

Features Used

deSolve(), expand(), [], fMax(), zeros(), [Y=], [WINDOW], [GRAPH], NewProb, tCollect(), limit(), ['], ZoomFit, **Trace**, [ANS], [°]

Setup

◆ 1, NewFold transym

Transient Circuit This chapter describes how to use the differential equation Analysis: Symbolic solver, deSolve(), to solve first- and second-order circuits containing resistors, capacitors, inductors, DC sources, and exponential sources. It also shows how to graph the solutions and find the zero crossing and peak values.

Topic 4: RC First-Order Circuit

In the circuit in Figure 1, the switch has been open for a long time. At time t = 0, the switch closes. Find v(t) after the switch is closed, that is, for t > 0.



Figure 1. A simple RC circuit

Kirchhoff's current law is applied to the circuit; therefore, the sum of the currents out of node 1 is zero. Current flow in the direction of the voltage drop across a capacitor is

$$i = C \frac{dv}{dt}$$
 or $i = Cv'$

Therefore, the nodal equation at node 1 is

$$\frac{v-36}{6} + \frac{1}{8}v' + \frac{v}{12} = 0$$

- 1. Clear the TI-89 by pressing [2nd [F6] 2:NewProb ENTER].
- 2. Enter this equation as shown in screen 1.

 $(v - 36) \div 6 + v 2nd ['] \div 8 + v \div 12 = 0 \text{ STO} n1$

3. Since the switch has been open for a long time, any charge originally on the capacitor has been discharged through the 12Ω resistor. This initial condition is expressed as v(0) = 0.

Enter the nodal equation, **n1**, and the initial condition, v(0) = 0, into the **deSolve()** command as shown in screen 2.

CATALOG deSolve(n1 CATALOG and v (0)) = 0 , t , v) STOP eqn

The solution is $v(t) = 24 - 24e^{-2t} V$, where t is in seconds.

Topic 5: Graphing First-Order Solutions

A graph of this solution may help you understand it — graph v(t) for $0 \le t \le 3$ seconds.

 Since y is always graphed versus x on the TI-89, v must be converted to y and t to x. This is accomplished in the Y= Editor (● [Y=]) using the "with" operator, □. To convert t to x and v(t) to y1(x), enter the expression as shown in screen 3.

v] eqn CATALOG and t = x

This expression causes v to be graphed using its value given in variable **eqn** and to use x instead of t.

2. Press ● [WINDOW] to set the graph range of t from 0 to 3 seconds and v from 0 to 25 V, as shown in screen 4.

The y-axes tick marks are set with **yscl**. The resolution or "closeness" of the pixels representing adjacent calculated values is set by **xres**. Picking **xres**=5 will complete the graphs more quickly, but also gives more jagged results.

3. Press ● [GRAPH] to see a graph of the solution (screen 5).



Note: To enter ['], press 2nd [']; it represents d/dt.









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4. Press F3 to trace the graph and to display the cursor location, **xc** and **yc**. The cursor can be moved with the cursor keys, () and () (screen 6).



Note: xc and *yc* are variables that can be accessed from the Home screen.



5. Tracing helps to verify that the voltage is approaching 24 V for large t as the equation shows, too. This can be verified with the **limit()** command as well (screen 5).

HOME CATALOG limit(v | eqn , t , \bullet [∞])

Topic 6: First-Order Circuit with an Initial Condition

Consider the circuit of Topic 5 (Figure 1) with an initial voltage on the capacitor of -10 V at time t =0. Figure 2 shows this circuit just after the switch has closed.



Figure 2. Circuit of Figure 1 just after the switch is closed

The circuit equation is the same as before. The initial condition is changed to v(0) = -10 V.

1. Enter the equation as shown in screen 8.

CATALOG deSolve(n1 CATALOG and v(0) = (-) 10, t, v) STO eqn

- 2. Since the capacitor voltage starts with v(0) = -10 V, use
 [WINDOW] and set ymin to -15. There is no need to reset the other window variables.
- **3.** The results are graphed with [GRAPH] as shown in screen 9.



4. The graphical form of the solution makes it easy to find the instant of time when the capacitor voltage is 0.

Move the cursor to the vicinity of the zero crossing by pressing F3 (the Trace tool) and using the cursor keys () and () (screen 10).

At time t =0.19 seconds, the voltage is small, 0.74 V, but not close enough.

More digits can be displayed by pressing MODE and changing **Display Digits** to **5:Fix 4**, but **yc** will not be any closer to 0.

