discussion about what happens to the distribution when *p*-value is closer to 0 and when the *p*-value is closer to 1.

