1-9 Perimeter, Circumference, and Area

What You'll Learn

- To find perimeters of rectangles and squares, and circumferences of circles
- To find areas of rectangles, squares, and circles

. . . And Why

To find the amount of fencing material needed to build a fence, as in Example 1

Check Skills You'll Need

GO for Help Skills Handbook page 757 and Lesson 1-8

Simplify each absolute value.

1. $|4 - 8|$ **2.** $|10 - (-5)|$ **3.** $|-2 - 6|$

Find the distance between the points to the nearest tenth.

4. $A(2, 3), B(5, 9)$ **5.** $K(-1, -3), L(0, 0)$

6. $W(4, -7), Z(10, -2)$ **7.** $C(-5, 2), D(-7, 6)$

8. $M(-1, -10), P(-12, -3)$ **9.** $Q(-8, -4), R(-3, -10)$

1 Finding Perimeter and Circumference

Vocabulary Tip

You can think of the perimeter of a polygon as the distance around it and the area as the number of square units it encloses.

Hands-On Activity: Finding Perimeter and Area

Draw each figure on centimeter grid paper.

- a rectangle with length 5 cm and width 3 cm
- a rectangle with length 8 cm and height 2 cm
- a rectangle with each side 4 cm

1. To find the perimeter of each rectangle, find the sum of the lengths of the sides. Record the perimeter of each rectangle.
2. To find the area of each rectangle, count the number of square centimeters in its interior. Record the area of each rectangle.
3. Do rectangles with equal perimeters have the same area?
4. Do rectangles with the same area have the same perimeter?
5. Use a piece of string and make a loop. Tie a slip knot. Adjust the loop and fix its total length at 36 cm. Use the loop to approximate different rectangles on your grid paper. Record their lengths, widths, perimeters, and areas. What do you notice?

Online active math

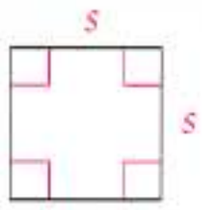
For: Perimeter/Area Activity
Use: Interactive Textbook, 1-9

The perimeter P of a polygon is the sum of the lengths of its sides. The area A of a polygon is the number of square units it encloses. For special figures such as squares, rectangles, and circles, you can use formulas for perimeter (called circumference in circles) and area.

Some formulas for perimeter and area are given in the chart at the top of the next page. You will also find the chart on pages 764 and 765 to be useful at times.

Key Concepts

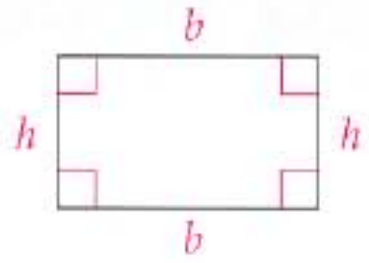
Summary — **Perimeter and Area**



Square with side length s

Perimeter $P = 4s$

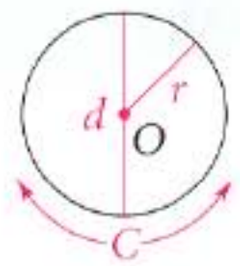
Area $A = s^2$



Rectangle with base b and height h

Perimeter $P = 2b + 2h$

Area $A = bh$



Circle with radius r and diameter d

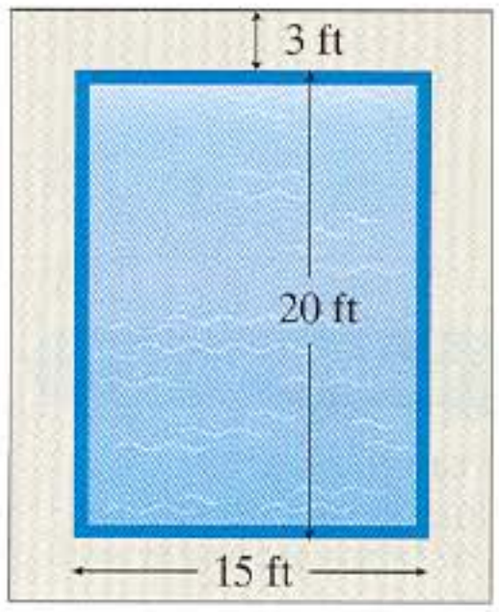
Circumference $C = \pi d$, or $C = 2\pi r$