- 5. To get closer, press [WINDOW] and set **xres** to a smaller value (try **xres**=2), but it will take longer to complete the graph (screen 11). This result may be closer to 0, but maybe not close enough due to graphical resolution.
- 6. A "cleaner" approach to finding the zero crossing is through the Zero command. Press F5 2:Zero (screen 12.)
- 7. Position the cursor to the left of the zero crossing to set a Lower Bound and press ENTER (screen 13). Note that the location is marked on the screen.
- **8.** Move the cursor to the right of the zero crossing and press <u>ENTER</u>.

A much more accurate value for the time at which the voltage is zero is now displayed in screen 14.

This more accurate time is t = .1742 seconds.









9. Return to the Home screen and find this value using the **zeros()** command (top of screen 15).

HOME CATALOG zeros(v | eqn , t)

Press ● [≈] to get a floating point approximation of the exact solution (bottom of screen 15).

Note that the more accurate graphical answer agrees with the floating point answer through the four significant figures.

11. Reset Display Digits (MODE) to Fix 2 and xres to 5 (

 [WINDOW]).

Topic 7: First-Order Circuit with a Time Varying Source

Modify the circuit of Topic 6 to include v(0) = -10 V with the battery and switch replaced by a source with $vs(t) = 36e^{-3t}$ V for $t \ge 0$ s.



Figure 3. The circuit of Figure 2 with a time varying source

 $(v - 36 \bullet [e^x] \oplus 3t) \oplus \div 6 + v 2nd ['] \oplus 8 + v \oplus 12 = 0 STO n1$

- 2. Use deSolve() to get the solution as shown in screen 17.

 CATALOG deSolve(n1 CATALOG and v (0) = (-) 10, t, v)

 STOP eqn
- **3.** Use **expand()** to put **eqn** in a more familiar form (screen 18).

CATALOG expand(eqn)

The solution is $v(t) = 38e^{-2t}-48e^{-3t} V$



Note: [e^{*x*}] is entered by pressing ● [e^{*x*}] and - is entered by pressing [...].

	F1+ F2+ Too1sA19eb	F3+ F4+ raCa1cOtherP	F5 r9ml0C1e	767 an Up
	- 3·t	- م∫_3·t.(u	8 7	12 ⁻ Գ u) - 45
		8		<u>, , , , , , , , , , , , , , , , , , , </u>
	■ de5ol× V÷	ve(n1 and = 2·(19·e ¹	v(0)= t-24)	=-10,⊅ . _e -3∙t
(17)	n1 and TRANSYM	0(0)=-1 RAD AUTO	0,t,v) FUNC	⇒eqn 9/30
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	F1+ F2+ Toots A19eb deSolv v: expand	$\frac{F_{3}}{r_{0}} \frac{F_{4}}{F_{4}} \frac{F_{4}}{F_{4}}$ $r_{0} contributer p$ $ve(n1 and$ $= 2 \cdot (19 \cdot e^{-1})$ $d(eqn)$ $v = -$ f_{1}	$\frac{F_{5}}{r^{9}\text{MID}(1)} = \frac{1}{24}$ $\frac{38}{e^{t}}^{2} = -\frac{1}{24}$	^{r67} ⁶⁷ ¹⁰

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 $\begin{array}{c} \hline F_{1*}^{F_{1*}} & F_{2*}^{F_{2*}} & F_{1*}^{F_{2*}} & F_{2}^{F_{2*}} & F_{2}^{F_{2}} \\ \hline \hline Tools also brack concluster [Primit] Clean up \\ \hline \upsilon = 24 - 34 \cdot e^{-2 \cdot t} \\ \bullet zeros(\upsilon \mid eqn, t) \\ & \left\{ \frac{1n(17/12)}{2} \right\} \\ \bullet zeros(\upsilon \mid eqn, t) & (.1742) \\ \hline \hline TEANSYM & Red Butto & FUNC & 7/30 \\ \hline \end{array}$

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- **4.** Press [GRAPH] to graph the solution since **eqn** is still defined as the equation to graph in the Y= Editor (screen 19). Note that the peak value is much different than previously.
- **5.** The range of the graph can be altered to see more of the behavior by pressing F2 A:ZoomFit to make the curve fit the window (screen 20).
- 6. Find the zero crossing using F5 2:Zero as before.
- **7.** Find a good first estimate of the peak value and its time with Trace, F3, where v = 3.52 V at time t = 0.66 seconds.
- 8. Find a more accurate value F5 4:Maximum. Use it the same way as F5 2:Zero to get v = 3.53 V at t = 0.64 seconds.
- **9.** Return to the Home screen and use the **fMax()** command to find an exact answer (top of screen 21).

