EXPLORATIONS

Activity 8

Storefront Signs

Teacher Notes

Concept

- Patterns and Functions

Skill

- Finding Areas
- Exploring a quadratic function

Applicable Calculator Functions

- [x^2], [$\sqrt{\ }$], [STO▶], [RCL]

Materials

- Student Activity Sheets (page 71)
- TI-30X IIS/TI-34 II calculator
- Geoboards and rubber bands (optional)

Objective

- Students will compare the areas and patterns of squares within a square

Prerequisites

Prior to this activity, students should have experience finding the areas of triangles and squares. They should be familiar with the [x^2] and [$\sqrt{\ }$] keys on the calculator.

Problem

Lee Lin convinced her boss to allow her to put a sign to advertise her club's upcoming car wash in the window of the store where she worked after school. The storefront display window was 60″ x 60″ with small hooks every 6″ around the border. The boss told Lee Lin there were three rules she had to follow:

1. The sign must not cover the entire display window.

2. The sign must be square.

3. Each corner of the sign must be attached to one of the hooks around the border of the display window.

Students will determine the area of the largest possible sign and the smallest possible sign.

Activity

Have students work in pairs to draw all the possible squares (or construct models of them using geo boards) and find the area of each. Most students will find the area of the four triangular regions outside the new square and subtract the total of those areas from the area of the large square. If students know the Pythagorean Theorem they may use it to find the side lengths of the new squares and then square that result for the corresponding area.

Completed Table

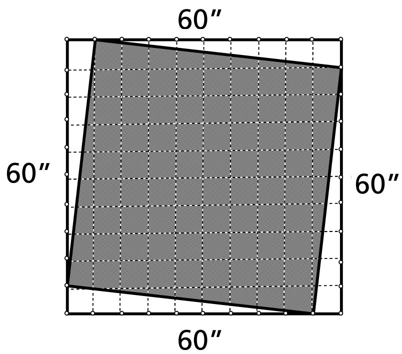
Distance of Hooks from Corners of Display Window	Area of Sign
6″	2952 in^2
12″	2448 in^2
18″	2088 in^2
24″	1872 in^2
30″	1800 in^2
36″	1872 in^2
42″	2088 in^2
48″	2448 in^2
54″	2952 in^2

Among the patterns that students may find from their tables are the following:

- There are pairs of squares with the same area (except for the one in the middle).
- The areas of the new squares get smaller and then get larger.
- The differences in the areas of successive squares diminish and then grow again.
- The differences in the areas of successive squares diminish and grow by a pattern (each difference is 144 more or less than the previous difference).

The area of the largest sign possible for this problem is 2952 inches.

Sample Sketch and Explanation



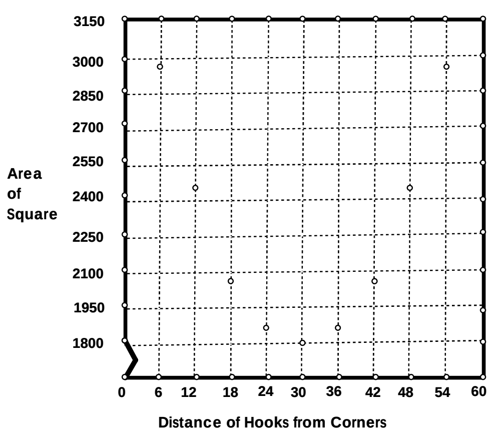
The area of each of the four small triangles surrounding the square is (6 x 54)/2 = 162. The total area of the four triangles is 4 x 162 = 648, so the area of the square is the area of the window (3600) – the total area of the four surrounding triangles (648) or 3600 – 648 = 2952

or

To find the length of one side of the square, consider it the hypotenuse of a right triangle with legs 6 and 54, so side = $\sqrt{6^2 + 54^2}$, and the area of the square = side^2 or $(6^2 + 54^2)$.

The graph of the results is a parabola.

Sample Graph and Explanation



The two biggest squares can be made using the hooks closest to the corners of the windows (6″ or 54″ away). The squares get smaller as you move farther from the corners until you get halfway (30″). So the coordinates form a symmetrical u-shaped curve with the largest areas at the outside and the smallest area in the middle.

Noticing the parabolic shape and understanding where it comes from is the heart of this activity. You may want students to make transparencies and share their explanations. Understanding the kinds of situations that create a quadratic function is an important step in developing algebraic thinking.