Area $= \pi r^2$

The units of measurement for perimeter and circumference include inches, feet, yards, miles, centimeters, meters, and kilometers. When measuring area, use square units such as square inches (in.^{2}), square centimeters (cm^2), square meters (m^2), and square miles (mi^2).

1 EXAMPLE **Real-World Connection**

Fencing Your pool is 15 ft wide and 20 ft long with a 3-ft wide deck surrounding it. You want to build a fence around the deck. How much fencing will you need?



To find the perimeter of the pool with the deck, first find the width and length of the pool with the deck.

Width of pool and deck $= 15 + 3 + 3 = 21$

Length of pool and deck $= 20 + 3 + 3 = 26$

Perimeter of a rectangle $= 2b + 2h$ — **Use the formula for the perimeter of a rectangle.**

$P = 2(21) + 2(26)$ — **Substitute.**

$P = 42 + 52$ — **Simplify.**

$P = 94$

You will need 94 ft of fencing.

Vocabulary Tip

For a rectangle, "length" and "width" are sometimes used in place of "base" and "height."

Quick Check 1 Suppose you want to frame a picture that is 6 in. by 7 in. with a $\frac{1}{2}$-in. wide frame.

a. Find the perimeter of the picture.

b. Find the perimeter of the outside edge of the frame.

Notice that the formulas for a circle involve π. Since the number π is irrational,

$$\pi = 3.1415926\ldots,$$

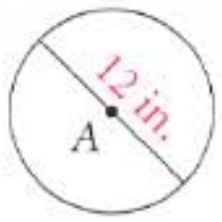
you cannot write it as a terminating decimal. For an approximate answer, you can use 3.14 or $\frac{22}{7}$ $\left(3.14 \approx \frac{22}{7}\right)$ for π. You can also use the rounded decimal you get by pressing $\boxed{\pi}$ on your calculator. For an exact answer leave the result in terms of π.

Vocabulary Tip

Read $\odot A$ as "circle A."

2 EXAMPLE Finding Circumference

Find the circumference of $\odot A$ in terms of π. Then find the circumference to the nearest tenth.



$C = \pi d$

$C = 12\pi$ This is the exact answer.

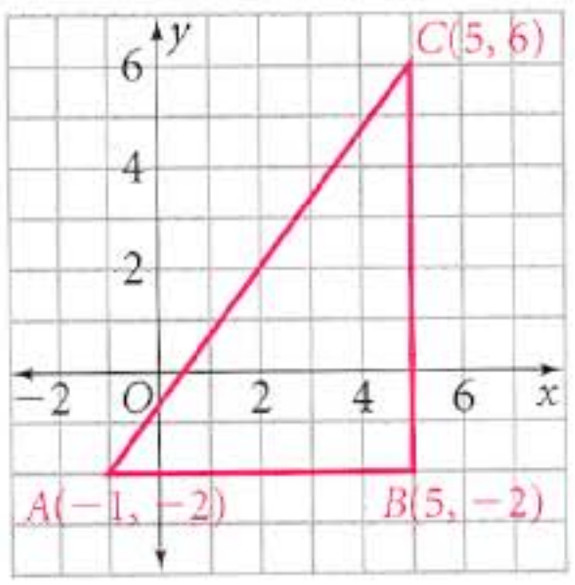
$12 \times \pi = 37.699112$ Use a calculator.

$C \approx 37.7$

- The circumference of the circle is 12π in., or about 37.7 in.