HOME CATALOG fmax(v \square eqn \neg t \bigcirc

10. Substitute the result of the **fMax()** command into the original equation (**eqn**) using the "with" operator, [] (bottom of screen 21).

eqn [] 2nd [ANS]

11. Find the floating-point solution using ● [≈] (screen 22). Note that the ans(2) command was used to get the answer from second line of the history area.

The accurate graphical answer agrees with the floating point answer.



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	■eqn t	, = 1n(36/1	9)	
			V =	= <u>6859</u> 1944
	■eqn t	, = 1n(36/1	9) v	= 3.53
(00)	egnlans	5(2)		
(22)	TRANSYM	DEGAUTO	FUNC	13/30

Topic 8: RLC Second-Order Circuit

Given the circuit in Figure 4 in which the current at time t=0 s is 10 A, (i(0)=10), and the time derivative of the current at t=0 s is 0, (i'(0)=0), find i(t) for t>0 s.



Figure 4: A simple RLC circuit

Kirchhoff's voltage law states that the sum of the voltages around every closed loop is zero. The voltage drop across a capacitor in the direction of current flow is

$$vc = \frac{1}{C} \int idt$$

and the voltage drop across an inductor in the direction of current flow is

$$vl = L \frac{di}{dt}$$

Therefore, Kirchhoff's voltage law for the single loop of the circuit above is given as

$$iR + L\frac{di}{dt} + \frac{1}{C}\int idt = 0$$

A derivative with respect to time of this equation and a rearrangement of terms gives

$$\frac{\mathrm{d}^2 \mathrm{i}}{\mathrm{dt}^2} + \frac{\mathrm{R}}{\mathrm{L}}\frac{\mathrm{d}\mathrm{i}}{\mathrm{dt}} + \frac{1}{\mathrm{LC}}\mathrm{i} = 0$$

The differential equation can be rewritten as

$$\mathbf{i''} + \frac{\mathbf{R}}{\mathbf{L}}\mathbf{i'} + \frac{1}{\mathbf{LC}}\mathbf{i} = 0$$

with the notation of i'(t) and i''(t) as the first and second time derivatives, respectively.

- 1. Clear the TI-89 by pressing 2nd [F6] 2:NewProb ENTER.
- 2. Enter the equation as shown in screen 23.

i[2nd ['] 2nd ['] + r ÷ ell × i[2nd ['] + 1 ÷ (ell × c) × i = 0 STO▶ eqn



Note: "ell" is entered instead of "I" because "I" and "1" look very similar.

- **3.** Enter the numeric values of **r**, **c**, and **ell** as shown in screen 24.
 - 1 [ST0▶] r
 - .1 ST0► c
 - 1 ST0► ell
- 4. Set the radian mode before running deSolve(); otherwise, the solution may appear differently than expected. 1 should have set the radian mode. To be sure, press MODE and set Angle to RADIAN.
- 5. Use deSolve() to find i as shown in screen 25.

 $\begin{array}{c} \hline \texttt{CATALOG} \ \text{deSolve(eqn} (\texttt{CATALOG} \ \text{and} \ i (\texttt{2nd} \ ['] (\ 0)) \equiv 0 \\ \hline \texttt{CATALOG} \ \text{and} \ i (\ 0) \equiv 10, \ t, \ i () \ \texttt{STO} \ \texttt{eqn} 2 \end{array}$

The result of **deSolve()** has been stored in a variable called **eqn2**, although only a small portion is visible in the entry line. To see the rest of the result, press O then O until the rest of the line is visible, as shown in screen 26. The complete result is

$$i(t) = 10.00e^{\frac{-t}{2}}\cos(3.12t) + 1.60e^{\frac{-t}{2}}\sin(3.12t) A$$

If **c** is entered as an exact value of 1/10 rather than 0.1, the answer will be in exact form. As a result, it may contain square roots and fractions that aren't always as easily interpreted. For decimal results, be sure that at least one of the element values is entered as a decimal number.

Engineers generally express values in terms of a single sinusoid, cosine or sine, and an angle rather than the sum of a cosine and a sine.