A valuable topic for discussion is whether the coordinates should be connected. Students at these grade levels often have dealt only with graphs for which the coordinates have been connected. This activity provides a nice context for introducing discrete versus continuous graphs. Students can easily conclude that the values between the coordinates for this situation have no meaning since the boss required hooks at the corners; therefore, this graph is discrete. Solicit from students examples of situations for which joining the coordinates for the graph of such an activity with line segments would be appropriate and explain that the graphs are called continuous.

For the 100cm x 100cm window, the same pattern of results would occur. A student might explain it in the following way:

> Following the rules, the area of the largest sign would be 8200 cm^2 and the area of the smallest sign would be 5000 cm^2 because the pattern would work the same way; the largest possible square would be formed by moving just one hook from each corner of the window and the smallest square would be formed by connecting the middle hook on each side. (Also, its area would be exactly half that of the original square.)

Wrap-Up

Have students share their paragraphs about what they found and learned. Be sure these questions are answered:

- ♦ *What patterns did you find in this activity? (See possible answers discussed earlier in these teacher notes.)*
- ♦ *What do the graphs of functions with rules which include squaring a quantity look like?*

  Some possible answers:

  - – The graph is shaped like a U.
  - – The graph is a u-shape, called a parabola.
  - – The points form a symmetrical curve that goes down and then back up.

Assessment

Use a related problem to have students repeat this process.

Example:

If the boss said Lee Lin's sign must be in the shape of a triangle and the other two rules remained the same, what would be the largest sign she could make? The smallest sign? Show your work and explain your thinking.

Extension

If your students have studied the Pythagorean Theorem, you can give them the following extension problem:

- ♦ If Lee Lin had fourteen strings of blinking lights, each fifteen feet long, and she followed her boss's rules, what is the largest size sign that could be bordered completely by lights?

Student Activity

Name ______________________

Date ______________________

Activity 8

Storefront Signs

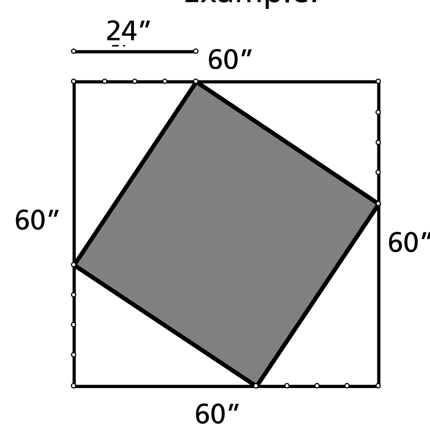
Objective: *You will explore the areas and patterns of squares within a square*

Problem: *Lee Lin convinced her boss to allow her to put a sign to advertise her club's upcoming car wash in the window of the store where she worked after school. The storefront display window was 60" x 60" with small hooks every 6" around the border. The boss told Lee Lin there were three rules she had to follow.*

Rules:

1. The sign must not cover the entire display window.
2. The sign must be square.
3. Each corner of the sign must be attached to one of the hooks around the border of the display window.

Example:

