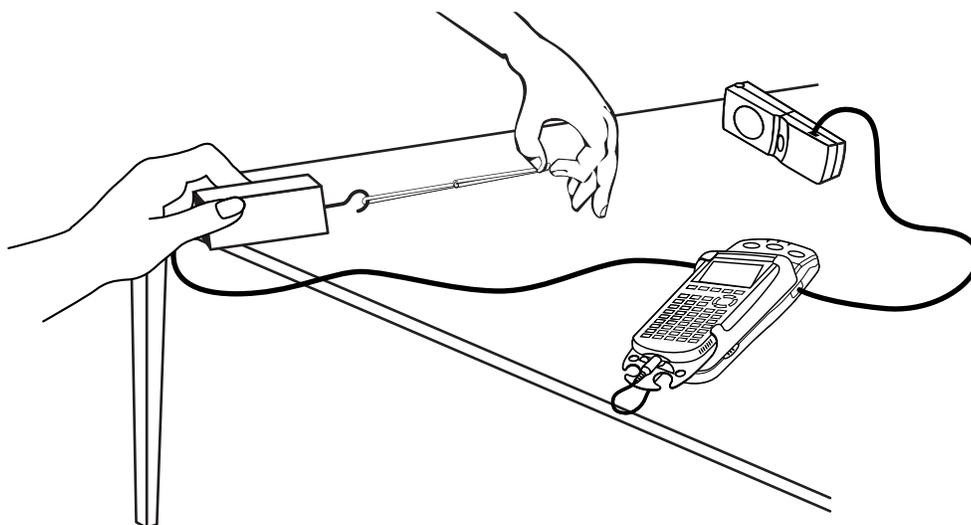


Stretch it to the Limit: The Linear Force Relation for a Rubber Band

When a force is applied to a rubber band, it stretches a certain amount. Exactly how much it stretches depends on the applied force and the characteristics of the rubber band. In general, the more force that is applied, the more it stretches. For rubber bands that are not stretched too much, if you double the force applied, it turns out that the stretch doubles as well. Two quantities, x and y , that change in this way are said to be *proportional*. x and y are related by the constant K in the equation

$$y = Kx$$

In this activity you will use a Force Sensor and a Motion Detector to investigate the relationship between the force applied to a rubber band and the distance that the rubber band stretches. To measure how much a rubber band has stretched, we will use the stretched length of the band minus the relaxed length of the band.



OBJECTIVES

- Record force versus stretch data for a rubber band.
- Model force versus stretch data with a proportional relationship.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
data-collection interface
EasyData application

Motion Detector
Vernier Force Sensor
1 long rubber band (or 3 smaller bands)

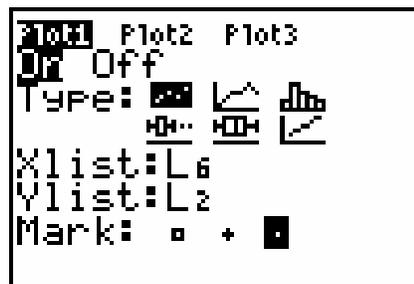
PROCEDURE

1. Prepare the Force Sensor and Motion Detector for data collection.
 - a. Turn on your calculator and connect it to the data-collection interface.
 - b. Set the Force Sensor range switch to 10 N.
 - c. Connect the Force Sensor to CH 1 of the data-collection interface.
 - d. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
 - e. Connect the Motion Detector to the DIG/SONIC port of the data-collection interface. (If there is more than one DIG/SONIC port, connect it to the first one.)
- 
- The diagram shows four icons: a calculator, a data-collection interface (represented by a circle with three vertical bars), a person walking, and a basketball.
2. For this activity, you will stretch and relax a rubber band with your hand. A Motion Detector will record the amount the rubber band is stretched while a Force Sensor measures the force of your pull. Be sure not to get closer than 50 cm to the Motion Detector during data collection. The rubber band being used should be flexible enough to stretch at least 15 cm. Several smaller rubber bands linked together also work well for this activity.
 3. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **File** from the Main screen, and then select **New** to reset the application.
 4. To account for the relaxed length of the rubber band, you need to zero both sensors. To do this you have to hold your hands in the position they will be in when data collection begins. You may need to have another person operate the calculator during these steps. Keep in mind that your hand must never get closer than 50 cm to the Motion Detector during data collection, so allow enough room to stretch the rubber band during the activity.
 - a. Hold the Force Sensor with your right hand as shown in the set-up drawing, or fix it to a ring stand. The Force Sensor should not move during data collection. With your left hand hold the rubber band so that it is just barely taut. Your left hand must be directly between the Force Sensor and the Motion Detector. Keep your hands in this position for now.
 - b. Select **Setup** from the Main screen, and then select **Zero...**
 - c. Keep your left hand still, and select **Zero** to zero the sensors. This will measure all future distance measurements from the position of your left hand, and all force measurements from the current force, which will be near zero. Do not move your hand from this position until you begin taking data in the next step.
 5. You are now ready to collect force versus stretch distance data.
 - a. Select **Start** to begin data collection.
 - b. Gently stretch and relax the rubber band, moving your left hand along a line between the Force Sensor and the Motion Detector. Do not let your hand get any closer than 50 cm from the Motion Detector.
 6. After data collection is complete, a graph of the force as a function of time will be displayed. To see the distance versus time data, select **Plots**, and then select **DIG1:Dist(m) vs Time**.
- If you want to repeat data collection, select **Main** and repeat Step 5.
7. If you are satisfied with the data, select **Main**. Exit EasyData by selecting **Quit** from the Main screen and then selecting **OK**.