6. Convert the solution from **deSolve()** to the more common form with **tCollect()** (screen 27).

CATALOG tCollect(eqn2) STO► eqn3

The complete result is

$$i(t) = 10.13e^{\frac{-t}{2}}\sin(3.12t + 1.41) A$$







To convert the angle from radians to degrees, multiply the angle by $180/\pi$, that is, $1.41*180/\pi=80.79^{\circ}$. Therefore, an alternate form of the current answer is

$$i(t) = 10.13e^{\frac{-t}{2}}\sin(3.12t + 80.79^{\circ}) A$$

Let's graph it.

1. Press • [Y=] and enter the equation as y2 (screen 28).

10.13 $\times \bullet$ [e^x] (-) t \div 2) \times 2nd [SIN] **3.12** t + **80.79** 2nd [°]) [] t = x

From the differential equation solution, it is clear that the time constant is 2, so the graph should extend for several time constants.

- **2.** Press \bullet [WINDOW] and set xmin = 0, xmax = 5, ymin = -10, and ymax = 10.
- **3.** Press [GRAPH].



Note: The ° symbol is essential for correct graphing as it "overrides" the radian mode setting.

Note: To see how to enter other special characters, press • EE.



In screen 29, **y1** graphs the results from the **tCollect()** command while **y2** graphs the simplification. They both should be the same graph if the simplification was done correctly; however, **y2** graphs more quickly. Refer to the Tips section to see how to speed up the graphing of **y1**.

The result looks like a nice, under-damped, second-order response.

Tips and Generalizations

Faster Graphs

Graphing **y1** as shown in the previous section is a handy way to graph the output of **deSolve()** (or **solve()**), but it graphs more slowly than just retyping the equation into the Y= Editor. One way to graph i more quickly without reentering it is to use the "with" operator, [1], to extract the equation for i prior to graphing and to store it as another variable. This appears to occur when the "with" operator, [1], is used for the **y** function within the Y= Editor. Screen 30 shows how to extract **i** from **eqn3** and rename it as **eqn4**.

i i eqn3 ST0> eqn4

Now press • [Y=] and enter **y3** as shown in screen 31.

In the Graph screen, the graph of **y3** should be the same as the last two graphs, but it should appear about twice as fast.

Since **y3** is the only function checked, it is the only one that is graphed.

$$\begin{array}{c} \hline & \hline \\ \hline F_{12}^{F_{12}} & F_{22}^{F_{22}} & F_{22}^{F_{22}} & F_{22}^{F_{22}} & F_{22}^{F_{22}} \\ \hline \hline F_{13} & F_{22}^{-1} & F_{22} & F_{22} & F_{22} \\ \hline & 1_{3} & F_{22}^{-1} & F_{22} & F_{22} & F_{22} \\ \hline & 1_{3} & F_{22}^{-1} & F_{22}^{-1} & F_{22}^{-1} \\ \hline & 1_{3} & F_{22}^{-1} & F_{22}^{-1} & F_{22}^{-1} \\ \hline & 1_{3} & F_{22}^{-1} & F_{22}^{-1} & F_{22}^{-1} \\ \hline & 1_{3} & F_{22}^{-1} & F_{22}^{-1} & F_{22}^{-1} \\ \hline & 1_{3} & F_{22}^{-1} & F_{22}^{-1} & F_{22}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} \\ \hline & 1_{3} & F_{3}^{-1} & F_$$



Note: F4 is used in the Y= Editor to select and deselect the functions to be graphed.

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Exact vs. Approx

If the exact/approx mode ($\boxed{\text{MODE}}$ $\boxed{\text{F2}}$) hasn't been changed, the TI-89 will produce exact (rational rather than decimal) results when it can. If a decimal value is used on the input, the output will be approximate (contains decimal values). If the input has no decimal values, the result will be exact (in terms of rationals). If a decimal result is desired, press \bullet [\approx] rather than just $\boxed{\text{ENTER}}$.

Summary

Although an RC circuit was shown here, any first-order equation can be solved and graphed using these techniques. **zeros()** and **fMax()** also were used. **fMin()** works that same way to find the minimum of a function. Check the other commands under the F5 menu in the Graph screen, which includes Minimums, Derivatives, Tangents, and so forth.

The **solve()** command also has a numeric version called **nSolve()** which finds a numeric solution rather than symbolic. This shouldn't be needed for linear equations, but **nSolve()** might be the only way of getting a solution of non-linear circuits.

In addition, solving for the current in a series RLC circuit has been shown, but these techniques can be used for finding voltage as well. A parallel RLC circuit could be solved for voltages or currents, too. In fact, any second-order circuit can be solved by these methods.

Although the TI-89 can find symbolic solutions for up to second-order differential equations, it can find numeric solution for systems of any number of first-order equations, as shown in Chapter 3.