Math Objectives

- Students will understand relationships between the horizontal intercepts of two linear functions and the horizontal intercepts of the quadratic function resulting from their product.
- Students will understand relationships between the slopes of two linear functions and the orientation and width of the quadratic function resulting from their product.
- Students will understand when the product of two linear functions will not be quadratic.
- Students will explore the relationship between linear factors and the resulting cubic function.
- Construct viable arguments and critique the thinking of others (CCSS Mathematical Practice).
- Look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

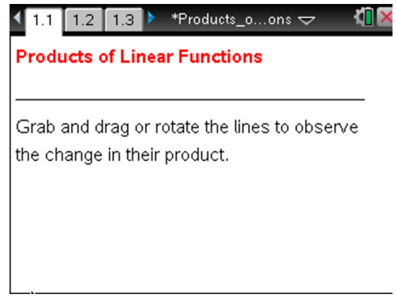
- linear function
- quadratic function
- cubic function
- roots
- intersect

About the Lesson

- This lesson involves polynomial functions viewed as a product of linear functions.
- As a result, students will:
  - Manipulate linear functions to change the graph of their product to meet certain constraints.
  - Make and test predictions about how the properties of linear functions affect the graphs of their quadratic or cubic product functions.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Products_of_Linear_Functions_Student.pdf

Products_of_Linear_Functions_Student.doc

TI-Nspire document

Products_of_Linear_Functions.tns

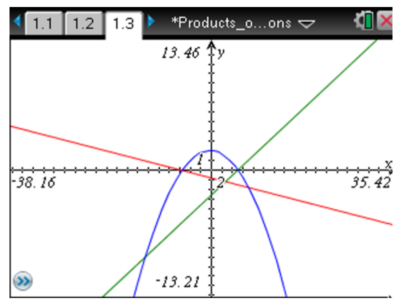
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a line, check to make sure that they have moved the cursor until it becomes a pair of intersecting lines, ✥. Then click and hold the mouse to drag the line.

If students experience difficulty rotating a line, check to make sure that they have moved the cursor until it becomes a pair of curved lines, ⟲. Then click and hold the mouse to rotate the line.

Move to page 1.3.



1. The graphs of linear functions **f1** and **f2** are plotted on Page 1.3. Their product, **f3**, is also plotted here. As you drag or rotate the linear functions, f3 changes dynamically.
   a. Before you move f1 and f2, predict what they will need to look like to create a product function f3 that crosses the x-axis at -3 and 2. Explain why you predicted this.

   <u>Sample Answers:</u> One of f1 and f2 must to cross the x-axis at -3, while the other must cross at 2. The linear factors of a quadratic with roots -3 and 2 would be constant multiples of x+3 and x-2. The first factor crosses the x-axis at -3, the second at -2.

   b. Move f1 and f2 to test your prediction. Were you correct? If not, what mistake did you make?

   <u>Sample Answers:</u> Common mistakes include using x-3 and x+2 as factors, or not recognizing that the linear functions whose product is a quadratic would also be the factors of the quadratic. Students might not recognize the connection between the roots of a quadratic function and the roots of its linear factors.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

2. a. Predict what f1 and f2 should look like in order for the graph of their product to be an upward opening parabola. Explain your reasoning.

Sample Answers: In order for the product of f1 and f2 to be an upward opening parabola, f1 and f2 need to be either both increasing or both decreasing. This is because the product of the coefficients of x in each of f1 and f2 will give the coefficient of x^2 in their product. A positive coefficient of x^2 corresponds to an upward opening parabola. If both f1 and f2 are increasing, the coefficients of x are positive, and their product is positive. If both f1 and f2 are decreasing, the coefficients of x are negative, and their product is positive.

b. Move f1 and f2 to test your prediction. Were you correct? If not, why do you think your prediction didn't work?

Sample Answers: Student answers will vary. Common mistakes include not recognizing the relationship between the leading coefficients of f1 and f2 and the leading coefficient of their product.

c. Are there any other possible arrangements of f1 and f2 that would result in the graph of their product being an upward opening parabola? Explain your reasoning.

Sample Answers: Students might not have recognized both relationships, and so might refer to the other relationship (i.e. both leading coefficients negative, both leading coefficients positive).

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

Teacher Tip: Teachers should ensure that students recognize that there is more than one possible arrangement of f1 and f2.

3. a. Predict what f1 and f2 should look like in order for the graph of their product to be a parabola that intersects the x-axis in only one place. Explain your reasoning.

Sample Answers: f1 and f2 must have the same *x*-intercept in order for their product to intersect the x-axis in only one place. Reasoning might include recognizing the relationship between roots of the linear factors and roots of the quadratic product. Intersecting the x-axis in only one place is equivalent to having only one root; in order for the quadratic to have only one root, the roots of its linear factors must be the same. This means the *x*-intercepts of f1 and f2 must be the same.

b. Move f1 and f2 to test your prediction. Were you correct? If not, why do you think your prediction didn't work?

Sample Answers: Student answers will vary. One possible mistake is assuming the product of a linear and constant function would result in a parabola with only one root. However, this product is a linear function with only one root.

c. Are there any other possible arrangements of f1 and f2 that would result in the graph of their product being a parabola that intersects the x-axis in only one place? Explain your reasoning.

