Simultaneous Equations Elimination

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Teacher Notes & Answers

7 8 9 10 11 12

TI-Nspire Activity Student 30 min

Introduction

Consider the following two equations:

Equation 1: $2x+3y=24$

Equation 2: $5x-2y=22$

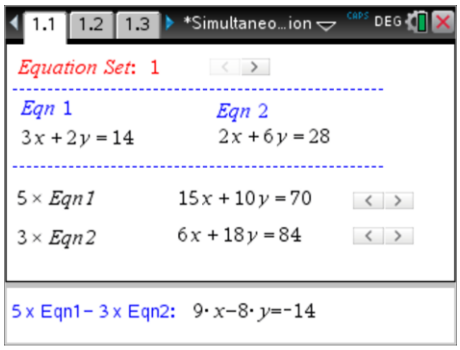
For each equation there are infinitely many combinations of x and y that satisfy the equality. There is however just one pair of values that satisfy both equations at the same time, 'simultaneously'. For example the point (3, 6) satisfies the first equation but not the second:

Equation 1: (3, 6) $2(3)+3(6)=24$

Equation 2: (3, 6) $5(3)-2(6)\neq 22$

There are many ways to determine the values for x and y that satisfy both equations at the same time. The elimination method involves using combinations of the two equations to eliminate one of the variables. The TI-Nspire document "Simultaneous Elimination" provides 20 worked examples to explore. The document does NOT provide the answers, just the first step of the elimination method.

Instructions

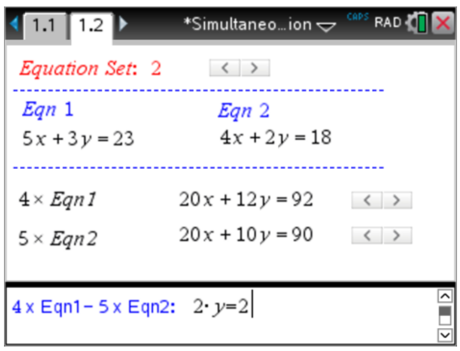


Open the TI-Nspire file: Simultaneous Elimination

Navigate to page 1.2 to see the elimination method in progress.

The slider at the top of the page generates a new set of equations, the two sliders on the right hand side of the page adjust the multiplication factors for equations 1 (Eqn1) and 2 (Eqn2)

The result of the multiplication and subtraction from equation 1 is shown at the base of the page.



Once x or y have been successfully eliminated, record the results and move on to the second equation set.

Example: Equation set 2. (Eliminating x)

$5x+3y=23$ $4x+2y=18$

4 x Eqn 1 $20x+12y=92$

5 x Eqn 2 $20x+10y=90$

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Question: 1.

For each equation set complete the following:

Equation Set 1: Note that elimination can occur for either x or y. Students may do both.

Equation		Multiplying Factor:	New Equation:
Equation 1:	$3x+2y=14$	$\times 2$	$6x+4y=28$
Equation 2:	$2x+6y=28$	$\times 3$	$6x+18y=84$
		m x Eqn 1 – n x Eqn 2:	$-14y=-56$
Solution for x:	$x=2$	Solution for y:	$y=4$
Verification Eqn1:	$3(2)+2(4)=14$	Verification Eqn2:	$2(2)+6(4)=28$

Equation Set 2: Eliminating y uses the same multiplication coefficients as equation set 1.

Equation		Multiplying Factor:	New Equation:
Equation 1:	$5x+3y=23$	$\times 2$	$10x+6y=46$
Equation 2:	$4x+2y=18$	$\times 3$	$12x+6y=54$
		m x Eqn 1 – n x Eqn 2:	$-2x=-8$
Solution for x:	$x=4$	Solution for y:	$y=1$
Verification Eqn1:	$5(4)+3(1)=23$	Verification Eqn2:	$4(4)+2(1)=18$

Equation Set 3: Eliminating y uses the same multiplication coefficients as equation set 1.

