

EXPLORATIONS

Chapter 2

Features Used

deSolve(), expand(), [],
fMax(), zeros(), [Y=],
[WINDOW], [GRAPH],
NewProb, tCollect(),
limit(), ['], ZoomFit,
Trace, [ANS], [°]

Setup

▶ 1, NewFold transym

Transient Circuit Analysis: Symbolic

This chapter describes how to use the differential equation solver, `deSolve()`, to solve first- and second-order circuits containing resistors, capacitors, inductors, DC sources, and exponential sources. It also shows how to graph the solutions and find the zero crossing and peak values.

Topic 4: RC First-Order Circuit

In the circuit in Figure 1, the switch has been open for a long time. At time $t = 0$, the switch closes. Find $v(t)$ after the switch is closed, that is, for $t > 0$.

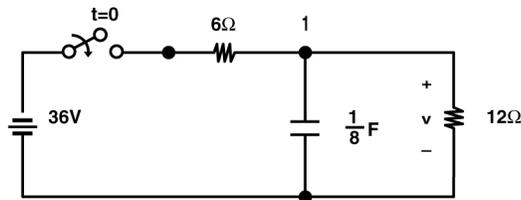


Figure 1. A simple RC circuit

Kirchhoff's current law is applied to the circuit; therefore, the sum of the currents out of node 1 is zero. Current flow in the direction of the voltage drop across a capacitor is

$$i = C \frac{dv}{dt} \text{ or } i = Cv'$$

Therefore, the nodal equation at node 1 is

$$\frac{v-36}{6} + \frac{1}{8} v' + \frac{v}{12} = 0$$

1. Clear the TI-89 by pressing $\boxed{2nd} \boxed{F6} \boxed{2:NewProb} \boxed{ENTER}$.

2. Enter this equation as shown in screen 1.

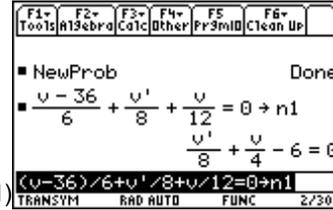
$$\boxed{(} \boxed{v} \boxed{-} \boxed{36} \boxed{)} \boxed{\div} \boxed{6} \boxed{+} \boxed{v} \boxed{2nd} \boxed{[']} \boxed{\div} \boxed{8} \boxed{+} \boxed{v} \boxed{\div} \boxed{12} \boxed{=} \boxed{0} \boxed{STO} \boxed{>} \boxed{n1}$$

3. Since the switch has been open for a long time, any charge originally on the capacitor has been discharged through the 12Ω resistor. This initial condition is expressed as $v(0) = 0$.

Enter the nodal equation, **n1**, and the initial condition, $v(0) = 0$, into the **deSolve()** command as shown in screen 2.

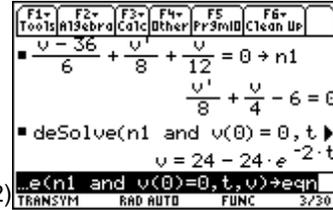
$$\boxed{CATALOG} \boxed{deSolve(n1 \ CATALOG \ and \ v(0) = 0, \ t, \ v)} \boxed{STO} \boxed{>} \boxed{eqn}$$

The solution is $v(t) = 24 - 24e^{-2t}$ V, where t is in seconds.



(1)

Note: To enter $[']$, press $\boxed{2nd} \boxed{[']}$; it represents d/dt .



(2)

Topic 5: Graphing First-Order Solutions

A graph of this solution may help you understand it — graph $v(t)$ for $0 \leq t \leq 3$ seconds.

1. Since y is always graphed versus x on the TI-89, v must be converted to y and t to x . This is accomplished in the Y= Editor (\blacklozenge $[Y=]$) using the “with” operator, $\boxed{[]}$. To convert t to x and $v(t)$ to $y1(x)$, enter the expression as shown in screen 3.

$$v \boxed{[]} \boxed{eqn} \boxed{CATALOG} \boxed{and} \boxed{t} \boxed{=} \boxed{x}$$

This expression causes v to be graphed using its value given in variable **eqn** and to use x instead of t .

