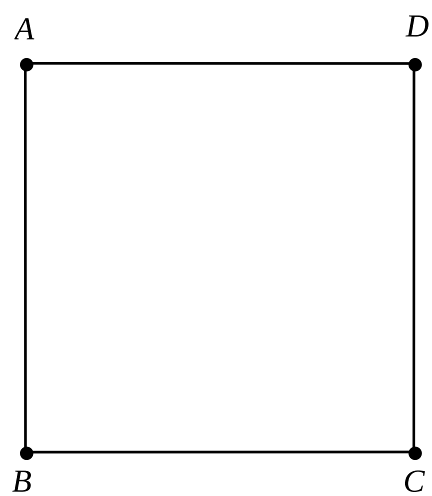
NUMB3RS Activity: The Four Bug Problem: Step on No Pets
Episode: "Spree, Part I"

Topic: Data collection, graphs, distance between two points, equation of a curve **Grade level:** 9 - 12
Objective: Find a mathematical model for a path and relate the model to the geometric context by solving a classic puzzle
Time: about 45 minutes
Materials: rope or string, pennies or other small markers, TI-83 Plus/TI-84 Plus graphing calculator, *What if - bugs, marching band, midpoints – Weeks.gsp* (for the Extensions)
To download this file, go to **http://education.ti.com/exchange** and search for "7423."

Introduction

In this episode, Charlie explains that Agent Edgerton has been chasing Crystal and Buck across the country. The path taken by Edgerton is known as the *pursuit curve*, and is formed when one object chases after another. If Crystal and Buck traveled in a predictable way, then Edgerton could just go to where they are headed and wait for them. However, because Crystal and Buck change their path, Agent Edgerton needs to anticipate their movements.

Pursuit curves were first studied by the French scientist Pierre Bouguer in 1732, and they have many modern-day applications in electricity and magnetism and the military. In this activity, your students are going to have an opportunity to try to form a pursuit curve by studying a classic problem that is commonly called the Four Bug Problem. In this problem four bugs (represented by *A*, *B*, *C*, and D) are located at the four corners of a square.



At exactly the same time, each bug moves towards the bug to its left. All of the bugs walk at the same rate. This means that bug *A* walks towards bug *D*, bug *B* towards bug *A*, and so on. The Four Bug Problem consists of the following questions: What does the path of each bug look like? What are the equations for these paths? Are the paths the same? What eventually happens to the four bugs? How far does each bug walk before "catching" the bug it is chasing?

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In the Extensions section, these questions will be asked for the Three Bug Problem (on an equilateral triangle), the Five Bug Problem (on a regular pentagon), and so on.

Discuss with Students

Prior to introducing this activity to your class, locate a large square on the floor of your classroom, hallway, or cafeteria. If the floor is covered with standard one foot by one foot tiles, then use a ten by ten set of them to form a 10-foot by 10-foot square. Alternatively, students could go outside and look for large squares on the pavement or they could draw squares with chalk.

To begin class, tell your students about the Four Bug Problem and that you need four of them to volunteer to be bugs. Each of the students should be given about ten pennies or markers. Review the instructions outlined in Question 1 with the class, and then head to the square for the activity.

Note that because students will be moving in a certain step size, they will not actually meet in the center of the square – the students will bump into each other because the 1-foot step size is small compared with the size of a person. In fact, the students (or bugs) will only meet in the center of the square if they walk continuously. You may want to allow students to make this realization themselves, or at least give them the opportunity to discover this before telling them.

Student Page Answers

1. The four paths are all spirals, and they meet in the center of the square. Students will obtain a set of line segments that closely approximate a spiral. The approximation improves if smaller step sizes are used. **2.** For the actual spiral path, the length of the path is equal to the length of the side of the square. When acting out the problem, students should find the length of their path to be approximately equal to the side of the square. **3.** Working on paper with a ruler should improve the accuracy. The total length should be closer to the length of the side of the square.

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Name: ___________________________________ Date: __________________________

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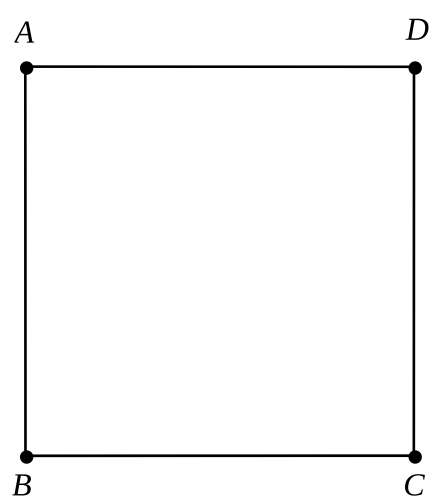
In this activity four people in your class will be taking a walk along a pursuit curve through the study of a classic problem that is commonly called the Four Bug Problem. The statement of this problem is given below.