Quick Check **2** **a.** Find the circumference of a circle with a radius of 18 m in terms of π.
b. Find the circumference of a circle with a diameter of 18 m to the nearest tenth.

3 EXAMPLE Finding Perimeter in the Coordinate Plane



Algebra Find the perimeter of $\triangle ABC$.

Find the length of each side. Add the lengths to find the perimeter.

$AB = |5 - (-1)| = 6$ Use the Ruler Postulate.

$BC = |6 - (-2)| = 8$

$AC = \sqrt{(5 - (-1))^2 + (6 - (-2))^2}$ Use the Distance Formula.

$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

$AB + BC + AC = 6 + 8 + 10 = 24$

- The perimeter of $\triangle ABC$ is 24 units.

Quick Check **3** Graph quadrilateral $KLMN$ with vertices $K(-3, -3)$, $L(1, -3)$, $M(1, 4)$, and $N(-3, 1)$. Find the perimeter of $KLMN$.

2 Finding Area

To find area, you should use the same unit for both dimensions.

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4 EXAMPLE Finding Area of a Rectangle

You are designing a rectangular banner for the front of the museum. The banner will be 4 ft wide and 7 yd high. How much material do you need?

$7 \text{ yd} = 21 \text{ ft}$ Change yards to feet using 1 yd = 3 ft.

$\text{Area} = bh$ Use the formula for area of a rectangle.

$A = 4(21)$ Substitute 4 for b and 21 for h.

$A = 84$

- The area of the banner is 84 square feet (ft^2). You need at least 84 ft^2 of material.

Quick Check **4** Find the area of the banner in Example 4 by first changing all units to yards. Compare your answer to the one in Example 4. How do they compare?

5 EXAMPLE Finding Area of a Circle

The diameter of a circle is 10 in. Find the area in terms of π.

radius $= \frac{10}{2}$ or 5 — $r = \frac{d}{2}$

$\text{Area} = \pi r^2$ — **Use the formula for area of a circle.**

$A = \pi(5)^2$ — **Substitute 5 for *r*.**

$A = 25\pi$

The area of the circle is 25π in.^{2}.

Quick Check 5 The diameter of a circle is 5 ft.
a. Find the area in terms of π.
b. Find the area to the nearest tenth.

The following postulates are useful in finding areas of figures with irregular shapes.

Key Concepts

Postulate 1-9

If two figures are congruent, then their areas are equal.

Postulate 1-10

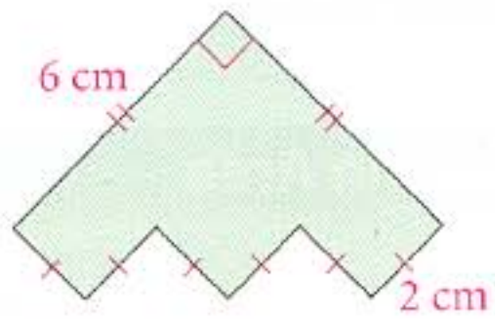
The area of a region is the sum of the areas of its nonoverlapping parts.

Example 6 applies Postulate 1-10 by summing the areas of the parts of a figure.

6 EXAMPLE Finding Area of an Irregular Shape

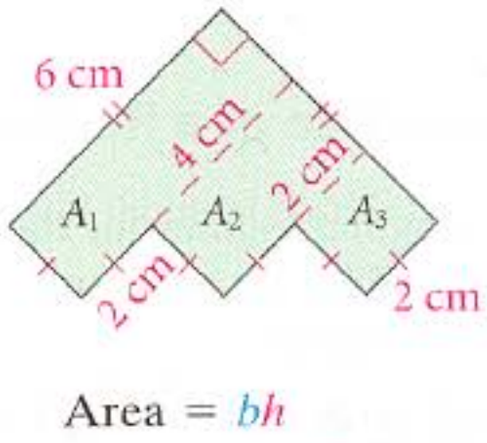
Multiple Choice What is the area of the figure at the right?

