

TI-Nspire ActivityTitle :

Transformations and Tessellations

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Activity Overview

In this activity you will explore various transformations and conclude by building a tessellation using these transformations.

Concepts included in Activity

- Reflections of geometric objects
- Translation of geometric objects
- Rotation of a geometric objects
- Dilation of a geometric object
- Tessellations

Teacher Preparation

- All instructions for students to complete this activity appear on the student worksheet and on this form
- The activity need not be teacher led as the activity is written so that the student can develop an understanding simply by reading the student worksheet. If teacher led, the teacher can enhance student understanding by making certain that students have the necessary background knowledge required
- The expected screen shots for the completed problems are given in the solutions.tns document
- Specific recommendations for the teacher to implement during the activity are included throughout this document

The Classroom.

- Students will need to download the activity to handheld and have a printed copy of student worksheet
- Many commands utilized can be entered by using the alphabet keys on the handheld or by using builtin commands listed in the library of commands – students need to be made aware of this.



The Activity

1.1 1.2 1.3 2.1 DEG AUTO REAL Transformtions and Tessellations

In this activity you will construct a variety of transformations. In Problem #1 you will create a reflection of a pentagon, in Problem #2 a translation of a regular hexagon, in Problem #3 a rotation of a quadrilateral in two ways, in Problem #4 a dilation of a triangle. In each case you will observe the results of these transformations and summarize them

You may work with your students on this activity. To begin have students open the activity *trans_tess.tns*

Page 1.1

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Inform students that to read the entire introduction they will need to scroll down the page using the down arrow on the NavPad (see *Figure #1*)

Figure #1

Problem #1 - Create a reflection of a pentagon.

Review the concept of a reflection with students. Then lead students through the problem as presented in student worksheet and below:

Page 1.2

Hexagon ABCDEF was constructed and then reflected about the line containing point G. Hexagon A'BC'D'E'F' is called the image of hexagon ABCDEF.

- Move your cursor over the hexagon on the left until the cursor becomes an "open" hand and the word "polygon ABCDEF" appears. (see *Figure #2*). Hold down the click key (*) until the "hand" closes down on the hexagon. (see *Figure #3*) The hand is "grabbing" the hexagon and now you can move the hexagon around using the arrow key to see what happens to its image. Press the ESC key, this un-grabs an object.
- 2. Move the cursor over vertex A of the hexagon and grab it once the open hand and the words "point A" appear. Move the point around using the arrow key to see what happens both to the hexagon and to its image. Try it with other vertices as well.

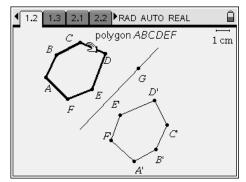


Figure #2

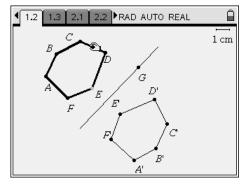
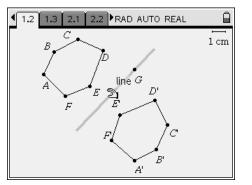


Figure #3







- 3. Move the cursor to point G, grab it, and move the point around and observe what happens to its image.
- 4. Move the cursor to a point on the line of reflection, but off G as shown in *Figure #4* until the word "line" appears, grab the line and move it around and observe what happens to its image.

Page 1.3

- 1. This is a Graphs and Geometry page. Change it to plane geometry view by selecting menu, 2:View, 2 Plane Geometry View.
- 2. Construct a pentagon to reflect as in *Figure #5*:
 - a. Press Menu, 6:Shapes, 4:Polygon You will now have a pencil and a point.
 - b. Arrow over to the left corner so your pentagon will not be in the middle of the screen and begin drawing your pentagon by pressing the click key to plot a point and then using the arrow buttons to draw a line segment. Continue this sequence until you have a pentagon, then either double click to set it or press enter to set it at the first point.

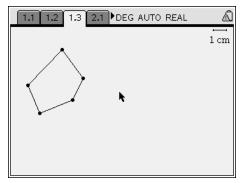


Figure #5

- 3. Construct a line of reflection
 - a. Press Menu, 6: Points and Lines, 4: Line.
 - b. Arrow to a point on the screen to place the first point for your line of reflection, then press the click key to set the point.
 - c. Move the arrow to where you want the line drawn and plot the second point. Double click or press enter to set the line.
- 4. Perform the reflection:
 - a. Press Menu and then A: Transformation followed by 2:Reflection
 - b. With the pointer move to the line of reflection, when you see the word "line" select it
 - c. Move the pointer to the pentagon you want to reflect, you should see a dotted line of the image, double click or press enter to set the line.
- 5. Press "ESC" to get out of the reflection mode. Repeat steps # 1 4 from **Page 1.2.** Remember, you need to move the cursor to the object you want to drag and hold down the pointer (click) key until the hand closes around the object. To re-open the hand, click on the pointer key again.

Grade level: 9-12 Subject: Transformations and Tessellations Time required: 30 to 40 minutes

Write below any observations you have made about reflections as you worked on this problem.