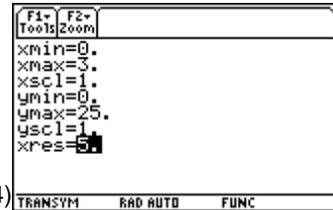
2. Press \blacklozenge $[WINDOW]$ to set the graph range of t from 0 to 3 seconds and v from 0 to 25 V, as shown in screen 4.

The y-axis tick marks are set with **yscl**. The resolution or “closeness” of the pixels representing adjacent calculated values is set by **xres**. Picking **xres=5** will complete the graphs more quickly, but also gives more jagged results.

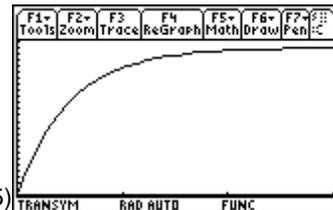
3. Press \blacklozenge $[GRAPH]$ to see a graph of the solution (screen 5).



(3)

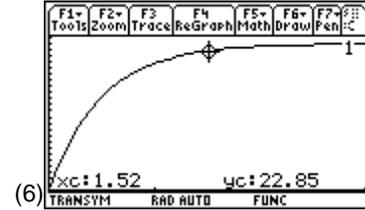


(4)



(5)

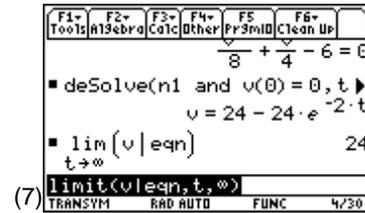
- Press **F3** to trace the graph and to display the cursor location, **xc** and **yc**. The cursor can be moved with the cursor keys, **←** and **→** (screen 6).



Note: *xc* and *yc* are variables that can be accessed from the Home screen.

- Tracing helps to verify that the voltage is approaching 24 V for large *t* as the equation shows, too. This can be verified with the **limit()** command as well (screen 5).

HOME **CATALOG** **limit**(**v** **|** **eqn** , **t** , **∞**)



Topic 6: First-Order Circuit with an Initial Condition

Consider the circuit of Topic 5 (Figure 1) with an initial voltage on the capacitor of -10 V at time *t* = 0. Figure 2 shows this circuit just after the switch has closed.

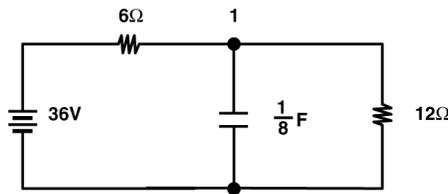


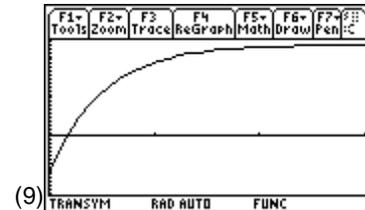
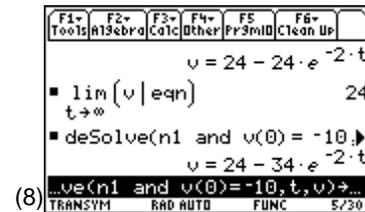
Figure 2. Circuit of Figure 1 just after the switch is closed

The circuit equation is the same as before. The initial condition is changed to $v(0) = -10$ V.

- Enter the equation as shown in screen 8.

CATALOG **deSolve**(**n1** **CATALOG** **and** **v** (**0**) **=** (**-**) **10** , **t** , **v**) **STO** **eqn**

- Since the capacitor voltage starts with $v(0) = -10$ V, use **◀** [WINDOW] and set **ymin** to -15. There is no need to reset the other window variables.
- The results are graphed with **◀** [GRAPH] as shown in screen 9.



- The graphical form of the solution makes it easy to find the instant of time when the capacitor voltage is 0.

Move the cursor to the vicinity of the zero crossing by pressing $\boxed{F3}$ (the Trace tool) and using the cursor keys \leftarrow and \rightarrow (screen 10).

At time $t=0.19$ seconds, the voltage is small, 0.74 V, but not close enough.

