NUMB3RS Activity: Magnetism
Episode: "Provenance"

Topic: Data collection, modeling **Grade level:** 11 - 12
Objective: To use transformations to find a mathematical model for the relationship between the strength of a magnet and distance.
Time: about 45 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

At one point in "Provenance," Larry and Amita watch as Charlie holds a battery with a screw attached by a tiny magnet to the positive (+) terminal. As he touches the negative (–) terminal with a copper wire, the screw spins. Like Larry and Amita, people have long been fascinated with the properties of magnets. In this activity, students will have an opportunity to explore what happens to the strength of a magnet as you move farther away from it.

Discuss with Students

This activity is based on the results of a lab designed for physics students, which involves the relationship between the strength of a magnet and the distance between the magnet and the place where the strength of the magnet was measured. The lab and a teacher's guide are available for download with this activity. To download these files, go to **http://education.ti.com/exchange**, and search for "7495." The "Magnetic Field of a Permanent Magnet" lab and the associated files are produced by Vernier Software & Technology.* Find additional materials, as well as a magnetic field sensor, at **www.vernier.com**.

Because the equipment needed for the lab is not found in most high school labs, it is suggested that you read through the lab with your students. Sample data has been provided in the activity for your students to analyze. This data was collected by Kenneth Appel and Clarence Bakken, authors of Physics with Calculators (Vernier).

The activity can be done without doing the lab, provided students have a basic sense for the goal of the lab and the results that are obtained. Be sure to emphasize the units (mT) used to measure the strength of a magnet. The mT stands for millitesla; informally speaking, tesla are used to measure a magnet's "power" divided by its area. Named in honor of the famous inventor and electrical engineer Nikola Testa, this unit was adopted in 1960. A more technical definition can be found at **http://www.answers.com/topic/tesla-unit**.

At the end of the lab students will discover that the magnitude of a small, powerful magnet varies inversely with the cube of the distance, measured from the center of the magnet and along the axis of the magnet. In the lab, the inverse cube relationship is determined using a power regression on the graphing calculator.

The main goal of this activity is to provide students with an alternate method to find the equation that does not make use of power regression. This method involves "linearizing" the data so a proportional model of the form $y = kx$ can be used.

* "Magnetic Field of a Permanent Magnet" files used with permission.

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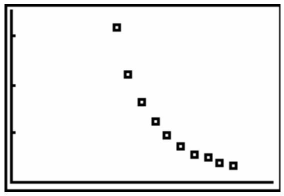
Student Page Answers:

1.

```
WINDOW
 Xmin=0
 Xmax=.05
 Xscl=.1
 Ymin=0
 Ymax=3.5
 Yscl=1
 Xres=1
```

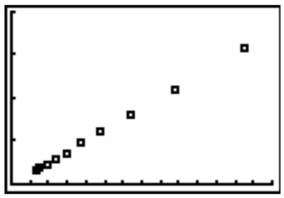


Graph of L_2 versus L_1

It is not immediately obvious what equation could be used to model this data. A linear model does not fit the data well

2a.

```
WINDOW
 Xmin=0
 Xmax=140000
 Xscl=10000
 Ymin=0
 Ymax=4
 Yscl=1
 Xres=1
```



Graph of L_2 versus L_3

The graph is linear since it has the form y = kz where, in this case, $z = \frac{1}{x^3}$.

2b. *$k \approx 0.000025$; Choose a point in the table of data to determine the slope k of the line:*
e.g., $\dfrac{3.165}{\dfrac{1}{0.02^3}} = 3.165(0.02^3)$ **2c.** *$k \approx A$* **3a.** *$p = 4$* **3b.** *$y \approx 10x$ fits the linearized data, so $y \approx 10x^4$ fits the original data.*

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Name: ________________________________ Date: ________________________

NUMB3RS Activity: Magnetism

At one point in "Provenance," Larry and Amita watch as Charlie holds a battery with a screw attached by a tiny magnet to the positive (+) terminal. As he touches the negative (–) terminal with a copper wire, the screw spins. Like Larry and Amita, people have long been fascinated with the properties of magnets. In this activity, you will have an opportunity to explore the relationship between the strength of a magnet and the distance between the magnet and the place where the strength of the magnet was measured.

This activity is based on a lab that makes use of a TI-83 Plus/TI-84 Plus graphing calculator and a Magnetic Field Sensor. The lab requires specialized equipment that is not found in most high schools, so instead of doing the lab, your teacher will guide you through a brief discussion of it. Sample data (Table 1) has been provided for your analysis. This data was collected by Kenneth Appel and Clarence Bakken, authors of Physics with Calculators (Vernier).

