Topic Areas

- Function Families
- "Stretch and Shift" Parameters
- Parametric Variation

Objectives

- To understand the effect of varying parameters in a class of functions
- To see individual functions as members of families

Families of Functions

Introduction

In this Lesson Concept we use whole-class participation in the Activity Center to cultivate students' understanding of how the graph of a basic function can vary through the change of a parameter. (For example, we explore how a linear function y = mx + b can vary through the change of one or both of the parameters m and b). The classroom-group environment can provide a motivating introduction to this concept, as the students' work can embody the "range of motion" of a base function, and this experience can offer a mental framework for more detailed investigation of particular parameters and values.

Fundamental concepts such as slope and parallelism in linear functions, or amplitude and phase shift in periodic functions can make their first appearances in the classroom through this kind of activity, since students will seek words to describe patterns that they see emerging in the public display space. As you lead the discussion, be ready for opportunities to push student thinking and build mathematical vocabulary.

Students often express surprise when they see the aggregated function families that they create in activities such as the ones described in this lesson concept. It appears that seeing the pattern of the function family emerge through a large number of instances that they themselves create triggers a new perspective on the concepts involved. The rich classroom dialogue that often ensues also suggests that this kind of activity can lead to increased engagement in learning difficult concepts.

Prerequisites and Materials

Prerequisites: Some introductory or working knowledge of the base functions whose families you will be exploring.

Materials: The Activity Settings files, `FunctionFamilies1.act`, `FunctionFamilies2.act`, `FunctionFamilies3.act`, `FunctionFamilies3a.act`, and `FunctionFamilies4.act` located on the CD.

Setting Up the Activity

From the TI-Navigator™ teacher computer home screen, open the Activity Center, 🌐. Then choose **File>Load>Load Activity Settings** to load the activity settings file, `FunctionFamilies1.act`.

In the example sequences that follow, we use linear functions and trigonometric functions, but any base function can be used.

Exploring Y-Intercept and Parallel Lines

The activity settings you loaded allow students to submit one equation, beginning from a blank, with the option to preview the graph of the equation and to resubmit.

Ask students to count off 1-20. (You may have multiple representatives of each number). Tell them that they will be submitting an equation of the form Y=X+([countoff number] - 10).

Start the activity, ▶, and watch as the submissions come in. Then pause the activity, ❚❚, and ask the students to describe the result. How are the graphs alike? How are they different? How are they related to each other? How could you turn one of them into any of the others? Take some time to let them come to a consensus description of what they see and what are the "family resemblances" between these functions.

Stop the activity, ■. Tell your students that their next task will be to submit an equation of the form Y=X+((½ * [countoff number]) - 5)). Ask them to predict what will happen, and how it will compare to what they have done previously. What will be different?

To get a fresh graph screen, clear the activity data by choosing **Edit>Clear Activity Data**.

Start the activity, ▶, and watch the result. Pause the activity, ❚❚, eliciting descriptions and watching for thinking that tends toward concepts of parallelism, vertical shifting, and so forth. Stop the activity, ■.

Exploring Slope

For this activity, load the activity settings file `FunctionFamilies2.act` by choosing **File>Load>Load Activity Settings**. Have your students count off

1-10 and tell them that they will submit one equation of the form, Y=[countoff number]*X. Ask them what the expect to see.

Start the activity, ▶, and let the equations flow in. Pause the activity, �II, and ask students to describe the result. How are the different functions in this picture alike? How do they differ? Why did the group's equations fill in only the part of the screen that it did? How might they change the prompt to fill in another part of the screen?

Stop the activity, ■. Explain that you are going to start the activity again without clearing activity data, giving them a chance to add lines of their own invention to the picture. Tell them to try other values of m, in Y=mX, that might fill in other parts of the screen. Ask them to jot down notes of their findings as they work. Tell the students you will give them 5 minutes to experiment, during which time you'd like them to come up with a theory about how the parameter m works. Start the activity, ▶, and let them work with the problem.

At the end of 5 minutes or so, pause the activity, II. Ask your students to turn to a neighbor and take the next 2 minutes sharing their findings. Let them know that at the end of that time they each should be ready to report one thing that their partner has discovered about m. When two minutes are up, ask for volunteers that are interested in sharing their partner's insight. Make a list of these insights on the board.

After you have a reasonable list, return to the graph. Select the equations from the experimental run and delete them from the display, so that you are left with only the initial set of 10 equations Y=X, Y=2X, …, Y=10X. Ask the students to return to their partner-pairs and formulate a rule for the class that uses the countoff number (like the rule "Y=[countoff number]*X") and that will systematically fill in another part of the screen. Give them 3 minutes.

