

Investigation of End Behavior

ID: 8284

Time required
45 minutes

Activity Overview

Students explore end behavior of rational functions graphically, algebraically, and by using tables. They will use multiple representations to look at values a given function approaches as the independent variable goes to positive or negative infinity. Tools are provided that support students in using a graphical approach, evaluating functions at different values, and using a function equation to find the end behavior of a rational function.

Topic: Rational Functions & Equations

- Graph a rational function to verify its domain and range.
- Evaluate a rational function of x at any value of x .

Teacher Preparation and Notes

- This investigation is intended to be used in an advanced Algebra 1 class as an introduction to the concept of end behavior of rational functions. It could also be used in an Algebra 2 class as a review of end behavior, or with students who might be struggling with some of these concepts.
- It is highly recommended that students have been introduced to rational functions and polynomial long division before beginning this activity.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8284" in the keyword search box.**

Associated Materials

- *EndBehavior_Student.doc*
- *EndBehavior.tns*

Suggested Related Activities

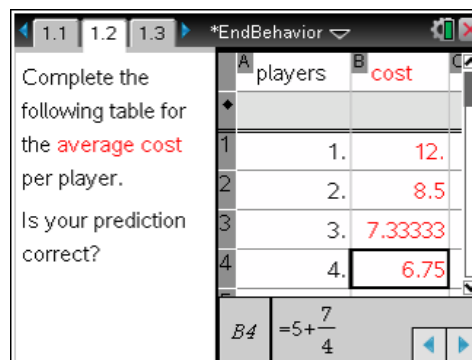
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *End Behavior of Polynomial Functions (TI-Nspire technology)* — 13268
- *Polynomial End Behavior (TI-Nspire technology)* — 12723
- *Graphs of Rational Functions (TI-Nspire technology)* — 16050

Exercise 1 – Cost per person for a pizza order

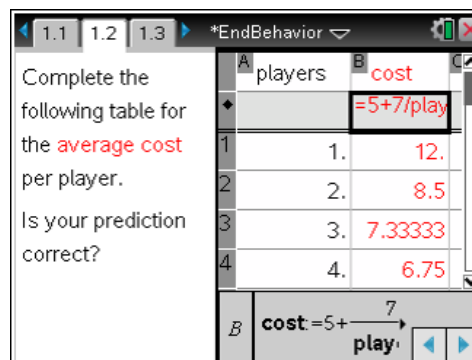
This problem provides a simple context to get students thinking about end behavior. Students should read the problem as posed on their worksheets and make a prediction about what they think will happen as more football players choose to order pizza. At this point, no formal responses are necessary—students should simply hypothesize that the cost will go down as more players decide to order.

On page 1.2, students will see a table with several values for the number of **players** in Column A. Have them use these values given to generate values in the **cost** column using what they know about the problem. Students should enter an expression preceded by an equals sign into each cell, such as $=5+7/1$, $=5+7/2$, $5+7/3$, etc. They should find that the cost does indeed decrease as more students order pizza.

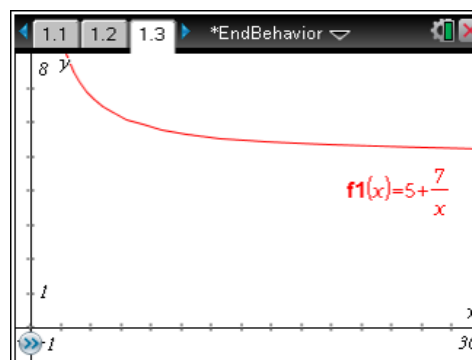


Students should now write a general equation based on the recursive process they just used. They can then test their equation in the formula cell (in gray) of Column C. An appropriate equation is:

$$cost = 5 + \frac{7}{players}$$



Next, students will define their equation as $f_1(x)$ on page 1.3. After displaying the graph, tell students to hide the Entry Line by pressing **ctrl** + **G** to allow them to see more of the graph. It should be clear that the value of $f_1(x)$ approaches 5 as x gets larger and larger. It may be beneficial for some students to graph the horizontal line $f_2(x) = 5$ so they can see that $f_1(x)$ is indeed approaching this line as the value of x increases.



