NUMB3RS Activity: A Breed Apart
Episode: "Nine Wives"

Topic: Inbreeding coefficients **Grade Level:** 9 - 12
Objective: Use probability to compute the inbreeding coefficient for a number of different situations.
Time: 20 - 25 minutes

Introduction

In "Nine Wives," the FBI enlists Charlie's help to find the leader of a polygamous cult, Abner "The Prophet" Stone. Charlie and the FBI find The Prophet's trailer, but it is empty except for a strange quilt and a picture. These both contain a tree diagram, or "family tree," that shows some inbreeding in the lineage of the cult. Usually, such family trees are only found in livestock, but Charlie is able to use these diagrams to calculate The Prophet's "inbreeding coefficient" to assist in determining his location.

Discuss with Students

Inbreeding occurs when an individual has one or more common ancestors. A common ancestor is one who is present on both the father's and mother's sides of the family. The degree of inbreeding is expressed by the *inbreeding coefficient,* which is calculated by a process developed by the American geneticist Sewall Wright in 1921.

Each individual being has two complete sets of chromosomes, one from each parent. Chromosomes contain a person's genes. Typically, genes exist in different versions, or alleles. When an individual receives the same allele for a gene from both parents, the individual is homozygous for that gene.

Some alleles can actually be harmful to an individual. For example, some recessive alleles, such as the allele for cystic fibrosis, can cause disease. However, in many cases, individuals will experience no harmful effects unless they are homozygous. That is, they must receive two copies of the particular allele to be affected negatively. Inbred individuals are more likely than individuals who are not inbred to be homozygous for a given gene. Thus, individuals in an inbred population are more likely to experience the negative effects of a harmful allele.

When none of the ancestors are inbred and a common ancestor appears on both the maternal and paternal lines, the inbreeding coefficient can be found by computing the probability that the same allele from this common ancestor is inherited through both the mother and the father.

This activity provides an opportunity to connect mathematics and biology. Livestock and pet breeders are very concerned about the effects of inbreeding because of the likelihood of undesirable results. A related topic is "linebreeding," where a breeder attempts to maintain a high relationship to some outstanding ancestor while keeping inbreeding as low as possible.

Students may know of examples of inbreeding from their study of history (Spanish royalty in the 19th century) or from their study of biology.

John F. Mahoney, Benjamin Banneker Academic High School, Washington, DC

Questions 1 – 5 on the student page make the assumption that B inherits the allele from D. The computations are directed toward determining if the same allele gets to A from the rest of the family tree.

Student Page Answers:

1. *50%, as it is one of his two alleles.* **2.** *Also 50%, since it is also one of his two alleles.* **3.** *P(B→A) × P(D→C) × P(C→A) = 0.5 × 0.5 × 0.5 = 0.125, where P(B→A) is the probability that B passes allele* a^1 *on to A.* **4.** *P(B→A) × P(D→G) × P(G→C) × P(C→A)= 0.5 × 0.5 × 0.5 × 0.5 = 0.0625* **5.** *This value is half the inbreeding coefficient in question 3. Therefore the inbreeding coefficient for the new situation should be 0.0625/2 = 0.03125.* **6.** *Each probability is 0.125, so the inbreeding coefficient for A is 0.25.* **7.** *P(B→C) × P(C→A) = 0.5 x 0.5 = 0.25*

Extensions:

1 - 3: *These answers are the same as those calculated in the activity.* **4.** $F_B = 0.25$ *and therefore the component of the coefficient where B is a common ancestor is* $\left(\frac{1}{2}\right)^{1+1}(1+0.25)=0.3125$ *, the component of the coefficient where E is a common ancestor is* $\left(\frac{1}{2}\right)^{2+1}(1+0)=0.125$ *, and the component of the coefficient where D is a common ancestor is* $\left(\frac{1}{2}\right)^{3+1}(1+0)=0.0625.$ *The sum of the three components yields the inbreeding coefficient of* $F_A = 0.5$

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Name: ________________________________ Date: ________________

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Each individual being has two complete sets of chromosomes, one from each parent. Chromosomes contain a person's genes. Typically, genes exist in different versions, or alleles. When an individual receives the same allele for a gene from both parents, the individual is homozygous for that gene.

Some alleles can actually be harmful to an individual. For example, some recessive alleles, such as the allele for cystic fibrosis, can cause disease. However, in many cases, individuals will experience no harmful effects unless they are homozygous. That is, they must receive two copies of the particular allele to be affected negatively. Inbred individuals are more likely than individuals who are not inbred to be homozygous for a given gene. Thus, individuals in an inbred population are more likely to experience the negative effects of a harmful allele.

Suppose that A's parents, father (B) and mother (C), both have the same father (D), but different mothers, as shown in the family tree in Figure 1.

Consider one of D's genes. It has two different alleles, a^1 and a^2 (from his parents). If he passes allele a^1 to B, then there is a 50% chance that allele a^1 will also be inherited by A, since it is one of B's two alleles.

