Derive Summation Formulas Creatively

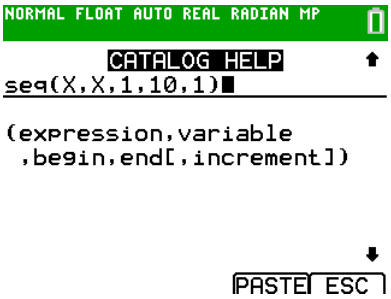
TI PROFESSIONAL DEVELOPMENT **TEACHER NOTES AND SOLUTIONS**

1. We are going to derive the formula for the sum of the first *n* perfect squares.
That is:

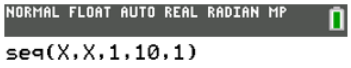
$1 + 4 + 9 + \ldots + n^2 = ?$

2. First enter the natural numbers from 1 to 10 into list L1 using the seq(command. To access it: 2nd 0 ln (for 'S'), scroll down to seq(.

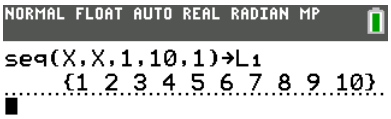
3. Press + to access the help screen and type the following:



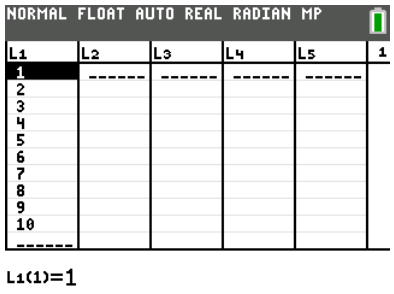
Press trace to PASTE.



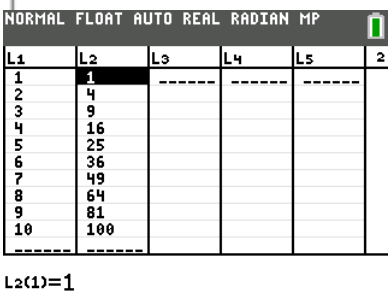
4. Store this list into L1:
sto→ 2nd 1 enter



5. View the list in L1:
stat enter

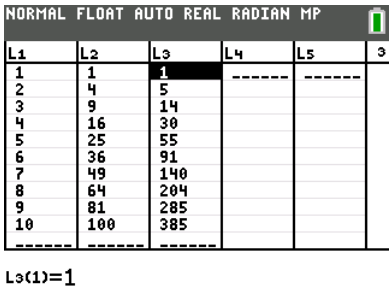


6. Place the squares of the values in list L1 into list L2.
Go to the very top of list L2 and enter the following:
2nd 1 x^2 enter

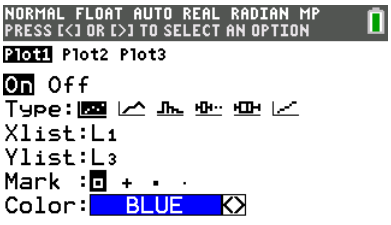


7. Place the cumulative sum of the perfect squares listed in list L2 into list L3. Go to the very top of list L3 and enter the following:
2nd stat ▶ 6 2nd 2 enter

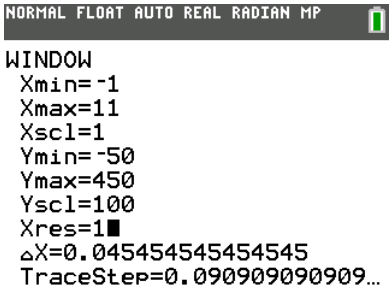
8. The results are listed below.



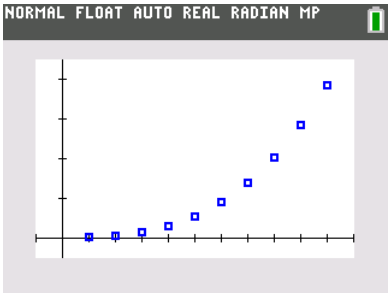
9. Plot list L3 versus list L1 using an appropriate window.
2nd y= enter . Set as shown below:



10. Set the window as shown below:



11. Press graph.

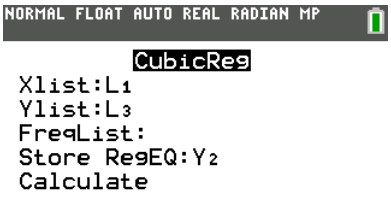


What type of curve can model this data?

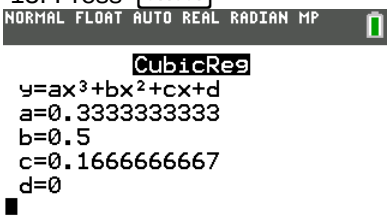
Cubic, other answers possible

12. To save time, we will model with a cubic regression equation. stat ▸ 6.

Set as shown below:



13. Press enter



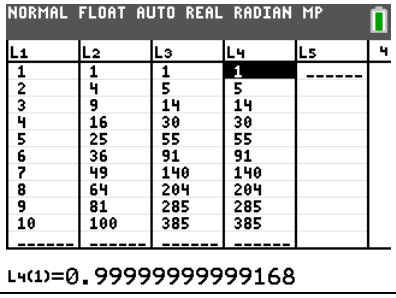
14. Notice that the decimals for a, b, and c can be written as common fractions. Write those values below:

$a = \frac{1}{3} \qquad b = \frac{1}{2} \qquad c = \frac{1}{6}$

Rewrite the regression equation using the fractions instead of the decimals.

$$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$$

15. We can check that this regression equation in Y2 is correct. Into the top of list L4, type L4=Y2(L1)



16. Using the equation in number 14, use some algebra to write the expression as a single rational expression and in factored form.

$$y = \frac{2}{6}x^3 + \frac{3}{6}x^2 + \frac{1}{6}x$$

$$y = \frac{1}{6}x\left(2x^2 + 3x + 1\right)$$

$$y = \frac{1}{6}x\left(2x+1\right)\left(x+1\right)$$

$$y = \frac{x\left(2x+1\right)\left(x+1\right)}{6}$$

17. With your teacher's assistance, write this as a formula using proper mathematical notation.

$$1 + 4 + 9 + \ldots + n^2 = \sum_{i=1}^{n}(i^2) = \frac{n(2n+1)(n+1)}{6}$$

18. Other summation formulas that you can derive the summation formulas include:

$1 + 2 + 3 + \ldots + n = ?$

$1^3 + 2^3 + 3^3 + \ldots + n^3 = ?$

$2 + 4 + 6 + \ldots + 2n = ?$

$3 + 6 + 9 + \ldots + 3n = ?$

$1 + 3 + 5 + \ldots + (2n-1) = ?$