(A) 12 cm^2 (B) 24 cm^2
(C) 30 cm^2 (D) 36 cm^2



Test-Taking Tip

Marking diagrams on a test can help you understand the problem. If you cannot mark on the test, make a sketch of the diagram on scratch paper.



Separate the figure into rectangles.

$\text{Area} = bh$ — **Use the formula for the area of a rectangle.**

$A_1 = 6 \cdot 2 = 12$ — **Find the area of each rectangle.**

$A_2 = 4 \cdot 2 = 8$

$A_3 = 2 \cdot 2 = 4$

$\text{Total Area} = 12 + 8 + 4 = 24$ — **Add the areas.**

The area of the figure is 24 cm^2. The correct choice is B.

Quick Check 6 Copy the figure in Example 6. Separate it in a different way. Find the area.

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

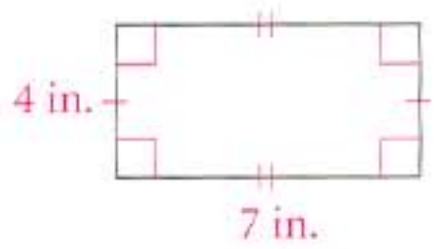
A Practice by Example

Example 1 (page 62)

GO for Help

Find the perimeter of each figure.

1.



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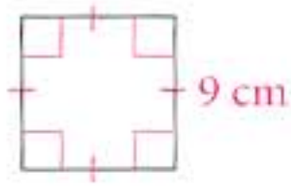


Find the perimeter of each rectangle with the given base and height.

3. 21 in., 7 in.

4. 16 cm, 23 cm

5. 24 m, 36 m

6. Framing A rectangular certificate 8 in. by 10 in. will have a frame $1\frac{1}{2}$ in. wide surrounding it. What is the perimeter of the outside edge of the frame?

7. Fencing A garden that is 5 ft by 6 ft has a walkway 2 ft wide around it. Find the amount of fencing needed to surround the walkway.

Example 2 (page 63)

Find the circumference of each circle in terms of π.

8.

9.

10.

11.

Find the circumference of the circle to the nearest tenth.

12. $r = 9$ in.

13. $d = 7.3$ m

14. $d = \frac{1}{2}$ yd

15. $r = 56$ cm

Example 3 (page 63)

Draw each figure in the coordinate plane. Find the perimeter.

16. $X(0, 2), Y(4, -1), Z(-2, -1)$

17. $A(-4, -1), B(4, 5), C(4, -2)$

18. $L(0, 1), M(3, 5), N(5, 5), P(5, 1)$

19. $S(-5, 3), T(7, -2), U(7, -6), V(-5, -6)$

Example 4 (page 63)

Find the area of each rectangle with the given base and height.

20. 4 ft, 4 in.

21. 30 in., 4 yd

22. 2 ft 3 in., 6 in.

23. 40 cm, 2 m

24. 3 m, 190 cm

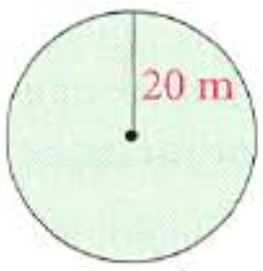
25. 240 cm, 5 m

26. Find the area of a section of road pavement that is 20 ft wide and 100 yd long.

Example 5 (page 64)

Find the area of each circle in terms of π.

27.