Four bugs (represented by *A*, *B*, *C*, and *D*) are located at the four corners of a square. At exactly the same time, each bug moves towards the bug to its left. All of the bugs walk at the same rate. So, bug *A* walks towards bug *D*, bug *B* towards bug *A*, and so on.

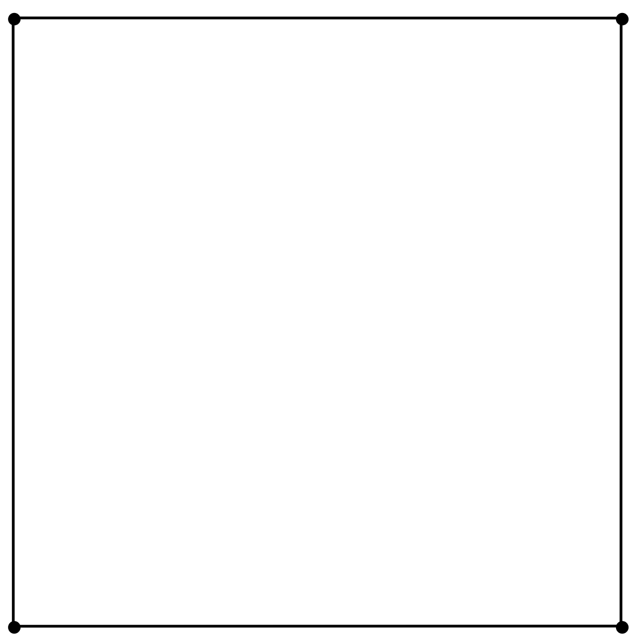


1. Four people from your class will be selected to be the “bugs” and will be positioned at the four corners of a square. Each person should then take a one-foot-long step in the direction of the person to his or her left, then stop and place a penny on the ground to mark his or her current location. (A ruler can be placed on the ground in front of each person to facilitate making a step of the right length.) Each person should now turn so that he or she directly faces the person to his or her left, take another one-foot-long step, and then stop and place another penny on the ground. The four students will continue this procedure until they meet.

At this point use the trail of pennies to develop a graph of the path followed by each person. Then use this graph to predict the actual paths if everyone had walked continuously instead of in 1-foot steps.

2. Find the distance between each successive pair of pennies, and use these lengths to estimate how far each student walked. Then compare the distance walked to the side length of the original square.

3. In order to obtain a more accurate record of what actually happens in the Four Bug Problem, follow the procedure in Question 1 on a piece of paper. On the diagram below, use a ruler to mark “steps” of 1 centimeter. Then measure the total length of the path taken by one of the “bugs.” Again, compare the length of the path to the side length of the square.



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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

- Suppose the bugs are located at the points (0, 0), (10, 0), (10, 10), and (0, 10) and that they move in steps of 1 unit at a time. Find the exact location after one step of the bug that starts from the point (0, 0). Use this result and transformations to find the exact location of the other three bugs after one step. Continue to find the exact locations for one of the bugs after 10 steps. Then use the distance formula to find the distance between each pair of consecutive points to estimate the total length of the path taken by that bug.

- Open the Geometer's Sketchpad® file *What if - bugs, marching band, midpoints – Weeks.gsp*. This file was created by Audrey Weeks from Calculus and Algebra in Motion (**http://www.calculusinmotion.com**) and gives an interactive approach to the Four Bug Problem. Write a report about the type of curve and its equation that describes the path taken by each bug. Use a TI-83 Plus/TI-84 Plus graphing calculator to explore these curves; make a square on the screen and use four different equations to create your own animation of the four bug problem.

- The Four Bug Problem can also be extended to any number of bugs greater than or equal to 3. Determine the equation of the path and the length of the path for three bugs on an equilateral triangle, five bugs on a regular pentagon, and so on.

Additional Resources

- These two Web sites have graphics and other information regarding the Four Bugs problem.
  http://mathworld.wolfram.com/MiceProblem.html
  http://www.mathpages.com/home/kmath492/kmath492.htm

- Gardner, Martin. Hexaflexagons and Other Mathematical Diversions: The First Scientific American Book of Puzzles and Games. Chicago: University of Chicago Press, 1988.

- Gardner, Martin. Martin Gardner's Sixth Book of Mathematical Diversions from Scientific American. Chicago. University of Chicago Press, 1984.

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