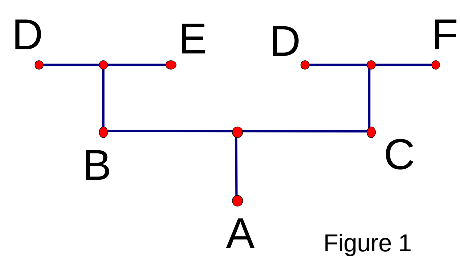

Figure 1

1. In Figure 1, D is the father of both B and C. What is the probability that he will pass allele a^1 on to C?

2. If D passes allele a^1 on to C, what is the probability that C will pass allele a^1 on to A?

3. To find the probability that A receives two copies of the same allele from D (recall that this is the meaning of inbreeding), find the product of the probabilities of B passing a^1 on to A, D passing a^1 on to C, and C passing a^1 on to A. This product is the inbreeding coefficient.

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As before, assume that D has passed the allele on to B.

Consider the situation where D is the *grandfather* of C (rather than the father), as shown in Figure 2.

4. Is A more likely or less likely to be inbred for the gene in this situation than in the previous situation? Use the reasoning from question 3 to find the inbreeding coefficient of A for Figure 2.

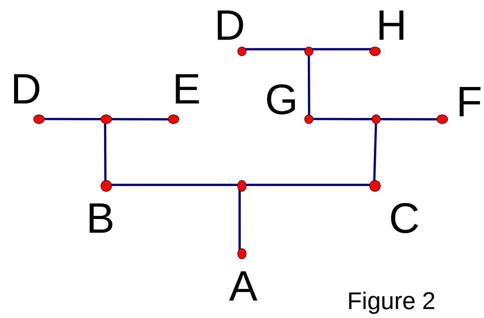

Figure 2

5. What is the relationship between your answer to questions 3 and 4? Find the inbreeding coefficient of A if D is the *grandfather* of **both** B and C.

Once again, assume that B has inherited the allele from D.

Livestock breeders worry about the possibility of siblings mating. This situation is shown in Figure 3.

6. In Figure 3, B would pass a copy of one of its alleles on to A. The other allele that A receives could come through C from D *or* E. To account for this when finding the inbreeding coefficient for A, add the probability that A could inherit two copies of the allele from D to the probability that A could inherit two copies of the allele from E. Find the inbreeding coefficient for A.

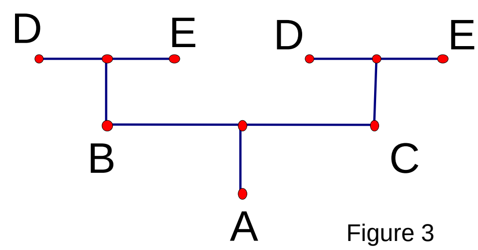

Figure 3

Livestock breeders also worry about the possibility of a sire-daughter mating, where the male is both the father and the mate. This situation is shown in Figure 4.

7. Compute the inbreeding coefficient of A for this diagram, given that A is guaranteed to inherit one copy of the allele from B as its parent.

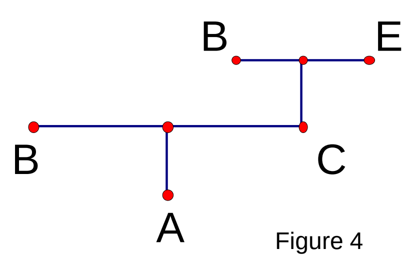

Figure 4

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

Sewall Wright’s formula for the inbreeding coefficient is

$$F_x = \sum\left[\left(\frac{1}{2}\right)^{n+1}(1+F_C)\right]$$

where F_x is the inbreeding coefficient of individual X, Σ is the summation sign (which indicates that the quantities for each common ancestor are to be added together), n is the sum of the number of generations between the parents of individual X and the common ancestor, and F_C is the inbreeding coefficient of the common ancestor, C.

In Figure 1, B and C are each one generation away from the common ancestor, D, so $n = 2$. No information is given about the inbreeding coefficient of D, so assume $F_D = 0$. There is only one common ancestor (D), so $F_A = \left(\frac{1}{2}\right)^{2+1}(1+0) = \frac{1}{8} = 0.125$.

1. Use Sewall Wright’s formula to calculate the inbreeding coefficient of A in Figure 2.

2. Use Sewall Wright’s formula to calculate the inbreeding coefficient of A in Figure 3.

3. Use Sewall Wright’s formula to calculate the inbreeding coefficient of A in Figure 4.

4. Use Sewall Wright’s formula to calculate the inbreeding coefficient of A in Figure 5.

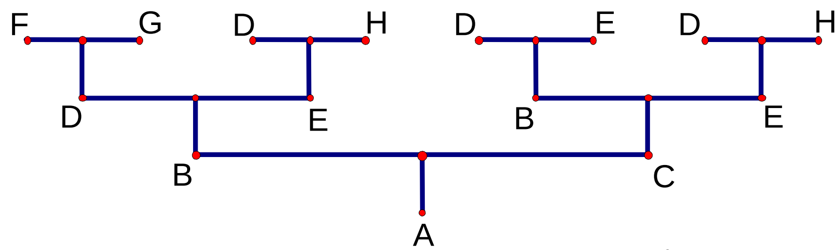

Figure 5

Additional Resources

The following web sites contain more information about the inbreeding coefficient:

- **http://freepages.genealogy.rootsweb.com/~jamesdow/wrigco.htm**
- **http://muextension.missouri.edu/explore/agguides/ansci/g02911.htm**
- **http://bowlingsite.mcf.com/Genetics/Inbreeding.html**

John F. Mahoney, Benjamin Banneker Academic High School, Washington, DC