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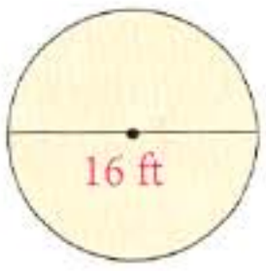

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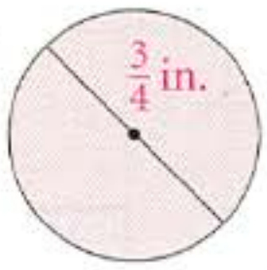

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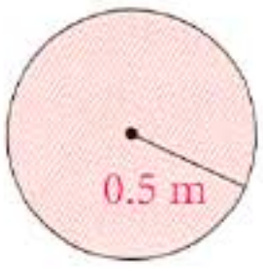

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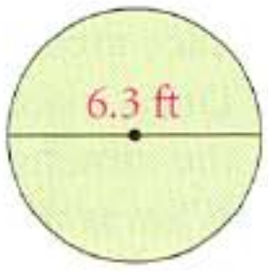
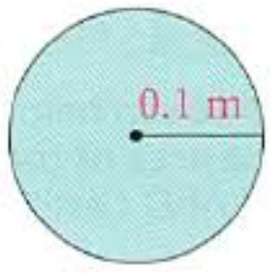


32.



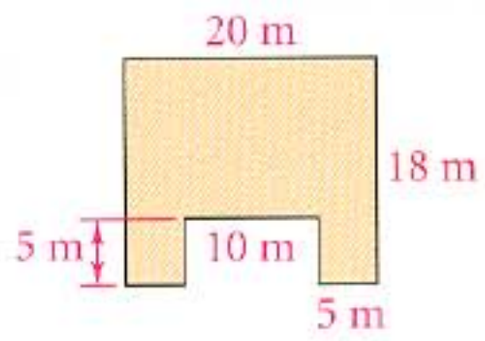
Find the area of each circle to the nearest tenth.

33. $r = 7$ ft **34.** $d = 8.3$ m **35.** $d = 24$ cm **36.** $r = 12$ in.

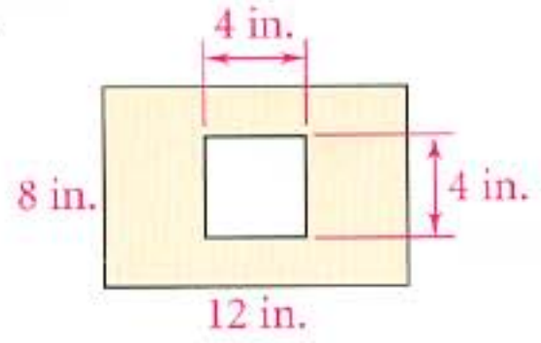
Example 6 (page 64)

Find the area of the shaded region. All angles are right angles.

37.



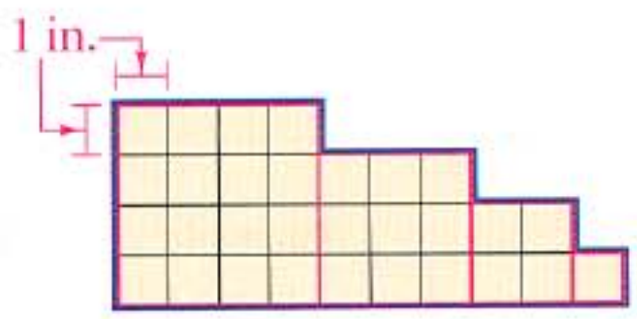
38.



