

Activity 9

**Testing Goodness-of-Fit and Two-Way Table Chi-Square**

Topic 47 tests to see if data follows a hypothesized distribution. Topic 48 builds on the descriptive two-way table analysis in Topic 18 to see if there is a significant relationship between two categorical variables.

**Topic 47—Chi-Square Goodness-of-Fit Test**

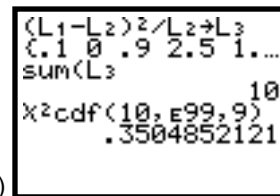
**Uniform Distribution (Check on  $\boxed{\text{MATH}}$  <PRB>1:rand)**

In Topic 26 (screen 56), you generated 100 random values between 0 and 10 and constructed a **Histogram** with ten classes of length 1. If the random generator was working effectively, we would expect ten values in each class. The observed and the expected values are given below. (Expected values are all greater than 5.)

Class	1	2	3	4	5	6	7	8	9	10
Observed (L1)	9	10	7	15	14	5	14	8	10	8
Expected (L2)	10	10	10	10	10	10	10	10	10	10

Test the hypothesis that the data comes from a uniform distribution:

- With the observed values in L1 and the expected values in L2, calculate  $\boxed{\square}$   $\boxed{2\text{nd}}$  L1  $\boxed{-}$   $\boxed{2\text{nd}}$  L2  $\boxed{)}$   $\boxed{x^2}$   $\boxed{\div}$   $\boxed{2\text{nd}}$  L2  $\boxed{\text{STO}}$  L3, as in the first two lines shown in screen 1. Notice that the smallest and largest values are 0 and 2.5; 0 to go with 10 (what we expected) and 2.5 to go with the biggest miss of 15.
- Press  $\boxed{2\text{nd}}$  [LIST] <MATH> 5:sum( L3  $\boxed{\text{ENTER}}$  for the chi-square statistic of 10. This is a one-tail test in that we are only concerned if the calculated chi-square is larger than we would expect for a uniform distribution. Degrees of freedom = 10 - 1 = 9.
- Press  $\boxed{2\text{nd}}$  [DISTR] 7: $\chi^2$ cdf( 10  $\boxed{,}$   $\boxed{2\text{nd}}$  [EE] 99  $\boxed{,}$  9  $\boxed{)}$   $\boxed{\text{ENTER}}$  for a p value = 0.35. (See the last line of screen 1.)



(1)

**Note:** Getting a statistic of 10 is a coincidence. If you generated another sample, the results would be different. All possible results would be approximately chi-square distributed.

### Activity 9, Testing Goodness-of-Fit and Two-Way Table Chi-Square (cont.)

A chi-square value of 10 is not that unusual for the sampling done at the beginning of this topic, so there is insufficient evidence for us to reject the hypothesis that the data came from a uniform distribution.

#### Binomial Distribution (Check on $\boxed{\text{MATH}}$ <PRB>7:randBin)

In Topic 25 (screen 45), **randBin** was used to simulate 100 binomial experiments with  $n = 10$  and  $p = 0.67$ . The **Histogram** plotted gave the following observed frequencies:

**X (L1):** 0 1 2 3 4 5 6 7 8 9 10  
**Observed (L2):** 0 0 0 2 3 12 23 26 24 10 0

Test the hypothesis that the data came from a binomial distribution with  $n = 10$  and  $p = 0.67$ .

With the random variable  $X$  (the number of successes) in **L1** and the observed frequency of each number of successes in **L2**, store the theoretical probabilities in **L3** by pressing  $\boxed{2\text{nd}}$  [DISTR] 0:binompdf( 10  $\boxed{.}$  0  $\boxed{.}$  67  $\boxed{\text{STO}}$   $\boxed{\rightarrow}$  **L3**, as shown in screen 2. Note that the probability of no successes is very small (**0.0000153**), so it does not surprise us when none of our 100 simulations produced ten failures (no successes).