Also, you may not be familiar with the units (mT) used for the magnetic field. The mT stands for millitesla. Informally speaking, the tesla is used to measure a magnet's "power" divided by its area. This unit was adopted in 1960, and was named in honor of the famous inventor and electrical engineer Nikola Tesla. A more technical definition can be found at the website **http://www.answers.com/topic/tesla-unit**.

1. On your calculator, enter the distance data into list L_1 and the magnetic field strength data into list L_2, or simply download the list files L1.8xl and L2.8xl from **education.ti.com** (search for "7495"). (Note that in the table view, your calculator will display only the first several digits of each value.) Graph the magnetic field strength (L_2) versus the distance (L_1). What types of functions could be used to model this data? How well does a linear model fit the data?

Table 1

x distance (m) L_1	*y* magnetic field (mT) L_2
0.02	3.165
0.0225	2.222880658
0.025	1.62048
0.0275	1.217490609
0.03	0.9377777778
0.0325	0.7375876195
0.035	0.5905539359
0.0375	0.4801422222
0.04	0.395625
0.0425	0.3298351313

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2. Many different types of functions could be used to model the data in Table 1. All that we really know is that the function decreases as x increases. Possible functions include $y = \frac{A}{x}$, $y = \frac{A}{x^2}$, $y = \frac{A}{x^3}$ and so on or even exponential functions such as $y = A(10^{-x})$. Had this data been linear, we would have known immediately that a linear function was involved. From reading the lab, you already know that the strength of the magnetic field y varies inversely with the cube of the distance x. That is, $y = \frac{A}{x^3}$ for some value of A. Because $y = A\left(\frac{1}{x^3}\right)$, we can "straighten out" this data by first entering the following command on your calculator:

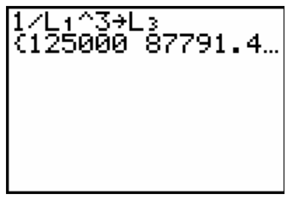

Figure 1

a. Now Graph L_2 (y-values) versus L_3 (x-values). Why is the graph linear?

b. Fit a line of the form $y = kx$ to this data. How did you find your value of k?

c. What is the relationship between your value of k and the value of A in the "inverse cube" model $y = \frac{A}{x^3}$?

3. Clear the data lists in your calculator. Then enter the data for x and y (Table 2) into lists L_1 and L_2 on your calculator.

a. Experiment with different integer values of p in the transformation $L_1^p \to L_3$ so that a linear graph is produced for the relationship between the transformed data in L_3 (x-values) and the data in L_2 (y-values).

b. Find the equation of this line and describe how to use it to find an equation fitting the data in L_1 and L_2.

Table 2

x L_1	y L_2
0.24	0.03
0.37	0.19
0.51	0.68
0.58	1.13
0.76	3.34
0.92	7.16

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

1. Use lists L_1 and L_2 from Question 1 of the Student Page. Clear the entries in L_3. Then enter the expressions shown in Figure 2.

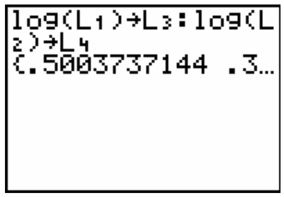

Figure 2

a. Graph L_4 (*y*-values) versus L_3 (*x*-values). Do you immediately recognize what type of function could be used to model this data? Do the results surprise you?

b. If you knew the specific equation that models the graph in part (a), describe how you could use it to find an equation that models the original data in L_1 and L_2?

c. Why did the transformations of the data in L_1 and L_2 lead to a situation in which the graph of L_4 versus L_3 was a straight line? What type of relationship needs to exist between the data in L_2 and L_1 in order for the relationship between the data in L_4 and L_3 to be linear?

2. The least squares formula for *k* in the proportional model $y = kx$ is $k = \frac{\sum x_i y_i}{\sum x_i^2}$.

a. Suppose for simplicity there are three data points: (x_1, y_1), (x_2, y_2), and (x_3, y_3). We seek to find *k* so the sum of the squared errors in minimized, i.e. the value of *k* for which $SSE(k) = (y_1 - kx_1)^2 + (y_2 - kx_2)^2 + (y_3 - kx_3)^2$ is the smallest. Now $SSE(k) = (x_1^2 + x_2^2 + x_3^2)k^2 - 2(x_1y_1 + x_2y_2 + x_3y_3)k + (y_1^2 + y_2^2 + y_3^2)$, which is a quadratic function of *k*. Why is $k = \frac{\sum x_i y_i}{\sum x_i^2}$?

b. Using this formula, compute the value of *A* in Question 2 and the value of the parameter in Question 3.

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