8. The graphs shown by EasyData are functions of time. That's not quite what you need, however, since you want force versus distance. To display that graph, you'll need to change your graph settings.

The distance data reflects the additional stretch of the rubber band. However, because the Motion Detector measures distances as increasing *away* from the detector, you will need to change the sign of the distance data stored in L6.

- Press $(-)$ 2^{nd} [L6] (\rightarrow) 2^{nd} [L6] to enter the expression $-L6 \rightarrow L6$.
 - Press (ENTER) to complete the operation.
9. Now you can plot the force used to stretch the rubber band versus the distance the band was stretched.
- Press 2^{nd} [STAT PLOT] and press (ENTER) to select Plot 1.
 - Change the Plot1 settings to match the screen shown here. Press (ENTER) to select any of the settings you change.
 - Press (ZOOM) and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the x and y ranges set to fill the screen with data.



ANALYSIS

As mentioned earlier, an approximate model for force and stretch holds that they are proportional. To test this, try plotting the line of proportionality $y = Kx$ with the data.

- First, enter the equation.
 - Press $(Y=)$.
 - Press (CLEAR) to remove any existing equation.
 - Enter $K*x$ in the Y_1 line.
 - Press 2^{nd} [QUIT] to return to the home screen.
- Next set a value for the parameter K , and then look at the resulting graph. To begin with use a slope of one, so $K = 1$.
 - Enter a value for the parameter K . Press (STO) K (ENTER) to store the value in the variable K .
 - Press (GRAPH) to see your data with the model graph superimposed.
 - Press 2^{nd} [QUIT] to return to the home screen.

To obtain a good fit, you will need to try several values for K . Repeat Steps a-c to store different values to the parameter K . Experiment until you find one that provides a good fit for the data.

\Rightarrow Answer Question 1 on the *Data Collection and Analysis* sheet.

Activity 9

3. The K value may also be determined algebraically by substituting the x and y values of one point into the variation equation, $y = Kx$.
- Press **GRAPH** **TRACE**.
 - Use the cursor keys to move the cursor to a point near the middle of the graph. If the cursor is on the line rather than the data points, press **▲** to shift the focus to the data points. Round the x and y values of this point to the nearest hundredth and record them in the Data Table on the Data Collection and Analysis sheet. Use these values to solve for K , and write this value in the Data Table.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

4. In addition to your two estimates of the parameter K , you can use the calculator to automatically determine the slope and intercept of a straight line. This line can be fit to the data using the steps below.

- Press **STAT** and use the cursor keys to highlight **CALC**.
- Press the number adjacent to **LinReg(ax+b)** to copy the command to the home screen.
- Press **2nd** **[L6]** **↵** **2nd** **[L2]** **↵** to enter the lists containing your data.
- Press **VAR** and use the cursor keys to highlight **Y-VARS**.
- Select **Function** by pressing **ENTER**.
- Press **ENTER** to copy Y_1 to the expression.

On the home screen, you will now see the entry **LinReg(ax+b) L6, L2, Y1**. This command will perform a linear regression with L_6 as the x -values and L_2 as the y -values. The resulting regression line will be stored in equation variable Y_1 .

- Press **ENTER** to perform the linear regression.

⇒ Record the regression equation with its parameters in Question 3 on the *Data Collection and Analysis* sheet.

- Press **GRAPH** to see the graph.

⇒ Answer Questions 4-7 on the *Data Collection and Analysis* sheet.

EXTENSION

As you were recording motion and force data for the stretched and relaxed rubber band, time values were being recorded simultaneously by the data-collection interface. In fact, this data was shown to you immediately after data collection.

Restart EasyData and inspect your force versus time graph. This plot shows how force values vary with time while you were pulling on the rubber band.

Make a prediction about what a plot of stretch distance versus time would look like. Check your prediction by displaying the distance graph. How is the stretch distance versus time plot related to the force versus time plot? Be specific.

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

<i>x</i>	
<i>y</i>	
<i>K</i>	

QUESTIONS

- Record the value of K that fits the data best.

$$K = \underline{\hspace{2cm}}$$

- Are the K values in Question 1 and the Data Table consistent? What might cause them to be slightly different?

- Record the regression equation with its parameters from Step 4.

- How does the slope in the linear regression compare with the K values in Question 1 and the Data Table? What should be the value of the y -intercept from the regression? Explain.