Equation		Multiplying Factor:	New Equation:
Equation 1:	$4x+2y=18$	$\times 3$	$12x+6y=54$
Equation 2:	$6x+4y=26$	$\times 2$	$12x+8y=52$
		m x Eqn 1 – n x Eqn 2:	$-2y=2$
Solution for x:	$x=5$	Solution for y:	$y=-1$
Verification Eqn1:	$4(5)+2(-1)=18$	Verification Eqn2:	$6(5)+4(-1)=26$

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Equation Set 4:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$3x+5y=2$	$\times 5$	$15x+25y=10$
Equation 2:	$5x+6y=1$	$\times 3$	$15x+18y=3$
		m x Eqn 1 – n x Eqn 2:	$7y=7$
Solution for x:	$x=-1$	Solution for y:	$y=1$
Verification Eqn1:	$3(-1)+5(1)=2$	Verification Eqn2:	$5(-1)+6(1)=1$

Equation Set 5:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$7x+4y=2$	$\times 1$	$7x+4y=2$
Equation 2:	$3x+2y=2$	$\times 2$	$6x+4y=4$
		m x Eqn 1 – n x Eqn 2:	$x=-2$
Solution for x:	$x=-2$	Solution for y:	$y=1$
Verification Eqn1:	$3(-1)+5(1)=2$	Verification Eqn2:	$5(-1)+6(1)=1$

Equation Set 6:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$8x+3y=2$	$\times 2$	$16x+6y=4$
Equation 2:	$5x+2y=3$	$\times 3$	$15x+6y=9$
		m x Eqn 1 – n x Eqn 2:	$x=-2$
Solution for x:	$x=-5$	Solution for y:	$y=14$
Verification Eqn1:	$8(-5)+3(14)=2$	Verification Eqn2:	$5(-5)+2(14)=3$

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Equation Set 7:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$7x+2y=21$	$\times 1$	$7x+2y=21$
Equation 2:	$5x-2y=-9$	$\times$ -1	$-5x+2y=9$
		m x Eqn 1 – n x Eqn 2:	$12x=12$
Solution for x:	$x=1$	Solution for y:	$y=7$
Verification Eqn1:	$7(1)+2(7)=21$	Verification Eqn2:	$5(1)-2(7)=-9$

Equation Set 8:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$3x+3y=12$	$\times 5$	$15x+15y=60$
Equation 2:	$5x-5y=10$	$\times 3$	$15x-15y=30$
		m x Eqn 1 – n x Eqn 2:	$30y=30$
Solution for x:	$x=3$	Solution for y:	$y=1$
Verification Eqn1:	$3(3)+3(1)=12$	Verification Eqn2:	$5(3)-5(1)=10$

Equation Set 9:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$2x+4y=-8$	$\times 3$	$6x+12y=-24$
Equation 2:	$6x-4y=24$	$\times 1$	$6x-4y=24$
		m x Eqn 1 – n x Eqn 2:	$16y=-48$
Solution for x:	$x=2$	Solution for y:	$y=-3$
Verification Eqn1:	$2(2)+4(-3)=-8$	Verification Eqn2:	$6(2)-4(-3)=24$

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Equation Set 10:

Equation		Multiplying Factor:	New Equation:
Equation 1:	$-2x+3y=11$	$\times 2$	$-4x+6y=22$
Equation 2:	$4x-5y=-21$	$\times$ -1	$-4x+5y=21$
		m x Eqn 1 – n x Eqn 2:	$y=1$
Solution for x:	$x=-4$	Solution for y:	$y=1$
Verification Eqn1:	$-2(-4)+3(1)=11$	Verification Eqn2:	$4(-4)-5(1)=-21$

Question: 2. (Extension Question – Generalisation)

It is possible to generalise the solution process.

Equation 1: $ax+by=c$

Equation 2: $dx+ey=f$

a) To eliminate x from the system of equations, what should Equation 1 and 2 each be multiplied by? Write down the new equations for each.

Eqn 1 $\times$ d $\quad adx+bdy=cd \qquad$ Eqn2 $\times$ a $\quad adx+aey=af$

b) Following from the previous step, what is the result for subtracting Equation 2 from Equation 1?

