Teacher Notes

Activity 2

Is There a Limit to Which Side You Can Take?

Objectives

- Given a function, state and explain the limit at a particular value
- Given a graph, state and explain the limit at a particular value

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 40 minutes

Abstract

This activity will examine one-sided limits.

Management Tips and Hints

Prerequisites

Students should:

- have knowledge of piecewise functions.
- be able to produce piecewise functions on the graphing handheld.
- be able to manipulate graphs and tables of values manually and use the [ZOOM] features of the graphing handheld.
- have a basic understanding of function language and rational, exponential, and trigonometric functions.

Student Engagement

Working in pairs or small groups is recommended for maximum student engagement.

Evidence of Learning

- Given a function, students will be able to state and explain the limit at a particular value using graphical and numerical analysis.

Common Student Errors/Misconceptions

- Students may not produce piecewise functions correctly.
- Students may misunderstand the behavior of a piecewise function.
- Students may incorrectly interpret the notation $\lim_{x \to 3^+}$ to mean moving toward the value (here 3) as it gets larger.
- Students may incorrectly interpret the notation $\lim_{x \to 4^-}$ to mean moving toward the value (here 4) as it gets smaller.
- Students may misinterpret the value of the limit of a function with the value of a function at a point.
- Care should be taken to make sure that students understand the difference between an actual graphing handheld error and the fact that the function is not defined at a particular point.

Activity Solutions

1. n/a

2. n/a

3. $f(x) = \{^-2, \text{ERROR}, 4\}$

4.

X	Y1	Y2
1.7	-1.3	ERROR
1.8	-1.2	ERROR
1.9	-1.1	ERROR
2	ERROR	ERROR
2.1	ERROR	3.1
2.2	ERROR	3.2
2.3	ERROR	3.3

X=2

5.

X	Y1	Y2
1.97	-1.03	ERROR
1.98	-1.02	ERROR
1.99	-1.01	ERROR
2	ERROR	ERROR
2.01	ERROR	3.01
2.02	ERROR	3.02
2.03	ERROR	3.03

X=2

6. There is no limit as x approaches 2. As the input value gets nearer and nearer to 2, the function value never gets near any one value.

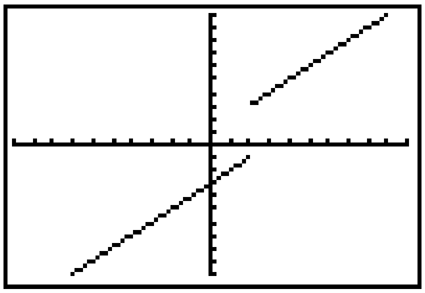
7. $\lim_{x \to 2^+} f(x) = 3$

As the input moves toward 2 from the right side, the function continues to get closer to a value of 3.

8. $\lim_{x \to 2^-} f(x)$

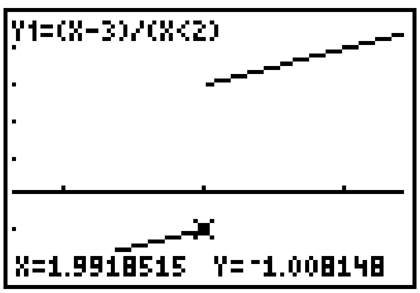
means finding the limit of the function as the input value is approached through values that are less than 2 (left side). The "useful" input values will vary, but 1.97, 1.98, 1.99 would make sense because they were used in the previous table. The limit from that side is $^-1$.

9.

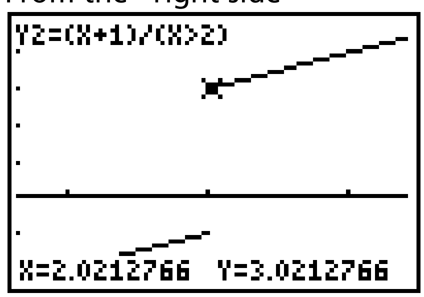


10. Possible graphs:

From the "left side"

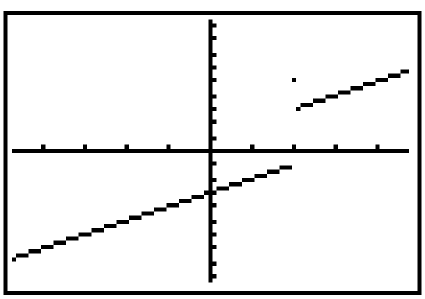


From the "right side"



Answers will vary. The goal is to have the student see the values approach the "one-sided" limits concluded in the numerical analysis.