More digits can be displayed by pressing $\boxed{\text{MODE}}$ and changing **Display Digits** to **5:Fix 4**, but yc will not be any closer to 0.

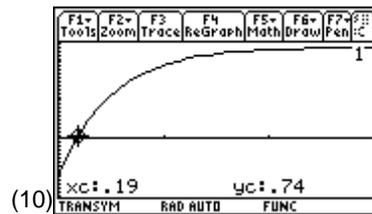
- To get closer, press $\boxed{\blacklozenge}$ [WINDOW] and set **xres** to a smaller value (try **xres=2**), but it will take longer to complete the graph (screen 11). This result may be closer to 0, but maybe not close enough due to graphical resolution.
- A “cleaner” approach to finding the zero crossing is through the **Zero** command. Press $\boxed{F5}$ **2:Zero** (screen 12.)

- Position the cursor to the left of the zero crossing to set a Lower Bound and press $\boxed{\text{ENTER}}$ (screen 13). Note that the location is marked on the screen.

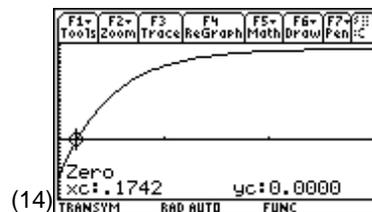
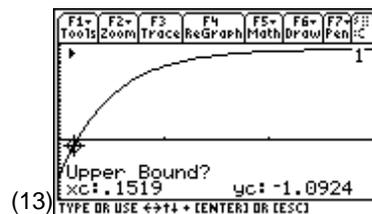
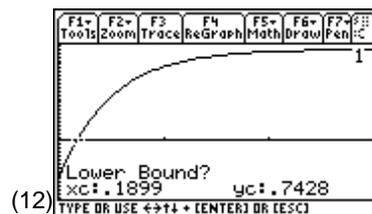
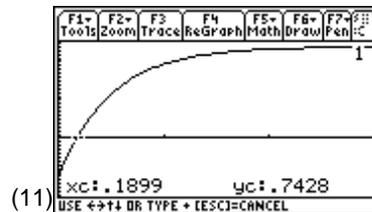
- Move the cursor to the right of the zero crossing and press $\boxed{\text{ENTER}}$.

A much more accurate value for the time at which the voltage is zero is now displayed in screen 14.

This more accurate time is $t = .1742$ seconds.



Note: Press $\boxed{2nd} \leftarrow$ or $\boxed{2nd} \rightarrow$ to take big steps, or hold down \leftarrow or \rightarrow .



Topic 8: RLC Second-Order Circuit

Given the circuit in Figure 4 in which the current at time $t=0$ s is 10 A, ($i(0)=10$), and the time derivative of the current at $t=0$ s is 0, ($i'(0)=0$), find $i(t)$ for $t>0$ s.

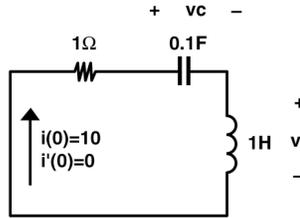


Figure 4: A simple RLC circuit

Kirchhoff's voltage law states that the sum of the voltages around every closed loop is zero. The voltage drop across a capacitor in the direction of current flow is

$$v_c = \frac{1}{C} \int i dt$$

and the voltage drop across an inductor in the direction of current flow is

$$v_l = L \frac{di}{dt}$$

Therefore, Kirchhoff's voltage law for the single loop of the circuit above is given as

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = 0$$

A derivative with respect to time of this equation and a rearrangement of terms gives

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

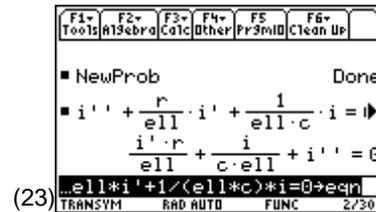
The differential equation can be rewritten as

$$i'' + \frac{R}{L} i' + \frac{1}{LC} i = 0$$

with the notation of $i'(t)$ and $i''(t)$ as the first and second time derivatives, respectively.

1. Clear the TI-89 by pressing $\boxed{2nd} \boxed{F6} \mathbf{2:NewProb} \boxed{ENTER}$.
2. Enter the equation as shown in screen 23.