Because there were 100 experiments, the theoretical frequencies are just 100 times the probabilities. These are stored in **L3** (shown in the last two lines in screen 2) for the results shown in screen 3. The first four rows have small expected frequencies, so we add them to the fifth row to get **7.32**, as shown in screen 4. (Many texts request the expected values be greater than 5 to assure the validity of the chi-square test.) The eleventh row had only 1.823 in it, so it was added to the 8.798 in the tenth row for 10.801. (See screen 4.) These additions must also be made in **L2**, so the first value will be  $0 + 0 + 0 + 2 + 3 = 5$  and the last value  $10 + 0 = 10$ .

```
binomPdf(10,.67→
L3
(.531578985e-5...
100*L3→L3
(.001531579 .03...
```

(2)

L1	L2	L3	3
0	0	.00153	
1	0	.0311	
2	0	.2841	
3	2	1.5382	
4	3	7.3200	
5	12	13.315	
6	23	22.528	
L3(5) = 5.46515073...			

(3)

```
L3(1)+L3(2)+L3(3
)+L3(4)+L3(5)→L3
(5)
7.320045674
L3(10)+L3(11)→L3
(10)
10.80099416
```

(4)

The results of the condensing on the previous page (with the help of the **DEL** key) are shown in screen 5, with the first row now representing four or fewer successes and the last row nine or ten successes. Note that the observed values in **L2** are fairly close to the expected values in **L3**.

Calculating the chi-square statistic (**1.78**), as shown in screen 6, we see that each contribution is small. Degrees of freedom =  $6 - 1 = 5$  because we reduced the number of cells to six by condensing.

The p-value of **0.879** (as on the last line in screen 6) indicates that our results are about what one would expect from the binomial distribution we hypothesized, so we certainly could not reject that hypothesis.

L1	L2	L3	1
4	5	7.32	
5	12	13.315	
6	23	22.528	
7	26	26.136	
8	24	19.889	
9	10	10.801	
-----			
L1(?)=			

(5)

$(L2-L3)^2/L3 \rightarrow L4$
C.7353249102 .1...
sum(L4)
1.780233835
$\chi^2cdf(1.78, 5$
.8786709693

(6)

## Activity 9, Testing Goodness-of-Fit and Two-Way Table Chi-Square (cont.)

### Topic 48—Two-Way Table Chi-Square Test

There was some concern that the wording of an exam was biased in favor of one sex. A random sample of the results for 263 test-takers is given in the table below. Test the null hypothesis that the grade on the exam is independent of the sex of the test-taker.

	Grade			Total
	Good	Fair	Poor	
Male	42	57	10	109
Female	43	95	16	154
Total				263

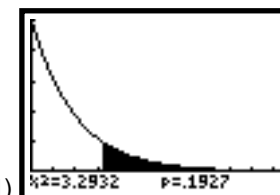
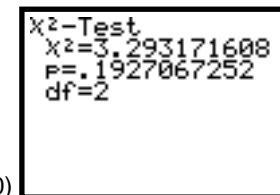
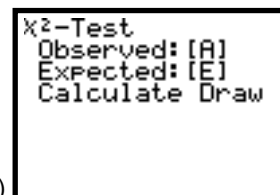
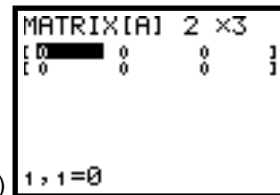
#### Storing Data in a Matrix

1. Press **MATRIX** **<EDIT>** **1:** **[A]** **2** **[ENTER]** **3** **[ENTER]** to display screen 7 with two rows and three columns. You may have values other than 0 in your matrix, but do not bother to change them because you will enter values over them.
2. Type **42** **[ENTER]** **57** **[ENTER]** **10** **[ENTER]** **43** **[ENTER]** **95** **[ENTER]** **16** **[ENTER]** to display screen 8. You can use the cursor control keys to edit any mistakes.