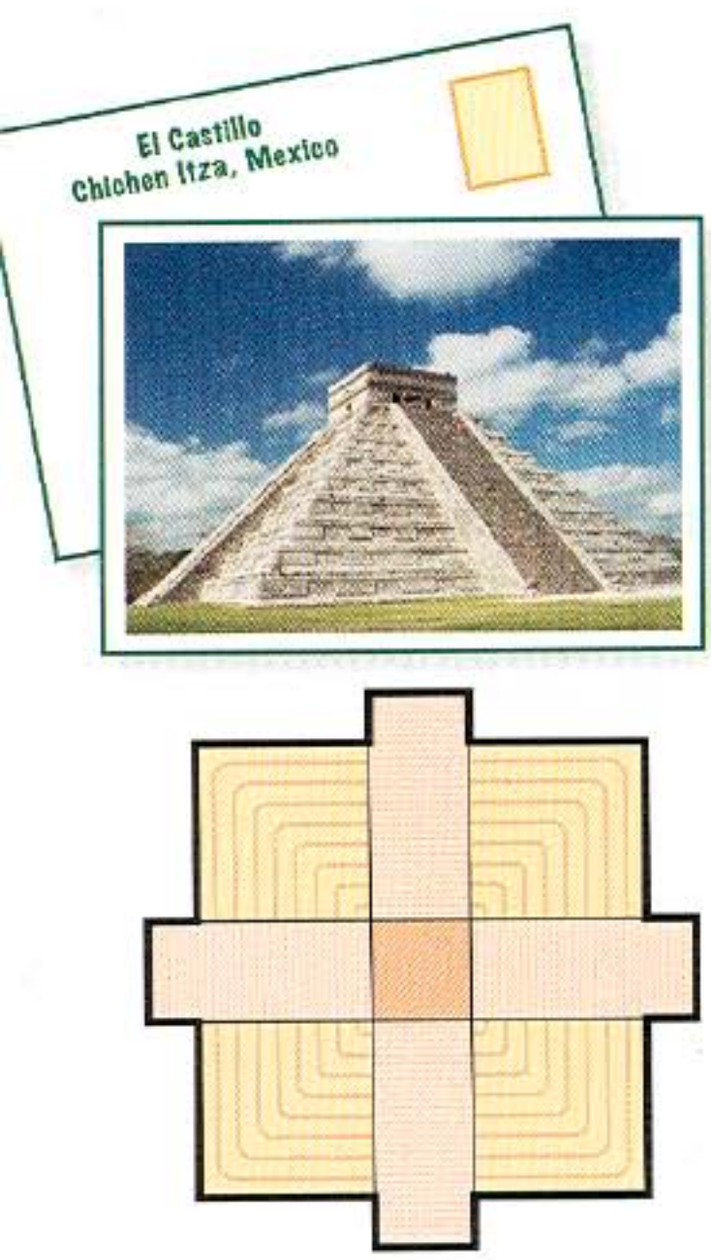
B Apply Your Skills

39. a. What is the area of a square whose sides are 12 in. long?
b. What is the area of a square whose sides are 1 ft long?
c. Reasoning How many square inches are in a square foot? Explain.

40. a. Count squares to find the area of the polygon outlined in blue.
b. Use a formula to find the area of each square outlined in red.
c. How does the sum of your results in part (b) compare to your result in part (a)? Which postulate does this support?



41. Estimation On a postcard from Mexico, Ky sketched the "footprint" of the pyramid known as El Castillo in the ancient Mayan city Chichen Itza. He said he estimated the three different lengths on each side to be 22 m, 6 m, and 11 m. Use those estimates to estimate the area of El Castillo's footprint.



Real-World Connection

Postulate 1-10 can help you estimate the area of the "footprint" of El Castillo.

Estimation Estimate the perimeter and area of each object.

42. the front cover of this book **43.** the front cover of your notebook

44. a classroom bulletin board **45.** the top of your desk

46. Writing Choose one exercise from Exercises 42–45 and explain why you chose your unit of length.

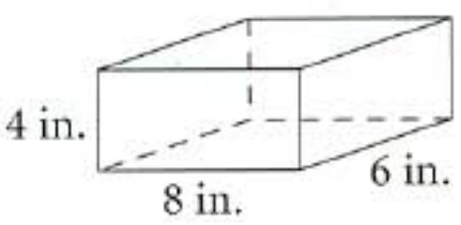
47. The area of an 11-cm wide rectangle is 176 cm^2. What is its length?

48. The perimeter of a rectangle is 40 cm and the base is 12 cm. What is its area?

49. A square and a rectangle have equal area. The rectangle is 64 cm by 81 cm. What is the perimeter of the square?

50. a. Critical Thinking Can you use the formula for the perimeter of a rectangle to find the perimeter of any square? Explain.
b. Can you use the formula for the perimeter of a square to find the perimeter of any rectangle? Explain.
c. Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.

