E X P L O R A T I O N S

A c t i v i t y   3

A Prime Investigation with 7, 11, and 13

In this activity you will:

♦ investigate the divisibility of 7, 11, and 13

♦ discover why special six-digit numbers are divisible by 7, 11, and 13

You will need to know this math vocabulary:

♦ divisible

♦ factors

♦ distributive property

Introduction

Mathematics can sometimes seem mysterious, sort of like the magic tricks a magician performs. Does the magician know why the tricks work? There are underlying reasons in the mystery of mathematics that explain why it works.

Problem

Choose any three-digit number. Then repeat those three digits in the same order to make a six-digit number, for example 459,459. (Do not use this number.) Write your original number down. Make sure no one in your group has the same number.

Your number is _____.

Activity

Part 1 - The Investigation

1. Find out if your number is **divisible** by 7. It is divisible by 7 if you divide it by 7 and get a remainder of 0. Use integer divide on the calculator to find out.
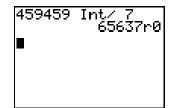   Press **459459** [2nd] [INT÷] **7** [ENTER].
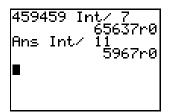
    Is your number divisible by 7?

```
459459 Int/ 7
          65637r0
■
```

2. Find out if your previous quotient is divisible by 11. Press [2nd] [INT÷] **11** [ENTER].

    Is your number divisible by 11?
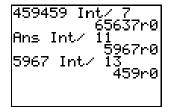
```
459459 Int/ 7
          65637r0
Ans Int/ 11
           5967r0
■
```

3. Integer divide the previous quotient by 13.

    Is your number divisible by 13? What do you notice about the new quotient?

```
459459 Int/ 7
          65637r0
Ans Int/ 11
           5967r0
5967 Int/ 13
            459r0
```

✎ Answer the questions on Part 1 of the Student Worksheet.

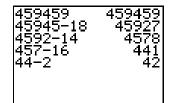### *Part 2 - Justify why these numbers are divisible by 7, 11, and 13*

1. Pick another three-digit number and enter it into your calculator. Repeat the same three digits to form a six-digit number. Test it for divisibility by 7, 11, and 13.

    ✎ Answer question 6 on the Student Worksheet.

2. Multiply 7 x 11 x 13.

3. Divide your number in #1 by the product in #2.

4. Multiply your three-digit number in **#1** by the product in **#2**. What answer did you get?

    ✎ Answer question 7 on the Student Worksheet.

5. Examine the screen shot at the right. Multiplying by 1000 is easy to do mentally because you just move the decimal point 3 places right. Multiplying by 1001 is also easy when you think of 1001 as 1000 + 1 and then multiply 459 x 1000 and add 459 x 1. This is known as the **distributive property.**

```
459*1000   459000
459*1001   459459
459(1000+1)
            459459
459*1000+459*1
            459459
```

    ✎ Answer questions 8 through 10 on the Student Worksheet.

Part 3 - Divisibility Tests

1. You are probably familiar with the divisibility tests for prime numbers 2, 3, and 5.

    ✎ Write the divisibility tests for 2, 3, and 5 in question 11 on the Student Worksheet.

2. There is also a divisibility test to tell if a number is divisible by 7, but it is somewhat tedious to use on large numbers. Here is how the test works:

    a. Take all but the last digit (units digit) and form a number.

    b. Subtract twice the units digit from the number you formed. Now you have a new number.

    c. Again, take all but the units digit and form a newer number.

    d. Subtract twice the units digit from this number.

    e. Continue this process until you are able to recognize whether the number is divisible by 7. See the screen shown at the right.

    ```
    459459        459459
    45945-18       45927
    4592-14         4578
    457-16           441
    44-2              42
    ```

    f. This divisibility test is more useful with three digit numbers. For example:

    $581 \Rightarrow$ 58 - 2 x 1 = 56 and since 56 is divisible by 7, then 581 is divisible by 7.

    Likewise:

    $635 \Rightarrow$ 63 - 2 x 5 = 53 and since 53 is not divisible by 7, then 635 is not divisible by 7.

3. There is a divisibility test for 11. For example, **6**5**6**3**7** is divisible by 11. To use the test, sum every other digit, then take the difference in the two sums: The sum of the digits in bold print is 6 + 6 + 7 = 19. The sum of the underlined digits is 5 + 3 = 8. Now take the positive difference in the two sums and see if the result is divisible by 11. Since 19 - 8 = 11 and 11 is divisible by 11, then 65,637 is divisible by 11.

    ✎ Answer questions 12 through 14 on the Student Worksheet.

Activity 3

A Prime Investigation with 7, 11, and 13

Part 1 - The Investigation

1. In step 1, was your number divisible by 7? _____

2. Was your number divisible by 11? _____

3. Was your number divisible by 13? _____

What did you notice about the new quotient?

_____

4. Would the order that you divided by 7, 11, or 13 affect your result? Try dividing your original number by these divisors in different orders. Write a sentence on your findings.

_____

_____

5. Did everyone in your group pick a number that was divisible by 7, 11, and 13?

_____

If so, discuss within your group why you think everyone's number was divisible by 7, 11, 13. Write a sentence or two explaining why you think these special six digit numbers are divisible by 7, 11, and 13.

