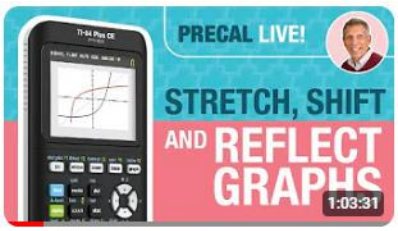
Thursday Night Precalculus
October 26, 2023

In this AP Precalculus Live session, we will explore inverses, transformations, and compositions.



New Functions from Old: Inverses, Transformations, and Compositions

About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand: https://www.youtube.com/watch?v=jZ94r3wv6mM&list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI&index=3&t=83s&pp=iAQB
    - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves functions and their inverses, transformations of functions, and function composition.
- The transformations include:
    - Additive transformations: horizontal and vertical translations,
    - Multiplicative transformations: horizontal and vertical dilations.
- Students should be able to use the TI-84 to explore inverses, compositions, and transformations.
- **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:

- Problems_10_26_23V3

Solutions

- Problems_solutions_10_26_23V3

YouTube

- https://www.youtube.com/watch?v=jZ94r3wv6mM&list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI&index=3&t=83s&pp=iAQB
- ***Documents and materials can be downloaded from this site.***

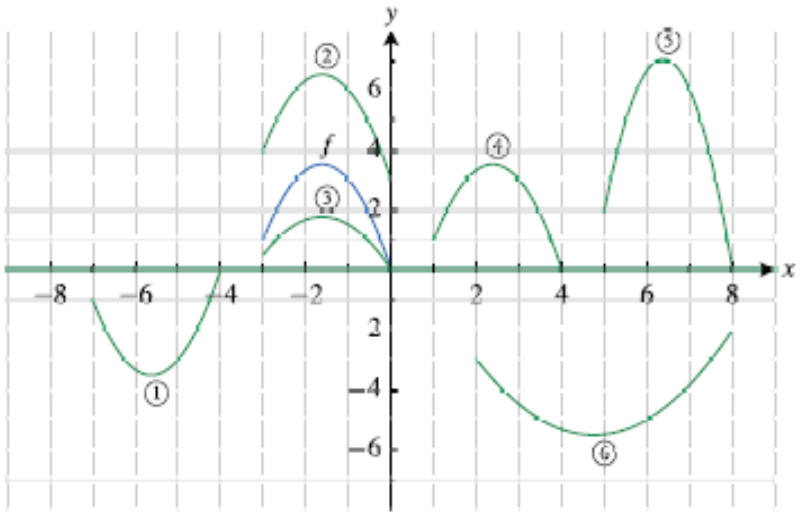
AP Precalculus Learning Objectives

- 1.12.A: Construct a function that is an additive or multiplicative transformation of another function.
- 2.7.A: Evaluate the composition of two or more functions for given values.
- 2.7.B: Construct a representation of the composition of two or more functions.
- 2.8.B: Determine the inverse of a function on an invertible domain.

Source: AP Precalculus Course and Exam Description, The College Board

Problem 1. (a) – (f)

The graph of f is given in the figure. Match each equation with its graph and give a reason for each choice.



(a) $y = f(x-4)$ (b) $y = f(x)+3$ (c) $y = 2f(x-8)$

(d) $y = \frac{1}{2} f(x)$ (e) $y = -f(x+4)$ (f) $y = -f\left(\frac{1}{2}(x-8)\right)-2$

Teacher Tip: The graph of $y = f(x)$ is blue in the figure above. The transformations are green.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Teacher Tip: AP Precalculus refers to these transformations as translations, dilations, and reflections.

Using the TI-84 with Problem 1. (a) & (f)

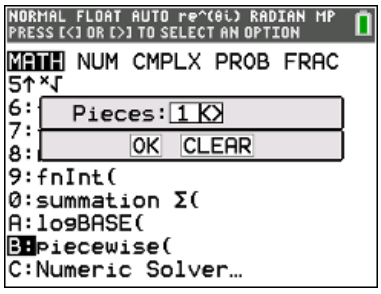
The use of the TI-84 for all parts of this problem is not discussed in the video. Screen shots of (a) – (f) from the TI-84 are shown on the next page.