51. The surface area of a three-dimensional figure is the sum of the areas of all of its surfaces. You can find the surface area by finding the area of a net for the figure.
a. Draw a net for the solid shown. Label the dimensions.
b. What is the area of the net? What is the surface area of the solid?



Real-World Connection

Four 6 in.-by-6 in. tiles will cover 1 ft^2.

52. Tiling The students in the Art Club are tiling a wall that is 8 ft by 16 ft at the entrance to the community center. They are using tiles that are 6 in. by 6 in. to create a multi-colored design. How many tiles do the students need?

Algebra Draw each rectangle in the coordinate plane. Find its perimeter and area.

53. $A(-3, 2), B(-2, 2), C(-2, -2), D(-3, -2)$

54. $A(-2, -6), B(-2, -3), C(3, -3), D(3, -6)$

Coordinate Geometry On graph paper, draw polygon $ABCDEFGH$ with vertices $A(1, 1)$, $B(10, 1)$, $C(10, 8)$, $D(7, 8)$, $E(7, 5)$, $F(4, 5)$, $G(4, 8)$, and $H(1, 8)$.

55. Find the perimeter of the polygon.

56. Divide the polygon into rectangles. Find the area of the polygon.

57. Biology In the Pacific Northwest, a red fox has a circular home range with a radius of about 718 meters. To the nearest thousand square meters, what is the area of the home range of a red fox?

58. Multiple Choice A rectangle has a base of x units. The area is $(4x^2 - 2x)$ square units. What is the height of the rectangle in terms of x?

(A) $(4 - x)$ units (B) $(4x^3 - 2x^2)$ units

(C) $(x - 2)$ units (D) $(4x - 2)$ units

Home Maintenance To determine how much of each item to buy, tell whether you need to know area or perimeter. Explain your choice.

59. wallpaper for a bedroom

60. weatherstripping for a door

61. fence for a garden

62. paint for a basement floor

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63. Coordinate Geometry The endpoints of a diameter of a circle are $A(2, 1)$ and $B(5, 5)$. Find the area of the circle in terms of π.

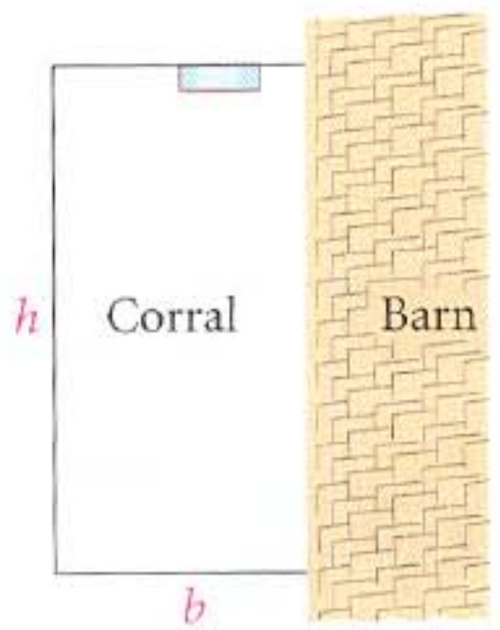
Challenge

64. Graphing Calculator You want to build a rectangular corral by using the side of a barn for one side and 100 ft of fencing for the other three sides.

a. Make a table on your graphing calculator listing integer values for the base and the corresponding values of the height and area.

b. Make a graph using your table values. Graph the base on the horizontal axis and area on the vertical axis.

c. What are the dimensions of the corral with the greatest area?



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65. How many circles with the given radius are needed for the sum of their areas to equal the area of a circle with the second given radius?

a. 1 in., 3 in. **b.** 2 in., 6 in. **c.** 3 in., 9 in.

d. Make a Conjecture How many circles with a radius of n in. are needed for the sum of their areas to equal the area of a circle with a radius of $3n$ in.?

