Introduction to The Fundamental Theorem

ID: 9779

Time required
45 minutes

Activity Overview

The activity builds student comprehension of functions defined by a definite integral, where the independent variable is an upper limit of integration. Students are led to the brink of a discovery of the Fundamental Theorem of Calculus, that $\frac{dy}{dx}\int_a^x f(t)dt = f(x)$.

Topic: The Fundamental Theorem of Calculus

- *Graph a function and use Analyze Graph > Integral to estimate the area under the curve in a given interval.*
- *Use Integral (in the Calculus menu) to obtain the exact value of a definite integral.*

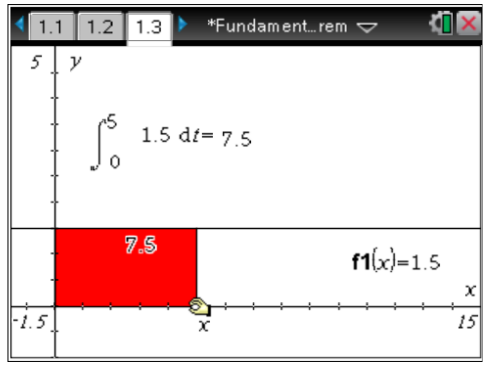
Teacher Preparation and Notes

- *This activity should follow coverage of the definition of a definite integral, and the relationship between the integral of a function and the area of a region bounded by the graph of that function and the x-axis. The first few problems deal with integrals where the upper limit of integration is greater than the lower limit, and where the integrand is a positive function. Subsequent problems deal with integrands that change sign. Before doing this activity, students should understand that if a < b and f(x) > 0, then:*
  - $\int_a^b f(x)dx > 0$
  - $\int_a^b -f(x)dx < 0$
  - $\int_b^a f(x)dx < 0$
  - $\int_b^a -f(x)dx > 0$
- *This activity could be used in Calculus after the definite integral is defined and its properties explored.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "9779" in the keyword search box.***

Associated Materials

- *FundamentalTheorem_Student.doc*
- *FundamentalTheorem.tns*

Problem 1 – Constant Integrand



Students explore the function $f(x) = \int_0^x 1.5dt$.

They can change the value of *x* by grabbing and dragging the point *x* (open circle) on the *x*-axis.

Students are to observe how the calculated value of the integral (the area under the curve) changes as the value of *x* changes.

They should notice that there is a constant rate of change in the graph of $f(x) = \int_0^x 1.5dt$. This rate of change is 1.5.

1. Look at the table to the right.

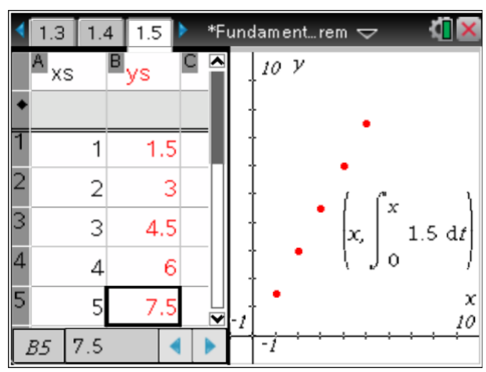
2. $\int_0^0 1.5dt = 0$; There is zero area under the graph of $y = 1.5$ from $x = 0$ to $x = 0$.

3. 1.5 units

4. The graph will be a line through the origin with slope 1.5.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

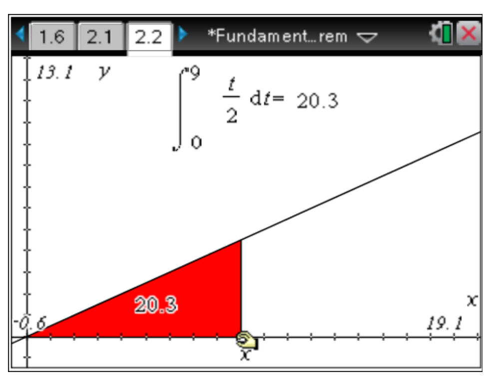


As students enter the outputs on page 1.5, a scatter plot of the ordered pairs will appear on the graph. They should observe that the graph of $\left(x, \int_0^x 1.5dt\right)$ is indeed a line with slope 1.5. Students are then asked to predict the shape of the graph of $\left(x, \int_0^x 0.5dt\right)$.