#### $\chi^2$ Test

1. Press **[STAT]** **<TESTS>** **C:**  **$\chi^2$ -Test** to display screen 9.
2. Paste **[A]** and **[E]** from **MATRIX** **<NAMES>** **1** and **5**.
3. Highlight **Calculate** or **Draw**, and press **[ENTER]** for screens 10 or 11.

With a large p-value of **0.1927**, there is insufficient evidence to reject the null hypothesis. If there were no relationship between grade and sex, it is not unusual to get this kind of result.



**Note:** If the sample size were twice as large and if the proportion in each cell the same (84 males and 86 females with good grades, and so on), this relationship would be significant with a p value of **0.037** (with males getting a significantly larger proportion of good grades than females and a significantly smaller proportion of fair grades—as you will see in the following section). Of course with a larger sample, the proportions could change.

## Expected Values

The expected values were calculated and stored in [E] and are shown in screen 12 with the original observed values in screen 13. Note that all expected values are greater than 5.

Some of the largest differences between observed and expected values are seen in the first column for those with the highest grades. If there were no relationship between grade and sex, we would expect about 35 good grades for the males. We had more than expected with 42. Because there were more females, we would expect more to receive good grades (about 50), but there were only 43. These differences are reasonable variations.

1. Transfer the data from matrix [A] to list L1, L2, and L3. You can do this by hand or by using  $\text{[2nd] [LIST] <OPS> A: Matr}\rightarrow\text{list( [A] [ ] L1 [ ] L2 [ ] L3 [ ] [ENTER] }$  for **DONE**.
2. Add the column totals into row 3 and the row totals into column L4 by hand or as explained on the first page of Topic 18.

The results are shown in screens 14 and 15.

As an example of how to calculate expected values from the spreadsheet, in step 3, we will calculate the second column of expected values on the home screen.

3. Proceed as shown in screen 16 by multiplying the sum of the first row times the sum of the second column and dividing by the grand total in L4(3) for **62.996**, and so on. Note that these agree with the values stored in matrix [E].

## Percents and Bar Charts

If you calculate the row percentages as done in Topic 18 and in screens 17 and 18, you see that 38.5 percent of males and 27.9 percent of females made good grades (32.3 percent of the sample total of 263 test-takers made good grades). These differences were not significant for this sample size. These percentages could be put in bar charts as you did in Topic 18.

## $\chi^2$ Contributions

If matrix [A] were transferred to L1, L2, and L3 and matrix [E] to L4, L5, and L6, then the  $\chi^2$  contributions could be calculated, as in screen 19, with the results shown in screen 20. The sum of all these values is the **3.293** of the  $\chi^2$ -TEST output. Notice that most of the contributions come from L1 (good grades).

(12)

MATRIX[E] 2 × 3			
[ 35.228	62.996	10.776 ]	
[ 49.772	89.004	15.224 ]	

(13)

MATRIX[A] 2 × 3			
[ 42	57	10	1
[ 43	95	16	1

(14)

L1	L2	L3	1	L4	3
42	57	10		109	
43	95	16		154	
85	152	26		263	
L1(4)=					

(15)

(16)

L4(1)*L2(3)/L4(3)	
62.99619772	
L4(2)*L2(3)/L4(3)	
89.00380228	

(17)

L1	L2	L3	1	L4	3
38.532	52.294	9.1743		100	
27.922	61.688	10.39		100	
32.319	57.795	9.8859		100	
L1(4)=					

(18)

(19)

(L1-L4)²/L4→L1	
(1.301747244 .9...	
(L2-L5)²/L5→L2	
(.570739003 .40...	
(L3-L6)²/L6→L3	
(.0558347712 .0...	

(19)

(20)

L1	L2	L3	1
1.3017	.57074	.05583	
.92137	.40396	.03952	
L1(3)=			

(20)

