LESSON

0.5

Out of Chaos

If you looked at the results of 100 rolls of a die, would you expect to find a pattern in the numbers? You might expect each number to appear about one-sixth of the time. But you probably wouldn't expect to see a pattern in when, for example, a 5 appears. The 5 appears **randomly,** without order. You could not create a method to predict exactly when or how often a 5 appears. As you explore seemingly random patterns, you'll review some measurement and fraction ideas.

Nothing in nature is random. . . . A thing appears random only through the incompleteness of our knowledge.

BARUCH SPINOZA

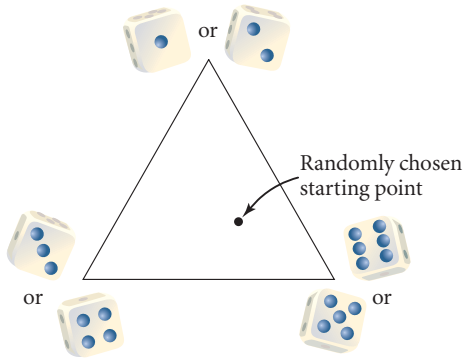
Investigation
A Chaotic Pattern?

You will need

- a die
- a centimeter ruler
- a blank transparency and marker
- the worksheet A Chaotic Pattern?

What happens if you use a random process recursively to determine where you draw a point? Would you expect to see a pattern?

Work with a partner. One partner rolls the die. The other measures distance and marks points.



Step 1	Mark any point inside the triangle as your starting point.
Step 2	Roll the die.
Step 3	In centimeters, measure the distance from your starting point to the corner, or *vertex*, labeled with the number on the die. Take half of the distance, and place a small dot at this midpoint. This is your new starting point.
Step 4	Repeat Steps 2 and 3 until you've rolled the die 20 times. Then switch roles with your partner and repeat the process another 20 times.
Step 5	How is this process recursive?
Step 6	Describe the arrangement of dots on your paper.
Step 7	What would have happened if you had numbered the vertices of the triangle 1 and 3, 2 and 5, and 4 and 6?

For best results, measure as accurately as you can.

Step 8 Place a transparency over your worksheet. Use a transparency marker and mark the vertices of the triangle. Carefully trace your dots onto the transparency.

Step 9 When you finish, place your transparency on an overhead projector. Align the vertices of your triangle with the vertices of your classmates' triangles. This allows you to see the results of many rolls of a die. Describe what happens when you combine everyone's points. How is this like the result in other recursion processes? Is the result as random as you expected? Explain.

A *random* process can produce ordered-looking results while an orderly process can produce random-looking results. Mathematicians use the term *chaotic* to describe systematic, non-random processes that produce results that look random. Chaos helps scientists understand the turbulent flow of water, the mixing of chemicals, and the spread of an oil spill. They often use computers to do these calculations. Your calculator can repeat steps quickly, so you can use the calculator to plot thousands of points.

Step 10 Enter the Chaos program into your calculator. [▶ See **Calculator Note 0E** for the program. To learn how to link calculators, see **Calculator Note 0F.** To learn about how to enter a program, see **Calculator Note 0G.**◀]

The program randomly "chooses" one vertex of the triangle as a starting point. It "rolls" an imaginary die and plots a new point halfway to the vertex it chose. The program rolls the die 999 more times. It does this a lot faster than you can.

Step 11 Run the program. Select an equilateral (equal-sided) triangle as your shape. When it "asks" for the fraction of the distance to move, enter $\frac{1}{2}$ or 0.5. It will take a while to plot all 1000 points, so be patient.

Step 12 What do you see on your calculator screen, and how does it compare to your class's combined transparency image?

Most people are surprised that after plotting many points, a familiar figure appears. When an orderly result appears out of a random process like this one, the figure is a *strange attractor*. No matter where you start, the points "fall" toward this shape. Many fractal designs, like the Sierpiński triangles on your calculator screen, are also strange attractors. Accurate measurements are essential to seeing a strange attractor form. In the next example, practice your measurement skills with a centimeter ruler.

Science CONNECTION

The growth and movement of an oil spill may appear random, but scientists can use chaos theory to predict its boundaries. This can aid restraint and cleanup. Learn more about the application of chaos theory with the Internet links at www.keymath.com/DA .