5. A line; yes (student answers may vary)

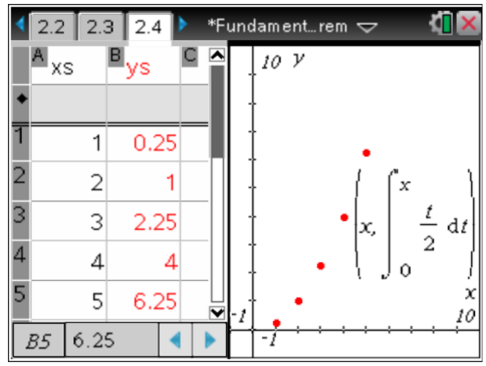
6. The same as before, except the slope would be 0.5 instead of 1.5.

Problem 2 – Non-Constant Integrand



In this problem, students investigate the behavior of $f(x) = \int_0^x \frac{t}{2}dt$. As in problem 1, they drag a point along the *x*-axis and observe the calculated value of $\int_0^x \frac{t}{2}dt$ change.

Students should note that this function changes at a non-constant rate and are asked to explain why this is so (from a geometric point of view).



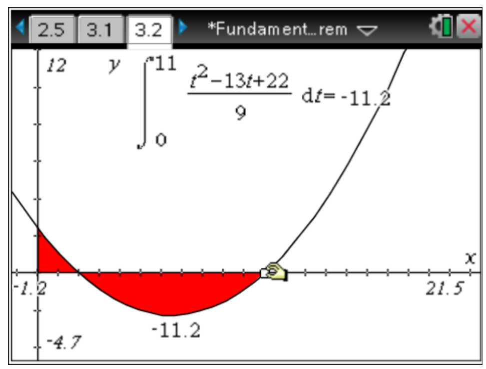
7. Look at the table to the right.

8. $\int_0^0 \frac{t}{2} dt = 0$; There is zero area under the graph of $y = \frac{x}{2}$ from $x = 0$ to $x = 0$.

9. As x gets larger, you add even larger additional "chunks" of area, since the integrand is positive and increasing.

10. The graph of $\left(x, \int_0^x \frac{t}{2} dt\right)$ increases at an increasing rate, and linear functions increase at a constant rate.

Problem 3 – An Integrand that Changes Sign



In Problem 3, students explore a function defined by an integral, where the integrand changes sign. They observe that the function $f(x) = \int_0^x \frac{t^2 - 13t + 22}{9} dt$ decreases where the integrand is negative, and that it reaches a minimum where the integrand changes sign from negative to positive. This connection is meant to remind students of the connections between the minimum of a function and a sign change in its derivative from negative to positive.

Similarly, the fact that $f(x) = \int_0^x \frac{t^2 - 13t + 22}{9} dt$ decreases where the integrand is negative is meant to suggest this same connection. Though the Fundamental Theorem is not explicitly stated in this activity, teachers could extend the activity to make this connection explicit.

TI-Nspire Navigator Opportunity: *Quick Poll* and *Live Presenter*

See Note 2 at the end of this lesson.

11. At $x = 2$.

12. a. All values from 2 to 11.

b. The integrand is negative.

13. a. $x < 2$ and $x > 11$.

b. It is positive.

14. a. The smallest value of the integral is –11.2037, reached at $x = 11$.

b. It changes sign from negative to positive.

15. A function reaches a minimum where its derivative changes sign from negative to positive.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Quick Poll*

Consider sending a *Quick Poll* for Questions 3 and 4 before having the students create the scatter plot on page 1.5.

Note 2
Problem 1, *Quick Poll, Live Presenter*

Consider sending *Quick Polls* for Questions 11 – 15. Then for any troubling responses, use the *Live Presenter* to illustrate and clear-up the misconception.