Eqn 1 – Eqn 2 = $bdy-aey=cd-af$

c) Following from the previous step, transpose your equation to make y the subject of the equation. This is the general solution.

Factor then transpose: $y=\dfrac{cd-af}{bd-ae}$

d) Use your result from the previous step to check your answers to a selection of equation sets from question 1.

Sample: (From Set 1) $y=\dfrac{14\times 2-3\times 28}{2\times 2-3\times 6}=4$

e) Determine a general solution for x .

$$x=\frac{ce-bf}{ae-bd}$$

Question: 3. (Extension Question – Beyond 2 Dimensions)

So far the equations covered in this activity have involved linear equations with two variables. These relationships can be represented on the Cartesian plane as straight lines. When a third variable is introduced it can form a 'plane' in three dimensions. Two planes can intersect along a line; three can intersect at a point. The process for finding this point can be summarised as follows:

Step 1: Combine two of the equations and determine the equation to the line of intersection.

Step 2: Combine a different pair of equations and determine the equation to the line of intersection.

Step 3: Use the two lines from Step 1 & 2 and solve these equations to find the single point of intersection.

Use this approach to find the point where the following lines intersect:

Equation 1: $2x+2y+4z=10$ Equation 2: $2x-y+2z=6$ Equation 3: $-x+2y+z=4$

Check out the graphs using the 3D graphing tool!

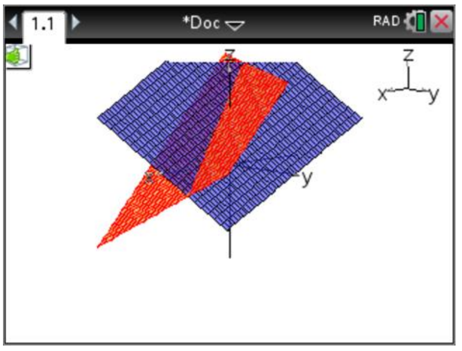
Teacher Notes:

The 3D graphing tool is an ideal tool to provide a visual demonstration of what is happening.

Equations must be of the form z = .

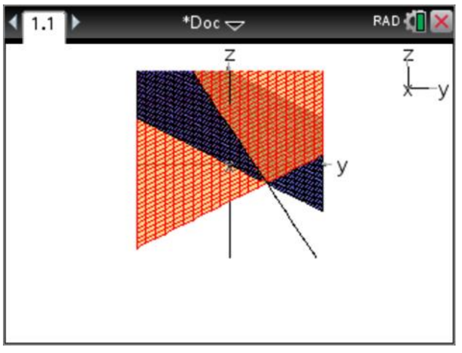
Eqn1: $z=\dfrac{10-2x-2y}{4}$

Eqn2: $z=\dfrac{6-2x+y}{2}$



Suppose x is eliminated from these two equations, the result would be: $3y+2z=4$, transposed to make z the subject:

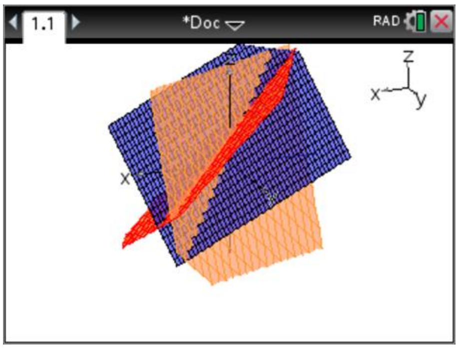
$$z=\frac{4-3y}{2}$$



Rotate the 3D view so that the x axis is along the line of sight by simply pressing 'x' on the keyboard. Graph the resulting equation above to show the line of intersection between the two planes. Rotating the 3D graph now will show one plane that passes through all points of intersection along the original two planes.

Try the same again by eliminating y.

Finally graph the third line Eqn 3: $z=4+x-2y$ to see how all three planes intersect at a single point.



Note that the graphs can be rotated using the arrow keys or auto rotated by pressing "A".

Solution: $x=-3,\quad y=-2,\quad z=5$

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