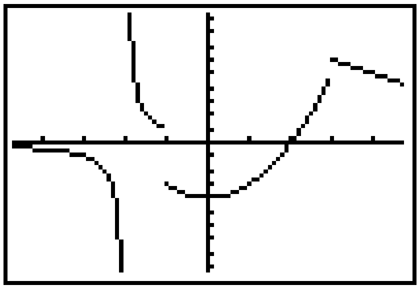
11.



12. The only difference is that the function $g(x)$ is defined at $x = 2$, and the function $f(x)$ is not.

13. $\lim_{x \to 2} g(x)$ = no limit $\quad \lim_{x \to 2^+} g(x) = 3$ and $\lim_{x \to 2^-} g(x) = {}^-1$.

Some students may say that at 2 there is a limit of 5 because the function is defined at that point. This is the opportunity to firm up the concept of limit as the behavior a function has as the input gets *very near* a particular value.

14. 

15.

a. No limit; there is a vertical asymptote at $^-2$.

b. $+\infty$; the function will continue in the positive direction.

c. No limit; there are two different one-sided limits.

d. 1; explanations will vary, but some students will trace, and some will use numerical conclusions.

e. No limit; two different one-sided limits.

f. 6; explanations will vary, but some students will trace, and some will use numerical conclusions.

16.

a. 6

b. $^-2$

c. $^-2$

d. 0

e. $^-2$

EXPLORATIONS™

Activity 2

Objectives

- Given a function, state and explain the limit at a particular value
- Given a graph, state and explain the limit at a particular value

Materials

- TI-84 Plus / TI-83 Plus

Is There a Limit to Which Side You Can Take?

Introduction

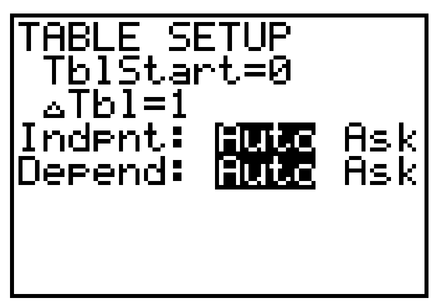
The limit describes the behavior of a function near a point. It represents how function outputs behave as inputs get very close to a value of interest. In some cases, the value of a limit depends on from which side the input value is approached. In this activity, you will investigate the idea of one-sided limits both graphically and numerically.

Exploration

1. Enter this piecewise function into your graphing handheld:

$$f(x) = \begin{cases} x - 3, x < 2 \\ x + 1, x > 2 \end{cases}$$

2. Set up your table as shown.



3. Using your knowledge of piecewise functions and the table output, record the function values for $x = \{1, 2, 3\}$.

4. Take a closer look at what happens as the input gets closer to 2. In other words, look at $\lim_{x \to 2} f(x)$.

Change the table to start at 1.7 and increment the table by 0.1. In the table at the right, record the values of $f(x)$ for the following inputs:

$x = \{1.7, 1.8, 1.9, 2, 2.1, 2.2, 2.3\}$

X	Y1	Y2
1.7		
1.8		
1.9		
2		
2.1		
2.2		
2.3		

X=2

5. Change the table to start at 1.97, and increment by 0.01. Record the values of $f(x)$ in the table shown.

X	Y1	Y2
1.97		
1.98		
1.99		
2		
2.01		
2.02		
2.03		

X=2

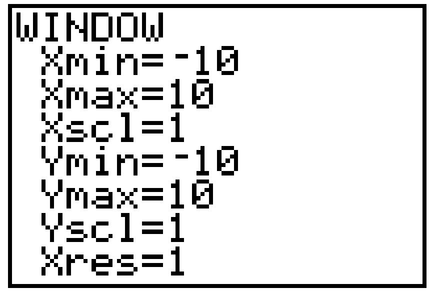
6. From your knowledge of limits, what is $\lim_{x \to 2} f(x)$? Explain.

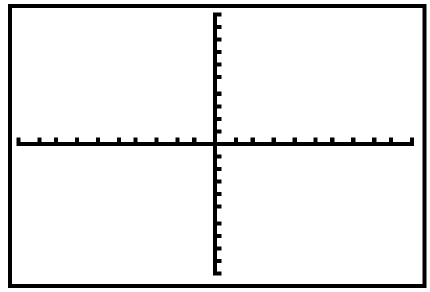
7. The notation $\lim_{x \to 2^+} f(x)$ means to investigate the limit of the function $f(x)$ as x approaches 2 through values that are greater than 2 (from the right). In this case, you would be looking at what happens as the input value gets *very near* 2 from values higher than 2. Using input values $x = \{2.3, 2.2, 2.1, 2\}$, what conclusion would you draw regarding $\lim_{x \to 2^+} f(x)$, and why?

