



Derive Summation Formulas Creatively

TI PROFESSIONAL DEVELOPMENT

TEACHER NOTES AND SOLUTIONS

1. We are going to derive the formula for the sum of the first n perfect squares.
That is:

$$1 + 4 + 9 + \dots + n^2 = ?$$

2. First enter the natural numbers from 1 to 10 into list L1 using the seq(command. To access it: **2nd** **0** **ln** (for 'S'), scroll down to seq(.

NORMAL FLOAT AUTO REAL RADIAN MP **0**

CATALOG
2-SampTInt
2-SampTTest
2-SampZInt(
2-SampZTest(
Scatter
Sci
Select(
Send(
▶seq(

3. Press **+** to access the help screen and type the following:

NORMAL FLOAT AUTO REAL RADIAN MP **0**

CATALOG HELP **↑**

seq(X,X,1,10,1) **↓**

(expression,variable
,begin,end[,increment])

Press **trace** to PASTE.

NORMAL FLOAT AUTO REAL RADIAN MP **0**

seq(X,X,1,10,1)

4. Store this list into L1:

sto→ **2nd** **1** **enter**

NORMAL FLOAT AUTO REAL RADIAN MP **0**

seq(X,X,1,10,1)→L1
{1 2 3 4 5 6 7 8 9 10}

5. View the list in L1:

stat **enter**

NORMAL FLOAT AUTO REAL RADIAN MP **0**

| L1 | L2 | L3 | L4 | L5 | 1 |
|----|----|----|----|----|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | | | | | |

L1(1)=1

6. Place the squares of the values in list L1 into list L2.
Go to the very top of list L2 and enter the following:

2nd **1** **x²** **enter**

L2=L1²

NORMAL FLOAT AUTO REAL RADIAN MP **0**

| L1 | L2 | L3 | L4 | L5 | 2 |
|----|-----|----|----|----|---|
| 1 | 1 | | | | |
| 2 | 4 | | | | |
| 3 | 9 | | | | |
| 4 | 16 | | | | |
| 5 | 25 | | | | |
| 6 | 36 | | | | |
| 7 | 49 | | | | |
| 8 | 64 | | | | |
| 9 | 81 | | | | |
| 10 | 100 | | | | |

L2(1)=1

7. Place the cumulative sum of the perfect squares listed in list L2 into list L3. Go to the very top of list L3 and enter the following:

2nd **stat** **▶** **6** **2nd** **2** **enter**

8. The results are listed below.

NORMAL FLOAT AUTO REAL RADIAN MP **0**

| L1 | L2 | L3 | L4 | L5 | 3 |
|----|-----|-----|----|----|---|
| 1 | 1 | 1 | | | |
| 2 | 4 | 5 | | | |
| 3 | 9 | 14 | | | |
| 4 | 16 | 30 | | | |
| 5 | 25 | 55 | | | |
| 6 | 36 | 91 | | | |
| 7 | 49 | 140 | | | |
| 8 | 64 | 204 | | | |
| 9 | 81 | 285 | | | |
| 10 | 100 | 385 | | | |

L3(1)=1

9. Plot list L3 versus list L1 using an appropriate window.
2nd **y=** **enter**. Set as shown below:

NORMAL FLOAT AUTO REAL RADIAN MP **0**

PRESS [◀] OR [▶] TO SELECT AN OPTION

Plot1 Plot2 Plot3

On Off

Type: **Eq** **Y=** **Ln** **Exp** **Log** **Ln**

Xlist: L1

Ylist: L3

Mark: **+** ***** **o** **x** **+** ***** **o** **x**

Color: **BLUE** **↔**



Derive Summation Formulas Creatively

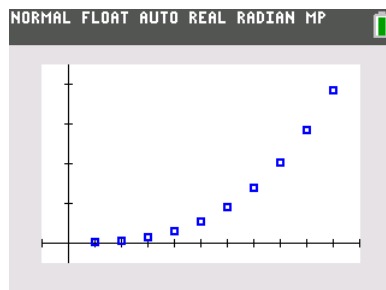
TI PROFESSIONAL DEVELOPMENT

TEACHER NOTES AND SOLUTIONS

10. Set the window as shown below:

```
NORMAL FLOAT AUTO REAL RADIAN MP
WINDOW
Xmin=-1
Xmax=11
Xscl=1
Ymin=-50
Ymax=450
Yscl=100
Xres=1
ΔX=0.045454545454545
TraceStep=0.090909090909...
```

11. Press **graph**.



What type of curve can model this data?

Cubic, other answers possible

12. To save time, we will model with a cubic regression equation.

stat **►** **6**.

Set as shown below:

```
NORMAL FLOAT AUTO REAL RADIAN MP
CubicReg
Xlist:L1
Ylist:L3
FreqList:
Store RegEQ:Y2
Calculate
```

13. Press **enter**.

```
NORMAL FLOAT AUTO REAL RADIAN MP
CubicReg
y=ax^3+bx^2+cx+d
a=0.3333333333
b=0.5
c=0.1666666667
d=0
```

14. Notice that the decimals for a, b, and c can be written as common fractions. Write those values below:

$$a = \frac{1}{3} \quad b = \frac{1}{2} \quad c = \frac{1}{6}$$

Rewrite the regression equation using the fractions instead of the decimals.

$$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$$

15. We can check that this regression equation in Y2 is correct. Into the top of list L4, type **L4=Y2(L1)**.

| L1 | L2 | L3 | L4 | L5 | |
|----|-----|-----|-----|----|--|
| 1 | 1 | 1 | 1 | | |
| 2 | 4 | 5 | 5 | | |
| 3 | 9 | 14 | 14 | | |
| 4 | 16 | 30 | 30 | | |
| 5 | 25 | 55 | 55 | | |
| 6 | 36 | 91 | 91 | | |
| 7 | 49 | 140 | 140 | | |
| 8 | 64 | 204 | 204 | | |
| 9 | 81 | 285 | 285 | | |
| 10 | 100 | 385 | 385 | | |

L4(L1)=0.999999999999168

16. Using the equation in number 14, use some algebra to write the expression as a single rational expression and in factored form.

$$y = \frac{2}{6}x^3 + \frac{3}{6}x^2 + \frac{1}{6}x$$

$$y = \frac{1}{6}x(2x^2 + 3x + 1)$$

$$y = \frac{1}{6}x(2x+1)(x+1)$$

$$y = \frac{x(2x+1)(x+1)}{6}$$

17. With your teacher's assistance, write this as a formula using proper mathematical notation.

$$1 + 4 + 9 + \dots + n^2 = \sum_{i=1}^n (i^2) = \frac{n(2n+1)(n+1)}{6}$$

18. Other summation formulas that you can derive the summation formulas include:

$$1 + 2 + 3 + \dots + n = ?$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = ?$$

$$2 + 4 + 6 + \dots + 2n = ?$$

$$3 + 6 + 9 + \dots + 3n = ?$$

$$1 + 3 + 5 + \dots + (2n-1) = ?$$