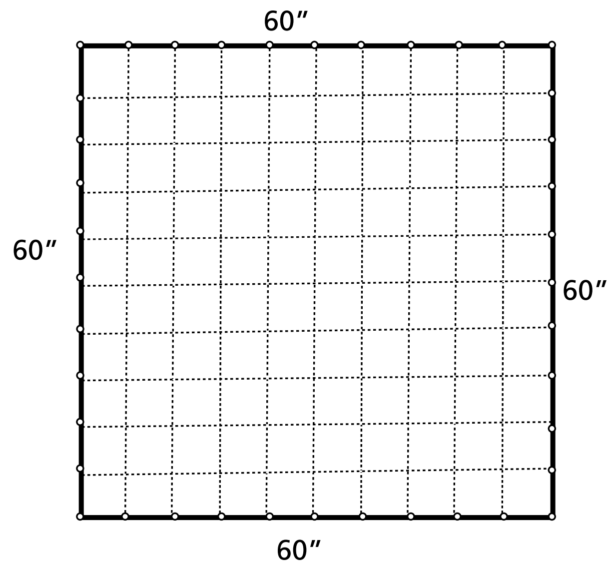


1. Complete the table to show the areas of all the signs Lee Lin could use.

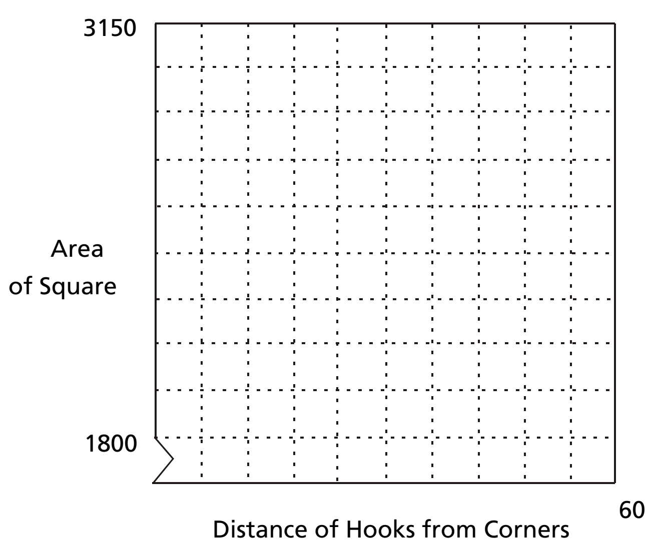
Distance of Hooks from Corners of Display Window	Area of Sign
6″	
12″	
18″	
24″	
30″	
36″	
42″	
48″	
54″	

2. What patterns did you find in your results?

3. Draw a sketch of the sign with the largest possible area and explain how you found its area.



4. Graph your results below.



5. Explain why the graph looks the way it does.

6. If the window was 100cm x 100cm with hooks every 10cm, what would be the area of the largest possible sign? What would be the area of the smallest possible sign? Explain how you know your answers are correct.

7. Write two paragraphs about this activity below.

a. What we found:

b. What I learned:

Storefront Signs
Keystrokes for the TI-34 II

To find the largest sign size, find the area of one of the triangles in the window not covered, then multiply that by 4, and subtract that amount from the area of the square window.

Example: $s^2 - 4 \cdot \frac{1}{2}bh$ or $60^2 - 4\left(\frac{1}{2} \cdot 54 \cdot 6\right)$ or $3600 - 4\left(\frac{1}{2} \cdot 54 \cdot 6\right)$ or $3600 - 648$

PRESS	DISPLAY
4 [×] .5 [×] 54 [×] 6 [STO▸]	A B C D E (use arrow key, if needed to underline A)
[ENTER]	4 X .5 X 54 X 6 → A 648
60 [x^2] [ENTER]	60^2 3600
[−]	Ans –
[2nd] [RCL]	A B C D E 648
[ENTER]	Ans – 648
[ENTER]	Ans – 648 2952

These are possible keystrokes for the largest sign size if you are familiar with the Pythagorean Theorem. You can find the hypotenuse of one of the triangles not covered and use that for the side of the square sign. In that case, you could find the hypotenuse by $c^2 = a^2 + b^2$ and then use area of the square A = s^2. s = c when $c = \sqrt{a^2 + b^2}$. The area of the square sign would be c^2.

PRESS	DISPLAY
54 [x^2]	54^2
[+] 6 [x^2]	$54^2 + 6^2$
[ENTER]	$54^2 + 6^2$ 2952
[2nd] [$\sqrt{\ }$]	$\sqrt{\ }($
2952 [ENTER]	$\sqrt{\ }(2952$ 54.33231083
[x^2]	Ans^2
[ENTER]	Ans^2 2952

Storefront Signs
Keystrokes for the TI-30X IIS

To find the largest sign size, find the area of one of the triangles in the window not covered, then multiply that area by 4, and subtract that product from the area of the square window.

Example: $s^2 - 4 \cdot \frac{1}{2}bh$ or $60^2 - 4\left(\frac{1}{2} \cdot 54 \cdot 6\right)$ or $3600 - 4\left(\frac{1}{2} \cdot 54 \cdot 6\right)$ or $3600 - 648$

PRESS	DISPLAY
4 [×] .5 [×] 54 [×] 6 [STO▸]	A B C D E (use arrow key, if needed, to underline A)
[ENTER]	4 ∗.5 ∗ 54 ∗ 6 → A 648
60 [x^2] [ENTER]	60^2 3600
[−]	Ans –
[2nd] [RCL] [ENTER]	A B C D E 648
[ENTER]	Ans – 648
[ENTER]	Ans – 648 2952

These are possible keystrokes for largest sign size if you are familiar with the Pythagorean Theorem. You can find the hypotenuse of one of the triangles not covered and use that for the side of the square sign. In that case, you can find the hypotenuse by $c^2 = a^2 + b^2$ and then use area of the square $A = s^2$. $s = c$ when $c = \sqrt{a^2 + b^2}$. The area of the square sign would be c^2.

PRESS	DISPLAY
54 [x^2]	54^2
[+] 6 [x^2]	$54^2 + 6^2$
[ENTER]	$54^2 + 6^2$ 2952
[2nd] [√]	√ (
2952 [ENTER]	√ (2952 54.33231083
[x^2]	Ans ^{2}
[ENTER]	Ans ^{2} 2952