Problem #2 - Create a translation of a regular hexagon.

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Review the concept of a translation with students. Then lead students through the problem as presented in student worksheet and below:

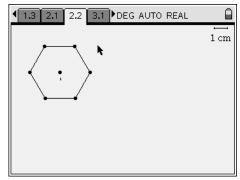
Page 2.1

Concave pentagon ABCDE was constructed and then translated a length of the line segment FG in the direction of FG. Pentagon A'B'C'D'E' is its image. (see *Figure #6*)

- 1. Grab pentagon ABCDE and move it around using the arrow key to see what happens to its image.
- 2. Move the cursor over any vertex of hexagon ABCDE and grab it once the open hand and the word "point" appears. Move the point around using the arrow key to see what happens both to the hexagon and to its image.
- 3. Move the cursor to point F, grab it, then move the point around both lengthening and shortening segment FG and changing its direction. Observe what happens to its image. Do the same for point G.

Page 2.2

- 1. As before change the page to a plane geometry view.
- 2. Construct a regular hexagon to translate as in *Figure #*7
 - a. Select menu, 8: Shapes, 5:Regular Polygon
 - b. Move the cursor to where you want your object to be and use click to select the center of your polygon.
 - c. Move the cursor outward to form the radius of the polygon and click when you are satisfied.
 - d. Select a vertex and move the cursor toward an adjacent vertex to reduce the number of sides in the polygon (it starts with 16) by lengthening a side. When you have the regular hexagon you want, press enter to set it.
- 3. Create a distance and direction for the translation
 - a. Select menu, 6: Points and Lines, 1: Point
 - b. Plot two points in the direction that you want to slide your object using the click button





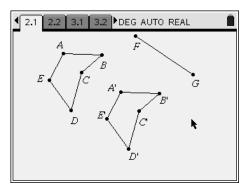


Figure #6



- c. Select menu, 6: Points and Lines, 5:Segment to construct a segment between the two points plotted.
- 4. Perform the translation
 - a. Press Menu, A:Transformation, 3: Translation
 - b. Select the two points in the direction you want the object to translate, then select the regular hexagon. You will see the image in dotted lines, press enter to set it.
- 5. Again press "ESC" to get out of the translation mode. Repeat steps # 1 3 from **Page 2.1.** Remember, you need to move the cursor to the object you want to drag and hold down the pointer (click) key until the hand closes around the object. To re-open the hand, click on the pointer key again.

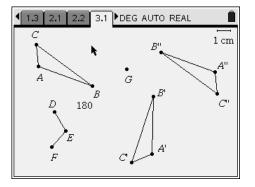
Write below any observations you have made about translations as you worked on this problem.

Problem #3 - Create a rotation of a triangle.

Page 3.1

Review the concept of a rotation with students. Then lead students through the problem as presented in student worksheet and below:

Triangle ABC was constructed and then rotated an amount equal to angle DEF about point G. Triangle A'B'C' is its image (see *Figure #8*). Then a text box with the number 180 was added and a second rotation of Triangle ABC was done about point G, this time using the text box to determine the rotation. Triangle A'B'C' is the image as the result of this rotation. For each of the steps below observe what happens:





- 1. Grab triangle ABC and move it around using the arrow keys to see what happens to its image.
- 2. Grab the rotation point G and move it around.
- 3. Move the cursor over any vertex of triangle ABC and grab it. Move the point around using the arrow key to see what happens to the triangle and to its images. Do the same for another vertex.
- 4. Move the cursor to point F, grab it, and move the point around to change the angle of rotation. Observe what happens to the image A'B'C' formed from ∠DEF.. Do the same for each point which defines the angle of rotation.
- 5. Move the cursor over the text box, and double click the box. Change the number in the box to see what happens to image of triangle A"B"C".

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Page 3.2

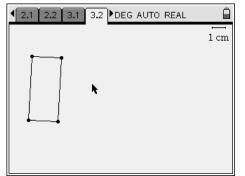
There are two ways to plot a rotation – one way is by designating the angle of rotation by plotting 3 points to form the angle

- 1. Change the page to a plane geometry view.
- 2. Construct a rectangle as in *Figure #9*
 - a. Press Menu, 8: Shapes, 3:Rectangle.
 - b. Move the cursor to where you want your rectangle to be, click on the pointer key to plot first vertex, move the cursor to begin drawing the rectangle, click on the pointer to plot second vertex, move again to finish the rectangle and then click on pointer to complete the rectangle. Press Enter to set it.
- 3. Perform the rotation:
 - a. Press Menu, A: Transformation, 4: Rotation
 - b. Using the arrow keys select the point you want as your center of rotation
 - c. Select the rectangle
 - d. Select 3 points "on the fly" to form the angle of rotation. You should see the rotation appearing as you are completing the selection. Press enter to set it. Segments connecting the 3 points could be constructed to better view the angle of rotation.
- 4. Once again, drag the objects around to see what happens to the image. ("Esc", move cursor to object and hold pointer key till hand closes around object.)