Algebra Find the area of each figure.

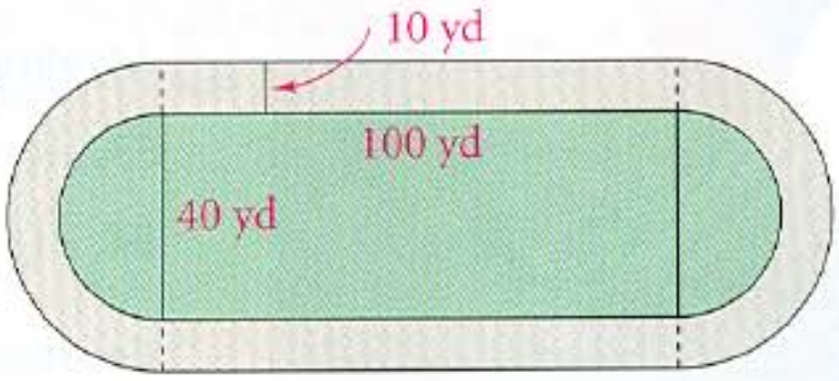
66. a rectangle with side lengths of $\frac{2a}{5b}$ units and $\frac{3b}{8}$ units

67. a square with perimeter $10n$ units

68. a square with side lengths of $(3m - 4n)$ units

69. Open-Ended The area of a 5 in.-by-5 in. square is the same as the sum of the areas of a 3 in.-by-3 in. square and a 4 in.-by-4 in. square. Find two or more squares whose total area is the same as the area of an 11 in.-by-11 in. square.

70. Track An athletic field is a rectangle, 100 yards by 40 yards, with a semicircle at each of the short sides. A running track 10 yards wide surrounds the field. Find the perimeter of the outside of the running track to the nearest tenth of a yard.



Test Prep

Gridded Response For Exercises 71 and 72, a rectangular garden has a rectangular walkway around it. The width of the walkway is 8 ft.

71. How many feet greater than the perimeter of the garden is the outside perimeter of the walkway?

72. If the garden is a square with a perimeter of 260 ft, what is the area of the walkway in square feet?

73. You need to tile a 12 ft-by-15 ft floor. The color you want allows you the choices found in the table at the right. How many dollars would it cost to tile the floor with 12 in.-by-12 in. tiles?

Size of Tiles	Cost
$12'' \times 12''$	$\$3/ft^2$
$11'' \times 11''$	$\$3/ft^2$
$10'' \times 12''$	$\$4/ft^2$
$6'' \times 8''$	$\$4.50/ft^2$

74. How many tiles would cover the 12 ft-by-15 ft floor if you choose the 10 in.-by-12 in. tiles?

75. How many dollars would it cost to cover the 12 ft-by-15 ft floor with the tiles that are 6 in. by 8 in.?

Mixed Review

Lesson 1-8 — GO for Help

76. The midpoint of $\overline{CD}$ has coordinates $(5, 6)$. Point C has coordinates $(-5, -1)$. Find the coordinates of point D.

Find (a) AB to the nearest tenth and (b) the coordinates of the midpoint of $\overline{AB}$.

77. $A(4, 1), B(7, 9)$ **78.** $A(0, 3), B(3, 8)$ **79.** $A(9, 2), B(-3, 9)$

80. $A(0, 1), B(-4, 6)$ **81.** $A(4, 10), B(-2, 3)$ **82.** $A(-1, 1), B(-4, -5)$

Lesson 1-7 — **$\overleftrightarrow{BG}$ is the perpendicular bisector of $\overline{WR}$ at point I.**

83. What is $m\angle BIR$? **84.** Name two congruent segments.

85. $\overline{WR}$ has length 124. What is the length of $\overline{IR}$?

Lesson 1-5 — **For the given coordinates, find PQ.**

86. P: 12, Q: -6 **87.** P: 3, Q: 9 **88.** P: -23, Q: 10