Students will then answer the question about what happens to the graph as the number of players increases. They should determine that the cost per player approaches \$5 as more pizzas are ordered. As a class, you can discuss why this is the case. The discussion should include the following:

The cost per pizza is \$5, but the \$7 delivery charge will be divided evenly between the people paying. Each person is going to pay \$5, plus his/her share of the delivery charge. Therefore, the cost will *approach* \$5 but will never be *equal* to \$5.

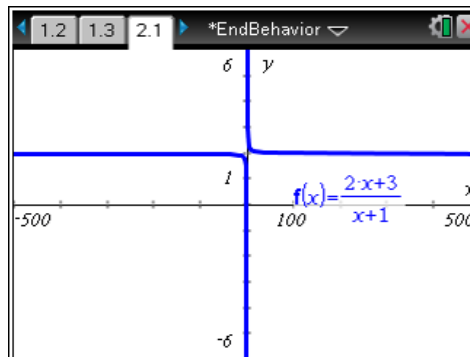
TI-Nspire Navigator Opportunity: Quick Poll

See Note 1 at the end of this lesson.

Exercise 2 – Investigating end behavior

On page 2.1, students will see the graph of the rational function $f(x) = \frac{2x+3}{x+1}$. Have students

change the window of the graph (**MENU > Window > Window Settings**) so that x ranges from -500 to 500 . They should see that the graph of the function ends up “looking like” a line. So as the x -values get very large in magnitude, the graph “looks” like its end behavior, which here is the line $y = 2$.



Next, students will investigate this behavior using a calculator and a table. Prompt them use 10, 100, and 1000 and then -10 , -100 , and -1000 to simulate the x -values going to positive and negative infinity, respectively. They should observe that for the positive x -values, the graph approaches 2 from *above*, but for the negative x -values, the graph approaches 2 from *below*. (Have them recall the shape of the graph.)

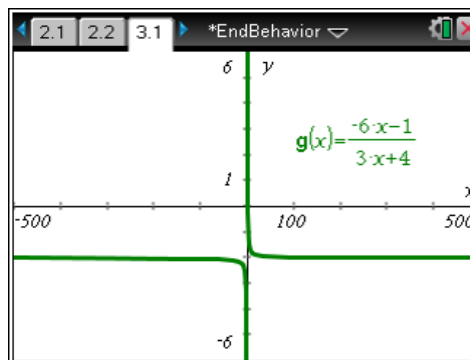
positive values	neg_values	=f(a[])
f(1)	1	-10. 1.88889
f(10)	2	-100. 1.9899
f(100)	3	-1000. 1.999
f(1000)	4	
	5	

Students can then summarize that $f(x)$ is approaching $y = 2$. This is supported by the graph because as the x -values go to positive and negative infinity the graph approaches two. They can also discuss that the graph is above the end behavior on the right and below the end behavior on the left.

Prompt students to think about how the values in the equation can be used to determine the end behavior. Rewrite the rational function using long division and encourage students to see that for values of x that are large in magnitude, the values of the fraction go to zero, and thus the function approaches 2.

$$x+1 \overline{) 2x+3} \rightarrow f(x) = 2 + \frac{1}{x+1}$$

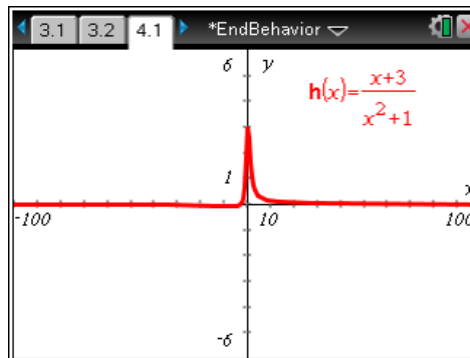
On page 3.1, students are shown the graph of another rational function, $g(x) = \frac{-6x-1}{3x+4}$. At this point, they should be able to complete this example individually, returning together as a class to see if they could determine the correct end behavior. Students should again extend the axes to see how the graph behaves for very large and very small values of x .



