Construction Tools: Interactive Function Graphs

You can explore quadratic functions of the form $y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$ by drawing multiple graphs using different values for the parameters a, b, c, h, and k. Each member of this family of graphs is an individual object. However, the general statement of a function implies that there is only one graph of the function that changes dynamically as the values of these parameters change. The graph of the function should change shape dynamically as the controlling parameters change interactively. The Voyage™ 200 and Cabri Geometry II allow you to do this for any function.

Task:

Draw a dynamic graph of the function $y = ax^2 + bx + c$ with interactive parameters a, b, and c.

Construction:

1. **Define the domain of the independent variable:** Show the rectangular coordinate axis on the screen. Draw a segment on top of the x axis from the left to the right edge of the screen. This segment controls the domain of the independent variable, x. Place a point on this segment near the origin, but not on it, and show its coordinates. Make sure the point is on the segment and not on the x axis.

2. **Scale the function to the graphics screen:** Place a point on the positive y axis and show its coordinates. Measure the distance between the origin and this point. Using the **Calculate** tool, create a **Scale Factor** by dividing the y coordinate of the point by the distance from the origin to the point (Figure 1). In the default scale, the Scale Factor = 1.00 because the coordinate system corresponds to the actual length in centimeters. If you drag a y axis unit to change the scale of the y axis, the scale value you created changes, giving you a value representing the number of coordinate units on the axis per centimeter of actual distance. Hide the point on the y axis, its coordinates, and its distance from the origin. Save the Scale Factor for later use.

Figure 1

3. **Write the function definition:** On the Voyage 200 with Cabri, select **Numerical Edit** and write the numbers 1.1, 1.2, and 1.3 in an open area of the screen. Use a **Comment** box to write **y =** . Drag the first parameter into position after this comment. Next, use another **Comment** box to write **x^2 +** . Drag the next parameter into place following this comment. Finally, add a third **Comment** box containing **x +** . Drag the last parameter into place immediately following this comment. The final result is a statement of the function definition with interactive parameters, as shown in Figure 2.
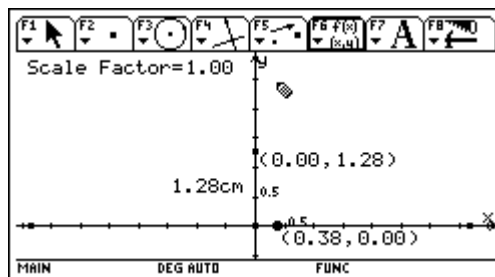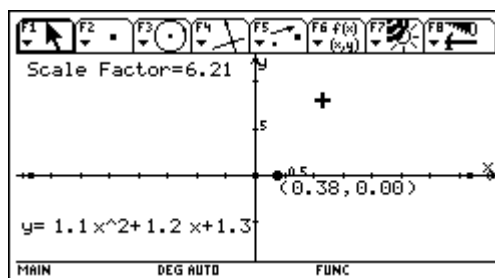

Figure 2

For Cabri Geometry II, select **Numerical Edit** and write the numbers 1.1, 1.2, and 1.3 in an open area of the screen. Create a **Comment** box, and start writing **y =** . When you need one of the parameters, point at it and click to include the number in the **Comment** box. Continue writing the function definition **x^2 +** (include number) **x +** (include number) in the **Comment** box (Figure 3). When you finish, drag the **Comment** box anywhere on the screen. The parameters in the **Comment** box are independent of the original values on the screen, so you can delete the original three numbers.
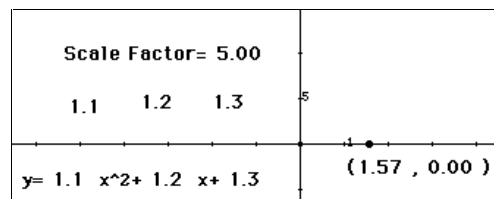
Figure 3: Cabri Geometry II screen

4. **Draw the function:** Using the **Calculate** tool, now define the numerical value of the function for the given point on the x axis and the interactive parameters. The letters assigned automatically to various numerical values by the **Calculate** tool on the Voyage™ 200 or Cabri Geometry II should not be confused with the function's parameters a, b, or c. Figure 4 shows the computation of the correctly scaled function value (result is **R: 0.87**) for the x coordinate 0.83, based on the function $y = 1.1x^2 + 1.2x + 1.3$. Each time the x value is needed in the computation, the x coordinate of the point on the x axis is used. The entire computation is divided by the Scale Factor to adjust the result to a measurement in centimeters.
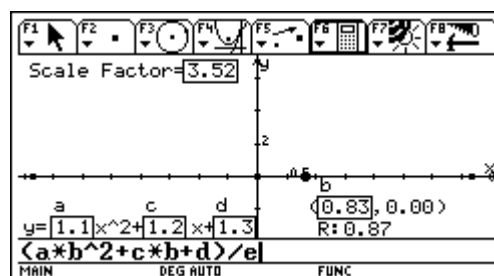
Figure 4

Using **Measurement Transfer**, transfer the result onto the y axis. This value transfers as a length in centimeters, not as a coordinate; this is why you needed to scale the y axis using the calculated Scale Factor. The other bonus for scaling the y axis in this manner is that you can now change the scale on the y axis at any time and the graph will still be drawn correctly.

The result of this computation is the scaled distance from the origin to a point on the y axis, representing the function value for the given point on the x axis. Because all of the parameters and the independent variable have been defined interactively, as you move the point on the x axis, the corresponding function value on the y axis changes, based on the current parameters.