Collect proposed prompts and have the class choose one to execute. Start the activity again, ▶, and collect the results. Repeat this (3 times total) until the class has filled out the whole "star" of linear graphs through the origin!

Stretch and Shift Parameters for a Trigonometric Function

Use this activity to introduce the concepts of amplitude. Load the activity settings file `FunctionFamilies3.act` by choosing **File>Load>Load Activity Settings**. Have your students count off 1-20.

If you have taught generalized sine and cosine functions, remind the class of these function families and the parameters you have introduced (Amplitude, Frequency, Phase Shift, Vertical Shift).

In each of the explorations below, ask your students to predict what the group graph will look like. This "what if" thinking is especially valuable: be sure to

give it a great deal of "air time" both in student thinking and in classroom discussion before you start the aggregation activity. Ask students to justify their hypotheses, and they will begin inventing or using mathematical language to describe their ideas. For example, many students will stretch the term "parallel" to describe their ideas about a family of vertically shifted sine functions.

First, explore amplitude. Tell your students they will contribute Y= (½*[countoff number])*sin(X) and ask them to predict what they will see. If they can visualize the result, ask them if they think the functions will intersect, and if so, where. Start the activity, ▶, and watch the results come in. Stop the activity, ■, and discuss.

Next, explore vertical shift. Tell your students they will contribute Y=sin(X)+[countoff number - 10] and ask them to predict what they will see. Ask them if they think the functions will intersect now, and if so, where. Start the activity, ▶, and watch the results come in. Stop the activity, ■, and discuss.

At this point, you may want to choose **File>Load>Load Activity Settings** to load the file `FunctionFamilies3a.act`, which has window settings better suited to exploring frequency and phase shift.

Now, explore frequency. Tell your students they will contribute Y=sin(½*[countoff number]X) and ask for predictions. Start the activity, ▶, and watch the results come in. Stop the activity, ■, and discuss.

Finally, phase shift. Prompt students for Y=sin(X + PI/6*[countoff number]) and ask for predictions. Start the activity, ▶, and watch the results come in. Stop the activity, ■, and discuss. As you walk through the results, make sure you point out the fact that the shifts are to the left.

Varying Both Slope and Y-Intercept in Linear Functions

In this more highly-structured aggregation activity, we use two countoff schemes. First, separate the class into groups of four or five, and number these groups. Then, have your students countoff within their groups. As a result, each student will have both a group number (between 1 and about 10) and a within-group countoff number (between 1 and 5).

Give the students a prompt in which you vary both the slope and y-intercept; this will lead to a complex figure in the shared space. Well-chosen graph window settings can help to focus the visual field and make the variation more salient. For the prompts below, you can use the file `FunctionFamilies4.act` by choosing **File>Load>Load Activity Settings**.

Ask your students to follow the prompt:

Contribute a line whose slope is equal to one-half your <u>group</u>
number and whose y-intercept is equal to your <u>countoff</u> number.

This will lead to a multi-branched figure where all the "1s" across groups have
y-intercept of 1. All the "2s" have a y-intercept of 2. At each "branch" of the
figure, the representative from group 1 will contribute the line with slope of .5;
the representative from group 2 will have the line with slope of 1, and so on.

If you do this activity with students contributing all at the same time, the graph
screen will be very chaotic. Since students can resubmit, however, you can
try the following.

After the whole class has submitted, Pause the activity, ‖. Then select an
item in the Equation Gallery and choose **Edit>Select All**. Choose
Edit>Delete to clear all of the class's contributions from the public display.

Now, tell group 1 (or, alternatively, all the "1s" across groups) that you want
them (and only them) to submit. Resume the activity, ‖, and let them
contribute their equations. If you've chosen group 1, then you will see a
family of parallel lines of slope .5 at different y-intercepts. If you've chosen
"all the 1s" then you'll see a family of lines with different slopes all with a y-
intercept of 1.

Pause the activity, ‖, for discussion, after each set of students submits.

Here are some more challenging prompts:

Contribute a line that has a y-intercept equal to your <u>countoff</u>
number and a slope of <u>group number</u> minus 5.

or

Contribute a line that has a y-intercept equal to your <u>countoff</u>
number and that goes through the point (6, [<u>group number</u> - 5]).

In both of these, lines of slope zero will be situated in families of functions
with changing slopes – a valuable and thought-provoking stimulus for
discussion. And in the second prompt, students will have to calculate slopes
based on two points.