Sample Answers: No. The only way the product of f1 and f2 can be both a parabola and intersect the x-axis in only one place is if both f1 and f2 are non-constant linear and both have the same *x*-intercept.

4. a. Predict what f1 and f2 should look like in order for the graph of their product to be a parabola that never intersects the x-axis. Explain your reasoning.

Sample Answers: This is not possible. In order for a parabola not to intersect the x-axis, it can not have any real roots. Therefore the function can not be factored into two linear factors with real roots. So as long as f1 and f2 intersect the x-axis, their product will as well. The only way f1 and f2 can not intersect the x-axis is if they are both horizontal. But in this case, their product would be constant, not quadratic.

b. Move f1 and f2 to test your prediction. Were you correct? If not, why do you think your prediction didn't work?

Sample Answers: Student answers will vary. A common mistake might be that students do not recognize the conditions under which the graph of a quadratic function would not intersect the x-axis.

c. What do you know about the roots of a quadratic function that never crosses the x-axis? How does this connect to your prediction and your test of your prediction?

Sample Answers: If the graph of a quadratic function does not cross the x-axis, it can not have any real roots. This indicates that any product of linear factors with real roots could not have as its graph a parabola that never crosses the x-axis.

5. Is there any way for the graph of the product of f1 and f2 to be something other than a parabola? Explain your reasoning.

   Sample Answers: Yes. If either of f1 or f2 are constant, or are vertical lines, their product can be constant or linear.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

6. a. Predict what f1 and f2 should look like in order for the graph of their product to be a very wide parabola.

   Sample Answers: Student answers will vary. Answers include: the *x*-intercepts of f1 and f2 need to be very far apart, the slopes of f1 and f2 have to be small in magnitude, or the product of the slopes of f1 and f2 has to be small in magnitude.

   b. Move f1 and f2 to test your prediction. Were you correct? If not, why do you think your prediction didn't work?

   Sample Answers: Student answers will vary.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 4 at the end of this lesson.

7. Write a paragraph summarizing the different arrangements of f1 and f2, and the effects of these arrangements on the graphs of their products.

   Sample Answers: The x-intercepts of the linear factors will be the intercepts of their quadratic product. If one or more of the linear factors are constant, the product will be linear or constant, not quadratic. It is not possible for the product of two linear factors to be a quadratic with no real roots. Linear factors with slopes which are small in magnitude, or for which the product of slopes is small in magnitude will result in a quadratic function whose graph is a wide parabola.

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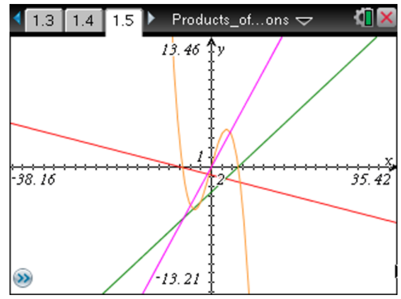

8. Make two predictions about the impact of three linear factors on the appearance of the graph of their product. Record your predictions, and explain your thinking.

   <u>Sample Answers:</u> Student answers will vary. Predictions might include that the cubic function will have the same roots as the linear factors, that if the lines share a common *x*-intercept, the cubic will have fewer than 3 intercepts, and that the relative slopes of the linear functions will impact the width of the resulting cubic.

9. Test your predictions by moving f1, f2, and f4. Were your predictions correct? If not, why do you think your prediction didn't work.

   <u>Sample Answers:</u> Student answers will vary.

10. Explore the results of moving f1, f2, and f4. Then write a paragraph summarizing the different arrangements of f1, f2, and f4, and the effects of these arrangements on the graphs of their products.

    <u>Sample Answers:</u> The roots of f1, f2 and f4 are the roots of their product. If one or more of f1, f2, or f4 is constant, the resulting product will be quadratic, linear, or constant. If the product of the slopes of f1, f2, and f4 is small, the resulting cubic function will have a very wide graph.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The relationship between the roots of linear factors of a quadratic and the roots of the quadratic.
- That a quadratic with no real roots can not be expressed as a product of linear factors with real roots.
- That the slopes of the linear factors of a quadratic function affect both the orientation of the resulting quadratic and the width of the resulting quadratic.

Assessment

Give students the equations of 2 linear functions and ask them, without graphing or multiplying, to describe the resulting quadratic.

TI-Nspire Navigator

Note 1

Question 1, Name of Feature: Screen Capture

A Screen Capture can be used to show multiple student screens and help students to observe that the roots of linear factors are the roots of their quadratic product.

Note 2

Question 2, Name of Feature: Screen Capture

A Screen Capture can be used to show multiple student screens and help students to observe that both cases (both linear functions increasing or both decreasing) will result in an upward facing parabola.

Note 3

Question 5, Name of Feature: Quick Poll

A Quick Poll can be used to gauge the extent of student exploration of the possible products of linear functions. If the Quick Poll reveals that students have not observed that the product of linear functions need not be quadratic, a Screen Capture can be used to display this.

Note 4

Question 6, Name of Feature: Screen Capture

A Screen Capture can be used to reveal different arrangements of the linear factors that will result in a wide parabola, and help students to generalize this idea.

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