$i \boxed{2nd} \boxed{[']} \boxed{2nd} \boxed{[']} \boxed{+} \boxed{r} \boxed{\div} \boxed{ell} \boxed{\times} i \boxed{2nd} \boxed{[']} \boxed{+} \boxed{1} \boxed{\div} \boxed{[]} \boxed{ell} \boxed{\times} \boxed{c} \boxed{[]} \boxed{\times}$
 $i \boxed{=} \boxed{0} \boxed{STO} \mathbf{eqn}$



(23) **Note:** "ell" is entered instead of "l" because "l" and "1" look very similar.

- Enter the numeric values of **r**, **c**, and **ell** as shown in screen 24.

1 [STO] r

.1 [STO] c

1 [STO] ell

- Set the radian mode before running **deSolve()**; otherwise, the solution may appear differently than expected. \blacktriangleright 1 should have set the radian mode. To be sure, press [MODE] and set **Angle** to **RADIAN**.

- Use **deSolve()** to find **i** as shown in screen 25.

[CATALOG] deSolve(eqn [CATALOG] and i [2nd] [,] [0] [] [=] 0
 [CATALOG] and i [0] [] [=] 10 [,] t [,] i [] [STO] eqn2

The result of **deSolve()** has been stored in a variable called **eqn2**, although only a small portion is visible in the entry line. To see the rest of the result, press \odot then \odot until the rest of the line is visible, as shown in screen 26. The complete result is

$$i(t) = 10.00e^{-\frac{t}{2}} \cos(3.12t) + 1.60e^{-\frac{t}{2}} \sin(3.12t) \text{ A}$$

If **c** is entered as an exact value of 1/10 rather than 0.1, the answer will be in exact form. As a result, it may contain square roots and fractions that aren't always as easily interpreted. For decimal results, be sure that at least one of the element values is entered as a decimal number.

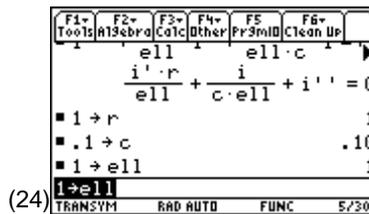
Engineers generally express values in terms of a single sinusoid, cosine or sine, and an angle rather than the sum of a cosine and a sine.

- Convert the solution from **deSolve()** to the more common form with **tCollect()** (screen 27).

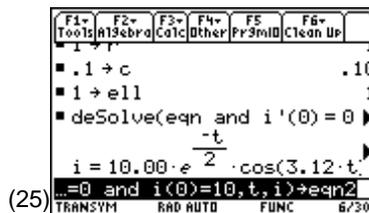
[CATALOG] tCollect(eqn2 [] [STO] eqn3

The complete result is

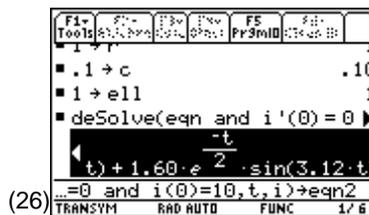
$$i(t) = 10.13e^{-\frac{t}{2}} \sin(3.12t + 1.41) \text{ A}$$



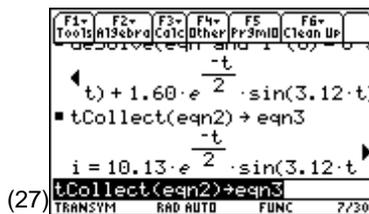
(24)



(25)



(26)



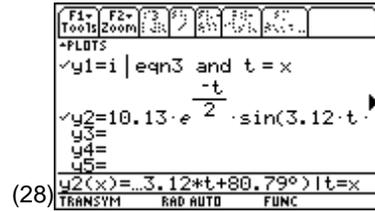
(27)

To convert the angle from radians to degrees, multiply the angle by $180/\pi$, that is, $1.41 \cdot 180/\pi = 80.79^\circ$. Therefore, an alternate form of the current answer is

$$i(t) = 10.13e^{-\frac{t}{2}} \sin(3.12t + 80.79^\circ) \text{ A}$$

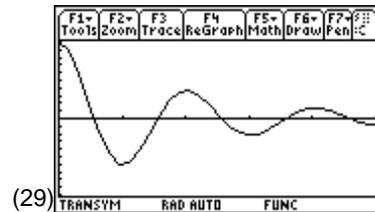
Let's graph it.