Draw lines perpendicular to the axes at the points on the x and y axis representing the independent and dependent values of the function. The intersection of these two perpendicular lines is a point on the graph of the function (Figure 5).
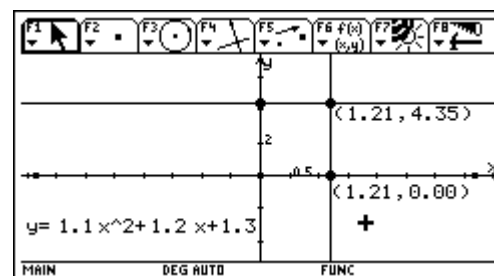
Figure 5

Drag the point on the x axis to see whether the point of intersection follows the path of the graph. The locus of all the points of intersection is the graph of the function. You can change the units on the y axis at any time; the path of the intersection point will still trace the graph of the function correctly.

5. **Exploring the graph of the function interactively:**

Hide the construction lines and the result so that only the function, the points on the x and y axes, and the point of intersection are showing. Using the **Trace** tool, trace the path of the point of intersection by dragging or animating the independent point on the x axis (Figure 6). This path is the graph of the function for the current parameters (coefficients) and y axis units (scale).

Figure 6

Turn **Trace** off. Select the **Locus** tool, and define the locus of the point of intersection (the graph tracing point) as the independent point on the x axis is moved along its path. The result of this command is the locus of points, constituting the graph of the function (Figure 7). Use **Numerical Edit** to change the parameters (coefficients) in the function definition. The graph of the function changes accordingly. On the Voyage™ 200, use ◆ and the cursor arrows to change the size of the number or to move the cursor in the **Numerical Edit** window. Use ◆ ⌃ to increase the value of the coefficient and ◆ ⌄ to decrease the value. For large changes, edit the whole number part of each

Figure 7

coefficient by placing the cursor to the right of the units digit. For finer changes in the graph, edit the decimal part of the coefficient by 0.1 units at a time by placing the cursor to the right of the tenths digit.

When using Cabri Geometry II, choose **Numerical Edit** and click on the value you want to change. Place the cursor to the right of the digit you want to change by pointing and clicking with the cursor (Figure 8). Use the up and down arrow keys to change the values. You can also use the **Animate** tool on both the Voyage 200 and Cabri Geometry II to change the value of the coefficients.
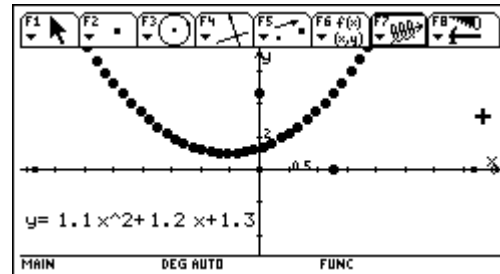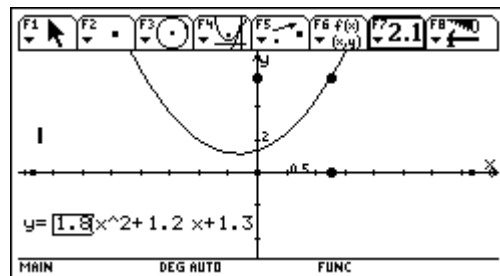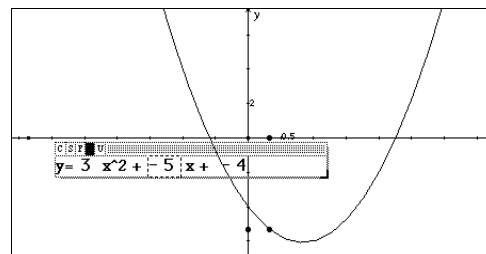

Figure 8: Cabri Geometry II screen

Extension:

1. Explain the effect of each of the parameters a, b, and c on the shape of the graph of the general quadratic function $y = ax^2 + bx + c$.

2. The vertex of a parabola of the form $y = ax^2 + bx + c$ is the point $\left(\dfrac{-b}{2a}, c - \dfrac{b^2}{4a} \right)$. Using the coefficients of the function on the screen, compute the location of the vertex of this parabola and use **Measurement Transfer** to map these values to the x and y axis. (Remember to scale the y coordinate using the Scale Factor.) Locate the vertex as the point of intersection of two perpendicular lines, and then hide the construction lines.

   What is the locus of points of the vertex of the parabola as the values of a, b, and c are changed in the general quadratic function $y = ax^2 + bx + c$? Explain algebraically why this happens.

3. The standard form of a quadratic function is given by $y = a(x - h)^2 + k$, where a is the same vertical stretch factor as in the general quadratic form, and h and k represent the horizontal and vertical translation factors of the function.

   Repeat the construction of the graph of a quadratic function using this standard form. Investigate the effects of changing a, h, and k interactively. How will you find the vertex of the parabola using this form of the function? Explain. What is the locus of the vertex as the various parameters are changed? What is the algebraic connection between these two forms of quadratic functions?

4. Using the same interactive graphing technique, plot the graph of a general cubic function of the form $y = ax^3 + bx^2 + cx + d$. Explain the effect of each coefficient on the shape of the graph.

5. Using the same interactive graphing technique, plot the graph of a general sine function of the form $y = a \sin (b(x - c)) + d$. Explain the effect of each parameter on the shape of the graph.