

EXTENDED LAW OF SINES
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INSTRUCTIONS FOR STUDENTS

NCTM EXPECTATIONS

GRADES 9-12

- explore relationships (including congruence and similarity) among classes two- and three-dimensional geometry objects, make and test conjectures about them, and solve problems involving them;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use trigonometric relationships to determine lengths and angle measures. (NCTM, 2000, p.397)

PUERTO RICO INDICATORS

- G.FG.11.5.5** Develop the Laws of Sine, the Law of Cosine and use them to determine lengths and angle measures in a triangle.
- G.FG.11.6.1** Establish conjectures based on the exploration of geometric situations with or without the use of technology.
- G.FG.11.6.2** Use direct or indirect proof to determine if a mathematical proposition is true.
- G.FG.11.6.5** Organize and present direct and indirect proof using tables of two columns, paragraphs, and flux diagrams.

INTRODUCTION

This activity is to be used with pre-service mathematics teachers or in service teachers in a project of professional development. The main goal is to demonstrate how to use the TI-nspire technology to enhance the students capacity to elaborate and demonstrate conjectures. The elaboration of conjectures is fundamental in the development of the mathematical reasoning. With the information provided and the previous knowledge, the students form a judgment on the content that are studying Beginning in the early years of schooling, teachers can help students learn to make geometric conjectures. The National Council of Teachers of Mathematics (NCTM) recommends that teachers formulate questions similar to the following ones in the mathematical situations presented: What do you think will happen next? What is the pattern? Is this always true? What will happen if...? (NCTM, 2000, p57).

INSTRUCTIONS

Students will work this activity in groups of three. Before beginning be sure that each one of your group mates has a copy of this document.

<ol style="list-style-type: none">1. Open the file Extended Law of Sines.tns on your TI-nspire handheld.2. Read pages 1, 2, and 3.3. Discuss with your group mates the question on page 4. Write your answer in the space that is provided.	
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4. Study the graphical and algebraic representations of the Extended Law of Sines that are provided on pages 5 and 6.
5. Read the instructions that are provided on page 7 and do the measurements that are required (Figure 1).
6. Verify that the Extended Law of Sines holds, moving the vertices of the inscribed circle on page 8 (Figure 2).
7. The proof of the Extended Law of Sines is based on two theorems that have been seen previously:
 - a. The angle inscribed in a semicircle is a right angle.
 - b. Given B and C, two fixed points on a circle, for two points A and J on the circle $\angle BAC$ and $\angle BJC$ are congruent or supplementary.
8. Make the measurements that are requested in page 10 (Figure 3).

1.4 1.5 1.6 1.7 DEG AUTO REAL

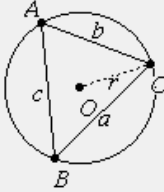
Exercise

On the next page measure angle A, the radius of the circle and the length of side a, and use *calculate* to verify that the Extended Law of Sines holds for angle A and side a.

Figure 1

1.5 1.6 1.7 1.8 DEG AUTO REAL

The Extended Law of Sines 1 cm



$\Delta A =$

$x = a =$

$r =$

$\frac{x}{\sin(A)} =$

$2r =$

Figure 2

1.7 1.8 1.9 1.10 DEG AUTO REAL

On the next page, measure $\angle FDE$, $\angle BAC$ and $\angle BJC$.

Move the points on the circle to verify that the two facts on the previous page really hold.

Figure 3

9. Verify that the two properties always hold true .
(page 11-Figure 4).

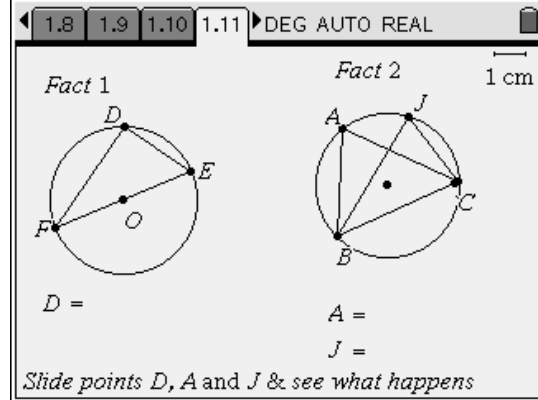


Figure 4

10. Describe thoroughly, with the help of your group mates, the figure on page 12 (Figure 5). Write your answer on the space bellow.

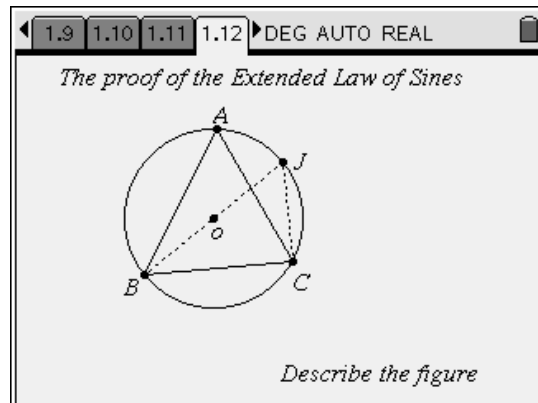


Figure 5

11. Discuss with your group mates the reason for the proof of the Extended Law of Sines that is provided (page 14-Figure 6).

Statements	Reasons
$\sin J = \frac{a}{BJ}$	
$\sin J = \frac{a}{2r}$	
$\frac{a}{\sin A} = \frac{a}{\sin J} = 2r$	

Figure 6

12. Explain why the proof does not fulfill all the cases.

13. Explain why the Extended Law of Sines also is true in the case that $\angle BAC$ and $\angle B/C$ are supplementary.

14. Explore with your group mates the relationship between the center of the circle and the circumcenter of the triangle (Figure 7).

15. Write a conjecture about this relationship.

EXTENSION

Prove that:

1. the perpendicular bisectors of the sides of a triangle concur in a point
2. the point of concurrence of the perpendicular bisectors of the sides of a triangle is the center of the circle circumscribed to the triangle.

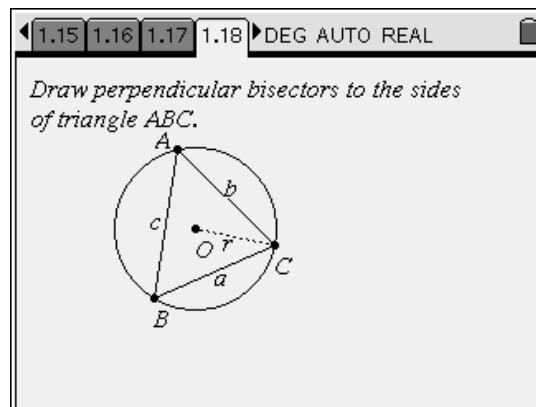


Figure 7

References

- Bulajich-Manfrino, R., & Gómez- Ortega, J. A. (2002). Geometría: Cuadernos de Olimpiadas Matemáticas. México: UNAM.
- Coxeter, H. S. M., and Greitzer, S. L. (1967). Geometry Revisited. Washington, D.C.: The Mathematical Association of America.
- National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: Author.