_____

_____

_____

Part 2 - Justifying why numbers are divisible by 7, 11, and 13

6. Write a sentence explaining your findings from Part 2 #1 (page 26).

_____

_____

_____

7. Describe the relationship between the product in step 4 (page 26) and these special six-digit numbers.

_____

_____

_____

8. Rewrite your three-digit numbers as the product of the number and 1001. Then write it out using the distributive property, as shown on page 26.

_____

_____

9. Explain why you know 468,468 is divisible by 7, 11, and 13.

_____

_____

10. Explain how to mentally multiply 369 by 1001.

_____

_____

Part 3 - Divisibility Tests

11. Write the divisibility tests for 2, 3, and 5.

_____

_____

_____

12. Show how to use the divisibility test for 11 to see if your original six-digit number is divisible by 11.

_____

13. Is 852,345 divisible by 11? _____

Show how to use this divisibility test to examine 852,345 for divisibility by 11.

_____

_____

14. Number theorists have developed divisibility rules or tests for many different numbers. Can you write a divisibility test for a six-digit number to be divisible by 1001?

_____

_____

♦ Number sense

Activity 3

♦ TI-73 calculator

♦ Student Worksheets (page 28)

A Prime Investigation with 7, 11, and 13

Students will investigate uncommon divisibility tests. They will also discover an interesting characteristic of six-digit multiples of 1001. The distributive property will be applied to unravel the mystery.

Vocabulary

divisible	A number is divisible by another if, when dividing the remainder is zero.
factors	The numbers multiplied in a multiplication problem.
distributive property	For any real numbers a, b, and c, a(b+ c) = ab + ac.

Classroom Management

Students should understand the concept of divisibility and know the divisibility rules for 2, 3, and 5. This activity can be student- or teacher-directed. If student-directed, group students in teams of 3 or 4. Students should be instructed to work independently but could confer with their team when prompted by the activity. You should monitor and observe the student groups to make sure they understand the task. Make sure students initially understand what kind of number to start with. A wrap-up may include questioning students on why these six-digit numbers are multiples of 7, 11, and 13.

Activity

Part 1 - The Investigation (sample answers with some teacher tips)

1. Make sure the students understand the function and display of integer divide [INT÷]. If they have not been exposed to this function, explain using a simpler problem. Yes, their number should be divisible by 7.

2. Make sure the students understand the display of the screen shot in number 2. If they are not familiar with using previous answers, show some simpler examples. Yes, their number should be divisible by 11.

3. Yes, their number should be divisible by 13. The new quotient is the original three-digit number.

Part 2 - Justify why these number are divisible by 7, 11, and 13
(sample answers with some teacher tips)

1. Their six-digit number abc,abc should be divisible by 7, 11, and 13.

2. 7 x 11 x 13 = 1001

3. abc,abc ÷ 1001 = abc

4. abc x 1001 = abc,abc
   The original six-digit number - abc,abc. It is the same.

Part 3 - Divisibility Tests (sample answers with some teacher tips)

1. A number is divisible by 2 if it is even or if the ones digit is 0, 2, 4, 6, or 8.
   A number is divisible by 3 if the sum of the digits are divisible by 3.
   A number is divisible by 5 if the ones digit is a 0 or a 5.

2. This procedure is presented as a bit of trivia. It is especially useless with a calculator in hand.

Answers to Student Worksheet

1. Yes, their number should be divisible by 7.

2. Yes, their number should be divisible by 11.

3. Yes, their number should be divisible by 13. The new quotient is the original three-digit number.

4. No, the order you divide by 7, 11, 13 would not affect the result. It doesn't matter what order you divide in because you are dividing by the same composite number of 1001.

5. Yes, they should have. If they did not, inspect their number. If you multiply 7 x 11 x 13 you get 1001 and if you take 1001 times any three-digit number you get a six-digit number where the original three-digit number repeats. Therefore, your original six-digit number would be divisible by 1001 or 7, 11, and 13.

6. Their six-digit number abc,abc should be divisible by 7, 11, and 13.

7. The original six-digit number - abc,abc. It is the same.

8. abc x 1001 = abc x (1000 + 1)

   = abc x 1000 + abc x 1

   = abc, 000 + abc

   = abc,abc

9. Since 468,468 = 468 x 1001 and 1001= 7 x 11 x 13 then 468,468 is divisible by 7, 11, and 13.

10. 369 x 1001 = 369 x (1000 + 1)

  = 369 x 1000 + 369 x 1

  = 369,000 + 369

  = 369,369

11. A number is divisible by 2 if it is even or if the ones digit is 0, 2, 4, 6, or 8.
A number is divisible by 3 if the sum of the digits are divisible by 3.
A number is divisible by 5 if the ones digit is a 0 or a 5.

12. In **A** <u>B</u> **C**, <u>A</u> **B** <u>C</u>, take the sum of every other digit and then subtract. If the difference is divisible by 11 then the original number is.
(A + B + C) - (<u>A</u>+ <u>B</u>+ <u>C</u>) = 0 and 0 is divisible by 11 so the original number is divisible by 11.

13. No. Since 8 + 2 + 4 = 14 and 5 + 3 + 5 = 13 and 14 - 13 = 1 and 1 is NOT divisible by 11, then 852,345 is not divisible by 11.

14. A six-digit number is divisible by 1001 if it is in the form abc,abc.