Use the graphing application to graph $y = f(x)$, where

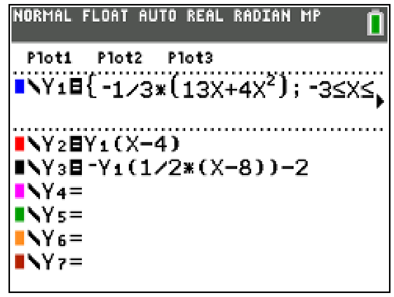
$$f(x) = -\frac{1}{3}\left(13x + 4x^2\right) \mid -3 \le x \le 0.$$

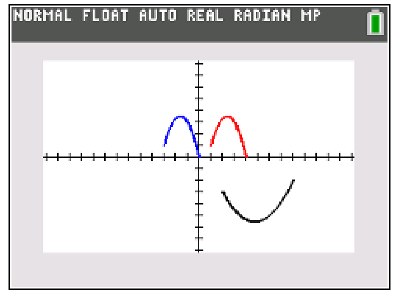
Select [y=], then [math] and scroll down to piecewise(. Use the arrow keys to select 1 piece. Graph the function with Zoom Decimal.

To check the matches of the functions to the graphs, type the new function in Y2 using Y1 with the various transformations. For instance, for 1 (a), enter Y2=Y1(x – 4) and for 1 (f), enter Y3=–Y1(1/2(x – 8)) – 2.



Technology Tip: Adjust the window settings to check the graphs given in Problem #1 by multiplying the existing minimums and maximums by 2.

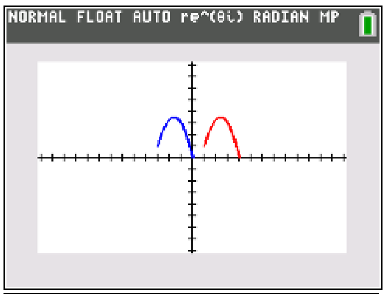


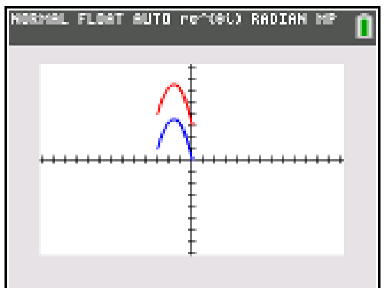


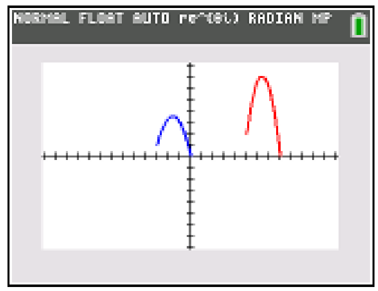
Teacher Note: The graph of 2. (f) in the video did not have the –2 vertical translation.

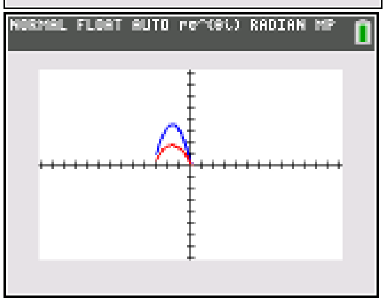
Showing grid lines may also help students visualize the transformations.

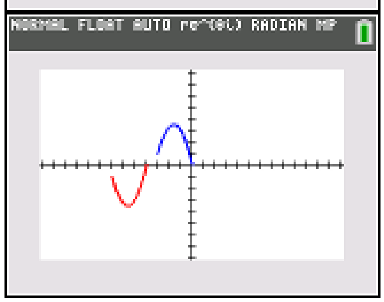
Menu – View – Grid – Lined Grid

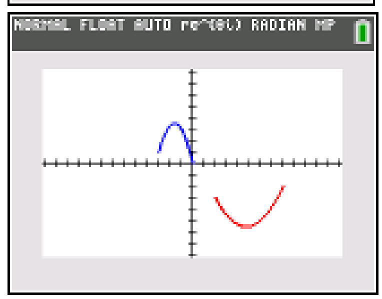












Problem 2. (a) - (d) (i)

Teacher Note: Before studying precalculus, all students should develop proficiency skills in topics typically found in the Algebra 1-Geometry-Algebra 2 (AGA) content sequence. The sum, difference, product, and quotient of functions are proficiency skills.

Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x)=2^x, \quad g(x)=3^x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What do we notice about the graphs of $(f+g)(x), (f-g)(x),$ *and* $(f\cdot g)(x)$ *as* x *decreases without bound?*

Possible Answers: The graphs of $(f+g)(x), (f-g)(x),$ and $(f\cdot g)(x)$ of $(f\cdot g)(x)$ approach zero as x decreases without bound. We could write these as a limits:
$\lim_{x\to-\infty}((f+g)(x))=0,\ \lim_{x\to-\infty}((f-g)(x))=0,\quad \lim_{x\to-\infty}(f\cdot g)(x)=0.$

Class Discussion:

What do we notice about the graph of $\left(\frac{f}{g}\right)(x)=\left(\frac{2}{3}\right)^x$ *as* x *increases without bound?*

Possible Answers: The graph of $\left(\frac{f}{g}\right)(x)$ approaches zero as x increases without bound. We would write this as $\lim_{x\to-\infty}\left(\frac{f}{g}\right)(x)=0.$

Teacher Note: Revisit the Class Discussion questions above using the TI-84.

Problem 2. (a) - (d) (ii)

Find (a) $f+g,$ (b) $f-g,$ (c) $fg,$ and (d) $f/g,$ and state their domains.
$f(x)=\log x,\quad g(x)=\ln x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What do we notice about the graph of $(f-g)(x)=\log x-\ln x$ *when* x *is between 0 and 1?*

Possible Answers: The difference would be positive.

Class Discussion:

Why do we have to exclude $x=1$ *from the domain of the function* $\left(\frac{f}{g}\right)(x)=\frac{\log x}{\ln x}$? *Why does the graph appear to be horizontal?*

Possible Answers: When $x=1$, the denominator would be zero. For $\left(\frac{f}{g}\right)(x)=\frac{\log x}{\ln x}$ we use the Change of Base rule and $\left(\frac{f}{g}\right)(x)=\log e$ which is a constant.

Teacher Note: Revisit the Class Discussion questions above using the TI-84.

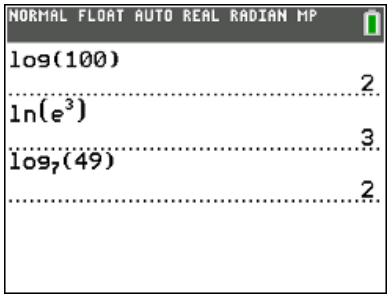
Using the TI-84 for Problem 2. (a) - (d) (ii)

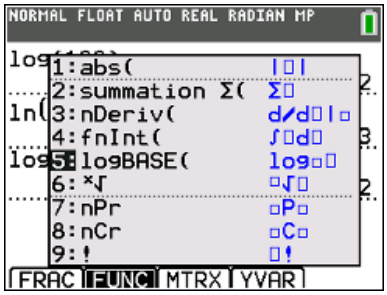
Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x)=\log x, \quad g(x)=\ln x$

Technology Tip: On the keyboard, there is [log] which is a common log and [ln] which is a natural log. To evaluate logarithms of other bases, select [math], scroll down to logBASE(.

As an alternate method, select f2 which is [alpha] [window].

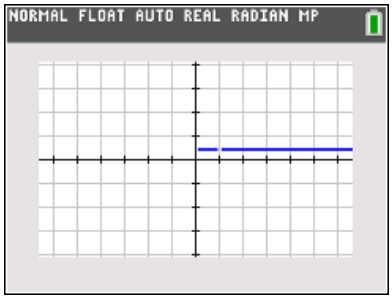




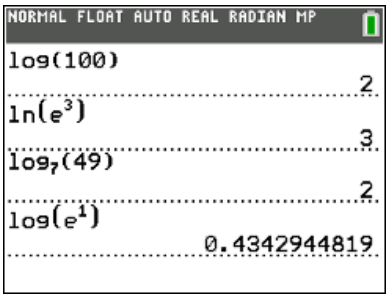
Graph Y1 = log(x) / ln(x). Use Zoom Decimal.