8. Explain what the notation $\lim_{x \to 2^-} f(x)$ means.

Give some examples of useful input and a value for this limit, if any.

9. Confirm what you concluded by sketching the graph on the axes at the right, using the standard viewing window.





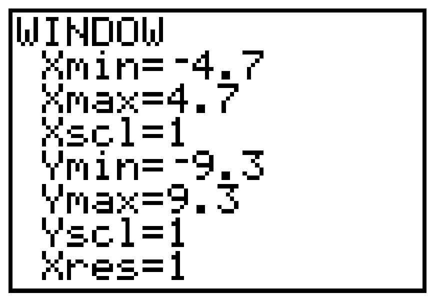
Imagine examining $\lim_{x \to 2^+} f(x)$ by "walking" from the right along the proper branch of the graph toward the value $x = 2$, and examining $\lim_{x \to 2^-} f(x)$ by walking from the left along the proper branch toward the input value $x = 2$.

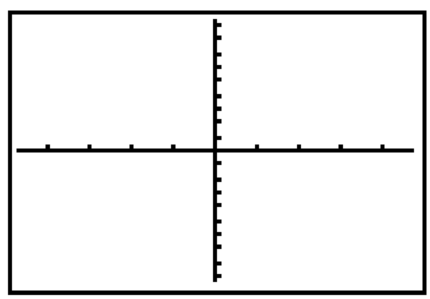
10. Use any zooming technique you prefer, keeping both branches visible and keeping $x = 2$ toward the center of the window. Trace along each branch. What do you see as the result?

11. Graph the function

$$g(x) = \begin{cases} x + 1, x > 2 \\ 5, x = 2 \\ x - 3, x < 2 \end{cases}$$

with the **WINDOW** settings shown. Sketch what you see.





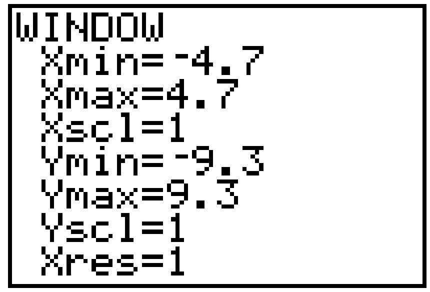
12. What difference, if any, is there in $g(x)$ from $f(x)$?

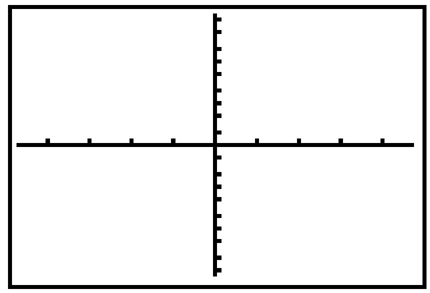
13. Find the following limits, and explain your results:
$\lim_{x \to 2} g(x)$, $\lim_{x \to 2^+} g(x)$, $\lim_{x \to 2^-} g(x)$.

14. Graph the function

$$h(x) = \begin{cases} \dfrac{1}{(x+2)}, & x < {}^{-}1 \\ x^2 + 2, & {}^{-}1 \le x < 3 \\ {}^{-}x + 9, & x \ge 3 \end{cases}$$

in the viewing window given, and sketch what you see.





15. Find each limit, and explain how you arrived at your conclusion.

a. $\lim_{x \to {}^{-}2} h(x)$

b. $\lim_{x \to {}^{-}2^{+}} h(x)$

c. $\lim_{x \to {}^{-}1} h(x)$

d. $\lim_{x \to {}^{-}1^{-}} h(x)$

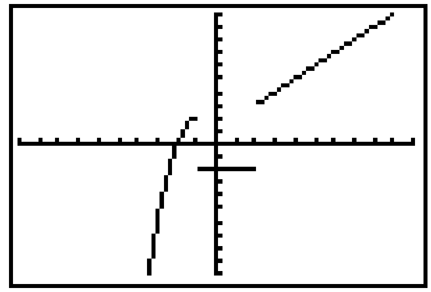
e. $\lim_{x \to 3} h(x)$

f. $\lim_{x \to 3^{+}} h(x)$

16. Estimate the limits from the given graph.

Note: *Each dot represents 1 unit.*

Be sure to write what each limit is asking for and then estimate its value.



a. $\lim_{x \to 5}$

b. $\lim_{x \to 0}$

c. $\lim_{x \to 2^-}$

d. $\lim_{x \to -2^+}$

e. $\lim_{x \to -1^+}$