On page 3.2, students should again use the *Calculator* and *Lists & Spreadsheet* applications to identify the end behavior for the function. They should find the end behavior to be $y = -2$.

a	g(a)
1	-1
10	-1.79412
100	-1.97697
1000	-1.99767
10000	-1.99977

On pages 4.1 and 4.2, students will similarly explore the end behavior of the function $h(x) = \frac{x+3}{x^2+1}$.



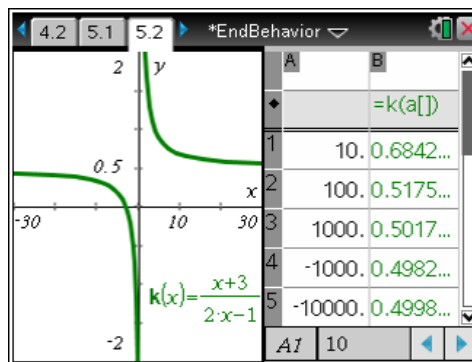
Here, they should determine that the function approaches $y = 0$.

a	h(a)
1	2
10	0.128713
100	0.010299
1000	0.001003
10000	0.0001

The pages in Problem 5 offer views of three different functions, and students are asked to simply identify the end behavior for each. If they have grasped the concepts explored so far, they should be able to move through these quickly and easily identify the end behavior. The end behavior of the function on page 5.1 is $y = 10$.

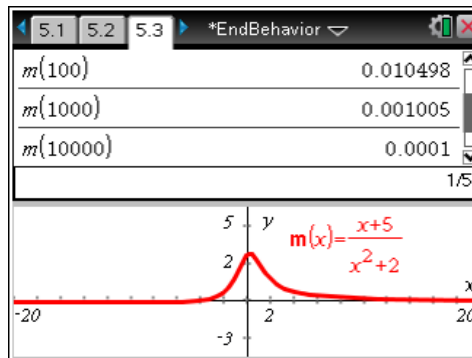
On page 5.2, the end behavior of is $y = \frac{1}{2}$.

The end behavior of the function displayed on page 5.3 is $y = 0$. (shown below, right)



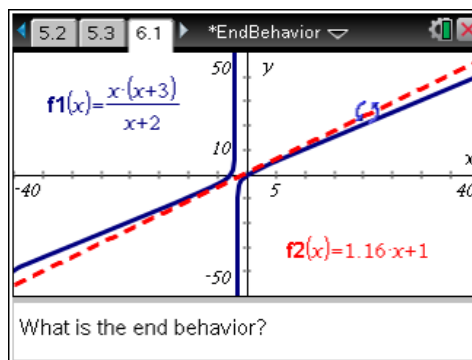
Bringing it all together

Exercises like these are important because students will begin to develop ideas about how to find asymptotes algebraically. For example, the end behavior of $f(x) = \frac{8x-1}{2x+3}$ is the quotient of the coefficients of x , i.e., $y = \frac{8}{2}$, or $y = 4$.



Extension

On page 6.1, students are given a rational function with an end behavior asymptote of $y = x$. This will introduce them to the idea that not all rational functions have horizontal end asymptotes. You can suggest they try different functions and asymptotes by altering the definition of $f1(x)$ and dragging and rotating the line given by $f2(x)$. Have them try to algebraically determine why this is the case.



TI-Nspire Navigator Opportunities

Note 1

Exercise 1, Quick Poll

You may choose to use Quick Poll to assess student understanding throughout the activity. The worksheet questions can be used as a guide for possible questions to ask.