In the video, the gridlines are visible.

Technology Tip: To show the gridlines, select format which is [2nd] [zoom].



Use Trace to verify the hole at $x=1$. Notice that the y-values are constant. That constant value is $\log(e)=0.43429$ which is a little less than $\frac{1}{2}$.

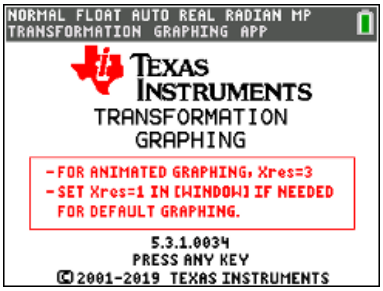


Using the Transformation App on the TI-84

Here is a Youtube Video in the Texas Instruments Education Channel on using the Transformation App.

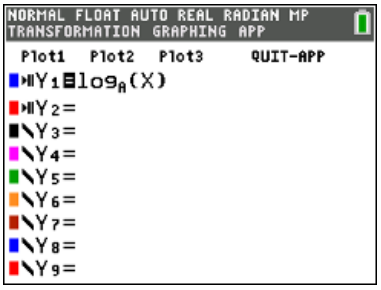
https://www.youtube.com/watch?v=WoBfrTDx2LQ

Technology Tip: Select [apps] then select Transfrm.

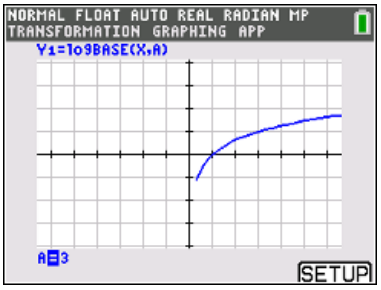


Select [y=] and notice that Y_1 and Y_2 have different symbols and can both be used for transformations.

Select [y=] and type $y1 = \log_A(x)$. A, B, C, and D are constants that can be used as parameters.



Graph using Zoom Decimal. Use the left and right arrows to change the base.



Class Discussion:

What point do all of the graphs, log functions with different bases, go through?

Possible Answers:

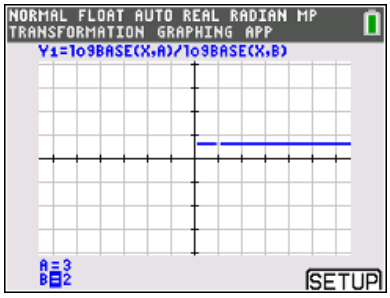
The point (0,1).

Class Discussion:

As you change the value of a, does every graph look like a logarithmic function?

Possible Answers: Every graph of a logarithmic function is a vertical dilation of another logarithmic function. Every logarithmic function is a multiple of another logarithmic function.

Graph of $y = \log_A(x) / \log_B(x)$. Graph using Zoom Decimal.



Class Discussion:

What do we notice about the graph as the values of A and B change?

Possible Answers: The graph is a constant function for $x > 0$ and $x \neq 1$.

Note: *Problems 2. (a) – (d) (iii) and 2. (a) – (d) (iv) are not discussed in the video.*

(iii) Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x) = x^3 + 2x^2, \quad g(x) = -2x^2 - 1$

(iv)) Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x) = \sqrt{4-x}, \quad g(x) = |x+3|$

Sample Solution:

Refer to the Teacher Solutions Document for the full solutions to these problems.

Problem 3. (a) - (d) (i)

Find (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$, and state their domains.

$f(x) = 2^x, \quad g(x) = 3x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What is happening with the graph of $(f \circ f)(x)$ *as* x *decreases without bound?*

Possible Answers: As x decreases without bound, $(f \circ f)(x)$ approaches 1.

Teacher Note: Revisit the Class Discussion questions above using the TI-84.