- Press \square [Y=] and enter the equation as **y2** (screen 28).
 $10.13 \square \times \square \square [e^x] \square (-) \square t \square \div \square 2 \square \square \times \square [2nd] \square [SIN] \square 3.12 \square t \square + \square 80.79 \square [2nd] \square [^\circ]$
 $\square \square t \square = \square x$
 From the differential equation solution, it is clear that the time constant is 2, so the graph should extend for several time constants.
- Press \square [WINDOW] and set **xmin** = 0, **xmax** = 5, **ymin** = -10, and **ymax** = 10.
- Press \square [GRAPH].



Note: The $^\circ$ symbol is essential for correct graphing as it "overrides" the radian mode setting.

Note: To see how to enter other special characters, press \square [EE].



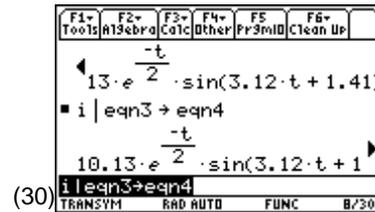
In screen 29, **y1** graphs the results from the **tCollect()** command while **y2** graphs the simplification. They both should be the same graph if the simplification was done correctly; however, **y2** graphs more quickly. Refer to the Tips section to see how to speed up the graphing of **y1**.

The result looks like a nice, under-damped, second-order response.

Tips and Generalizations

Faster Graphs

Graphing **y1** as shown in the previous section is a handy way to graph the output of **deSolve()** (or **solve()**), but it graphs more slowly than just retyping the equation into the Y= Editor. One way to graph **i** more quickly without reentering it is to use the "with" operator, [I], to extract the equation for **i** prior to graphing and to store it as another variable. This appears to occur when the "with" operator, [I], is used for the **y** function within the Y= Editor. Screen 30 shows how to extract **i** from **eqn3** and rename it as **eqn4**.

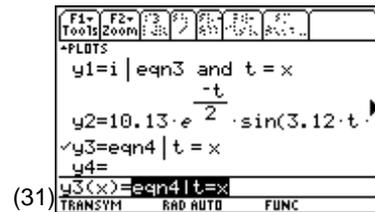


$i \square [I] \square eqn3 \square [STO] \square eqn4$

Now press \square [Y=] and enter **y3** as shown in screen 31.

In the Graph screen, the graph of **y3** should be the same as the last two graphs, but it should appear about twice as fast.

Since **y3** is the only function checked, it is the only one that is graphed.



Note: [F4] is used in the Y= Editor to select and deselect the functions to be graphed.

Exact vs. Approx

If the exact/approx mode (MODE F2) hasn't been changed, the TI-89 will produce exact (rational rather than decimal) results when it can. If a decimal value is used on the input, the output will be approximate (contains decimal values). If the input has no decimal values, the result will be exact (in terms of rationals). If a decimal result is desired, press 2nd $\text{[}\approx\text{]}$ rather than just ENTER .

Summary

Although an RC circuit was shown here, any first-order equation can be solved and graphed using these techniques. **zeros()** and **fMax()** also were used. **fMin()** works that same way to find the minimum of a function. Check the other commands under the F5 menu in the Graph screen, which includes Minimums, Derivatives, Tangents, and so forth.

The **solve()** command also has a numeric version called **nSolve()** which finds a numeric solution rather than symbolic. This shouldn't be needed for linear equations, but **nSolve()** might be the only way of getting a solution of non-linear circuits.

In addition, solving for the current in a series RLC circuit has been shown, but these techniques can be used for finding voltage as well. A parallel RLC circuit could be solved for voltages or currents, too. In fact, any second-order circuit can be solved by these methods.

Although the TI-89 can find symbolic solutions for up to second-order differential equations, it can find numeric solution for systems of any number of first-order equations, as shown in Chapter 3.