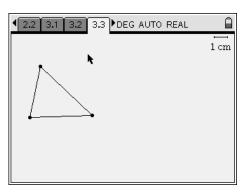
Page 3.3

Another way to plot a rotation is to enter the angle of rotation in a text box.

- 1. Change the page to a plane geometry view.
- 2. Construct a triangle as in *Figure #10*
 - a. Press Menu, 8: Shapes, 2:Triangle.
 - b. Move the cursor to where you want your triangle to be, click on the pointer key to plot first vertex, move the cursor, click on the pointer to plot second vertex, move again to finish the triangle and then click on pointer to complete the triangle. Press Enter to set it.
- 3. Construct a text box to enter the angle of revolution
 - a, Press Menu, 1:Actions, 6:Text
 - b. Move the cursor to a blank part of the screen and double-click
 - c. Enter the number 180 in the textbox .











- d. Be sure the handheld is set to degrees: Home, 8:Document Settings, then tab to angle in window, set it to degrees if necessary, then press Enter to return to Page 3.3
- 4. This time we will use the midpoint of one side of the triangle to serve as the point of rotation.
 - c. Press Menu, 9:Construction, 5:Midpoint
 - d. Move the cursor over the triangle until the words "side (1)" appear. The midpoint will appear, hit Enter to set it
- 5. Perform the rotation:
 - a. Select keys: menu, A: Transformation, 4: Rotation
 - e. Using the arrow keys select the midpoint as your center of rotation by hovering over until the words "point (b)" appear, then select it
 - f. Select the triangle
 - g. Select the text box containing the angle of rotation. You should see the rotation appearing as you are completing the selection. Press enter to set it.
- 6. Once again, drag the objects around to see what happens to the image. ("Esc", move cursor to object and hold pointer key till hand closes around object.)

Write below any observations you have made about rotations as you worked on this problem.

Problem #4 - Create a dilation of a quadrilateral

Review the concept of a dilation with students. Then lead students through the problem as presented in student worksheet and below:

Page 4.1

- 1. Change the page to a plane geometry view
- 2. Create any quadrilateral
 - a. Press Menu, 8: Shapes, 4:Polygon
 - b. Construct the quadrilateral
- 3. Create a center of dilation and a scale factor
 - a, Press Menu, 6: Points and Lines, 1: Point then plot a point to be the center of dilation
 - b. Insert a text box to provide a scale factor, Press Menu, 1: Actions, 6:Text then move the cursor to where you want to type a number and click on the pointer key. Type in a scale factor of 0.5 and press enter to set it.



- 4. Perform the dilation
 - a. Press Menu, A: Transformation. 5: Dilation
 - b. Select the scale factor, the point, and then move toward the quadrilateral you want to dilate and you should see the image. Press enter to set it.
- 5. Once again, drag the objects around to see what happens to the image. ("Esc", move cursor to object and hold pointer key till hand closes around object.) Change the scale factor as well.

Write below any observations you have made about dilations as you worked on this problem.

Problem # 5 – Have students create the tessellation shown in *Figure #11*Page 5.1

- 1. Change the page to a plane geometry view
- 2. Create a triangle somewhere toward the left of the page and with one side relatively horizontal
- 3. Color in the triangle by selecting the triangle, then press (err) (err) (err), choose 2: Attributes, a dialog box will appear, arrow to the right to change the shading. Select medium gray.

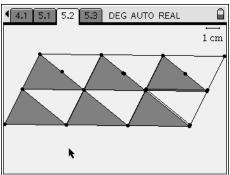


Figure #11

- 4. Construct a midpoint on the right side of the triangle. Press Menu, 9: Construction, 5: Midpoint. Press enter to set it.
- 5. Rotate the triangle around the midpoint 180 degrees.
 - a. Construct a text box containing the number 180 near the bottom of the screen to enter the angle of revolution
 - b. Press Menu, A: Transformation, 4: Rotation, then select the midpoint as the point of rotation, the textbox and the triangle to create the rotation.
- 6. Drag a point of the original triangle to see that the shape is retained. What shape have you formed?
- 7. Using the two vertices on the horizontal side of the original triangle as a translation vector, translate the triangle That is, press Menu, A: Transformation, 3: Translation, select the two vertices on the horizontal side of a triangle, then select the triangle which will set the direction and distance for the translation.



- 8. Continue translating until you have the first row of triangles in *Figure #11*.
- 9. Drag to confirm that the top and bottom edges of this row of triangles are always straight lines. What does that demonstrate about the sum of the angle measures in the original triangle?
- 10. Now translate to form the first triangle of the second row using another side of the original triangle as a translation vector. Continue until the entire row is complete. Once *Figure #11* is complete, drag to confirm that no matter what shape your original triangle has, it will always tessellate.
- 11. Look at a point in the tessellation that is completely surrounded by triangles. How many angles come together at this point? What is the sum of the angles surrounding this point? Why?
- 12. Consider the activity you just completed. Give an explanation for how you know that all triangles tessellate.

Problem #6 – have students create the tessellation shown in *Figure #12*

Once you have completed the activity, provide an explanation for how you know that all quadrilaterals tessellate.