Problem 3. (a) - (d) (ii)

Find (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$, and state their domains.

$f(x) = 2^x, \quad g(x) = \log x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

(a) $f(x) = 2^x, g(x) = \log x,$ and $(f \circ g)(x) = 2^{\log x}$

Class Discussion:

What is happening with the graph of $(f \circ g)(x) = 2^{\log x}$ *as x approaches 0 from the right?*

Does the graph of $(f \circ g)(x)$ *have a hole at* $x = 0$?

Possible Answers: As x approaches 0 from the right, $(f \circ g)(x) = 2^{\log x}$ approaches 0.

$\lim_{x \to 0^+} (f \circ g)(x) = 0$

The graph of $(f \circ g)(x)$ does have a hole at $x = 0$.

(d) $f(x) = 2^x, g(x) = \log x,$ and $(g \circ g)(x) = \log(\log x)$

Class Discussion:

What is happening with the graph of $(g \circ g)(x)$ *as x approaches 1 from the right?*

Does the graph of $(g \circ g)(x)$ *have a vertical asymptote at* $x = 1$?

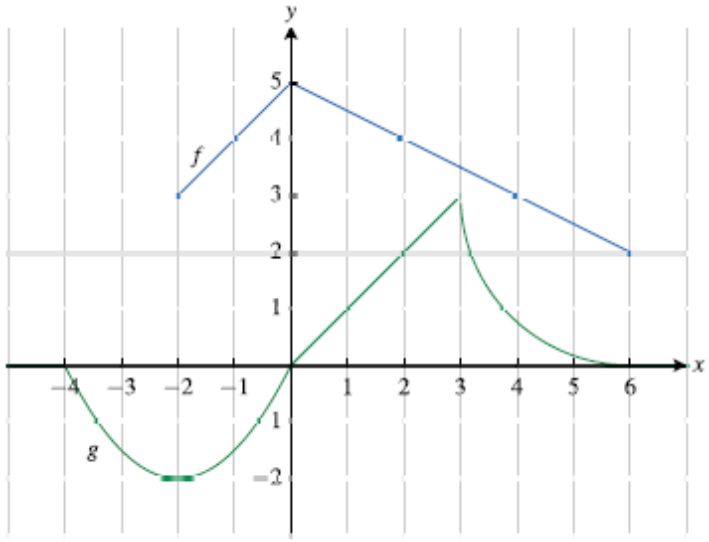
Possible Answers: The graph of $(g \circ g)(x) = \log(\log x)$ approaches negative infinity as x approaches 1 from the right. The graph has a vertical asymptote $x = 1$.

$\lim_{x \to 1^+} (g \circ g)(x) = -\infty$

Teacher Note: Revisit the Class Discussion questions above using the TI-84.

Note: *The following problems, 4 and 5, are not discussed in the video.*

Problem 4. (a) – (i)
Use the graphs of f and g to evaluate each expression or explain why it is undefined.



(a) $f\left(g\left(2\right)\right)$ (b) $g\left(f\left(4\right)\right)$ (c) $\left(f\circ g\right)\left(-2\right)$

(d) $\left(g\circ f\right)\left(6\right)$ (e) $\left(g\circ g\right)\left(-2\right)$ (f) $\left(f\circ f\right)\left(0\right)$

(g) $\left(g\circ f\right)\left(5\right)$ (h) $\left(f\circ g\circ f\right)\left(4\right)$ (i) $\left(g\circ f\circ g\right)\left(-2\right)$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (a) – (d)

Problem 5. (a)
Find a formula for the inverse of the function.

$f\left(x\right)=1+\sqrt{3+7x}$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (b)
Find a formula for the inverse of the function.

$f\left(x\right)=\dfrac{4x-1}{2x+3}$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (c)

Find a formula for the inverse of the function.

$f(x) = \sqrt{1-x^2},\ 0 \le x \le 1$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (d)

Find a formula for the inverse of the function.

$f(x) = 3 + \log_2 x, \qquad x > 0$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to verify transformations.
- The calculator application can be used to evaluate compositions.
- The graphing application can be used to explore the graphs of compositions.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI