NUMB3RS Activity: Galton Board
Episode: "Spree, Part II – Daughters"

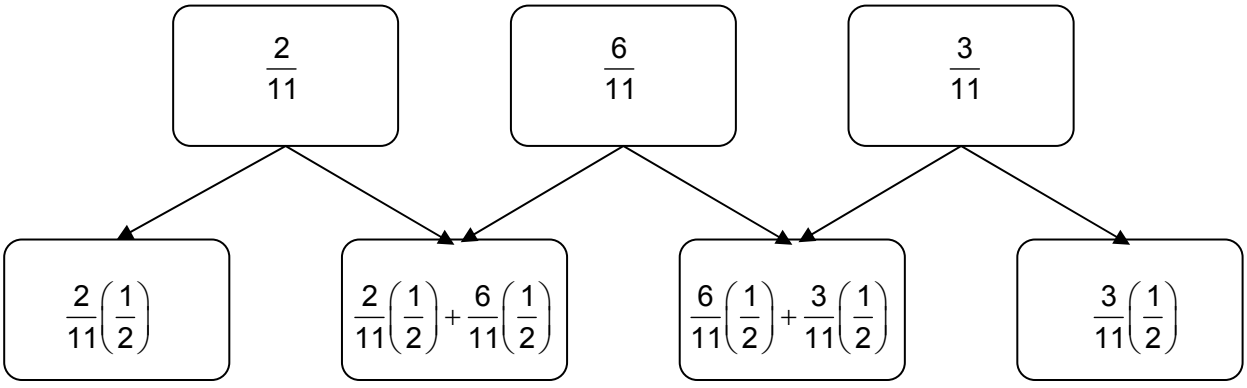
Topic: Probability **Grade Level:** 9 - 12
Objective: Determine probabilities on a Galton board
Time: 20 - 30 minutes

Introduction

In "Spree Part II," the FBI is trying to rescue a kidnapping victim. Charlie discusses how this problem is similar to balls falling through a Galton board. A Galton board is a vertical (or inclined) board with a series of staggered pins arranged in a triangle. As a ball is dropped through the board it strikes the pins. At each pin, half of the time the ball will fall to the right and half of the time the ball will fall to the left. At the bottom of the board, the balls land in numbered (or lettered) bins. If enough balls are dropped, the results approximate a bell curve. This activity examines the probability of balls ending up in specific bins and how that probability is affected by placing obstacles on the board.

Discuss with Students

To better see the calculations, have the students look at a section of our Galton board with some unusual fractions. Completing the calculations can aid the students to determine what value would be in the boxes. Point out to students that when a ball hits a pin, there are two possibilities – the ball falls to the left or the ball falls to the right. Thus, the probability of following any one path is $\frac{1}{2}$, which is why we multiply by $\frac{1}{2}$.



Student Page Answers:
1. *A = 1/64, B = 3/32, C = 15/64, D = 5/16, E = 15/64, F = 3/32, G = 1/64* **2.** *Students should notice that the distribution is symmetric with D being the largest and decreasing as it moves towards the edges.* **3.** *A = 1/8, B = 1/8, C = 3/4, D = 0, E = 0* **4.** *D and E* **5.** *Answers will vary.*

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Name: _____ Date: _____

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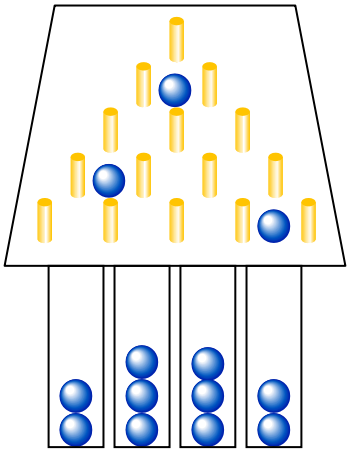
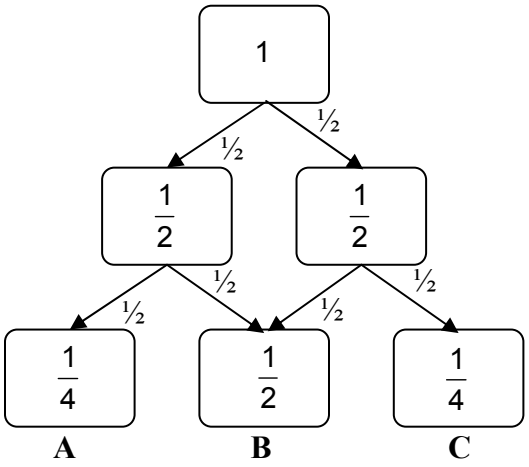
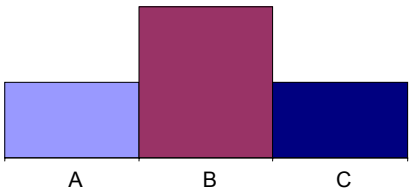


The theoretical probability of a ball following a particular path can be represented using an arrangement of boxes as shown at the right. At each pin, the probability is multiplied by $\frac{1}{2}$. Notice that the sum of each row of boxes equals one.



The distribution of cells A, B, and C approximate a normal curve.



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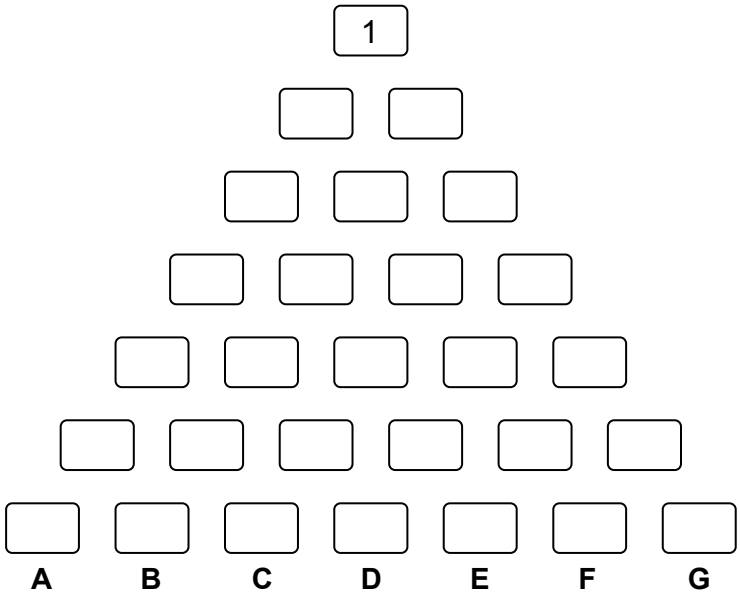
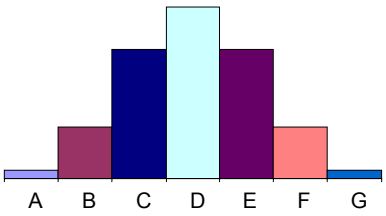
1. The diagram below represents a Galton board with 7 rows of pins. In each box, calculate the probability that a ball will fall in that direction. Remember to use a probability of $\frac{1}{2}$ for each direction, and remember that the sum of each row must equal 1.

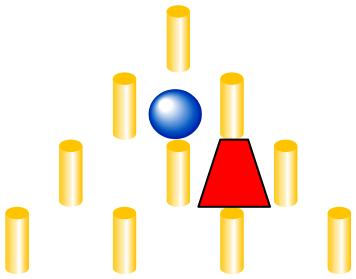


2. Your distribution should match the graph below. What conclusions can you draw from this distribution?



Now that you have explored basic Galton boards, consider a more interesting scenario: What would happen if some of the paths were obstructed?

For example, look at the Galton board to the right. If a pin is blocked, the number of possible paths for the ball changes; the ball can now move only to the left. This means the probability that the ball will move to the left is now 1, instead of $\frac{1}{2}$.



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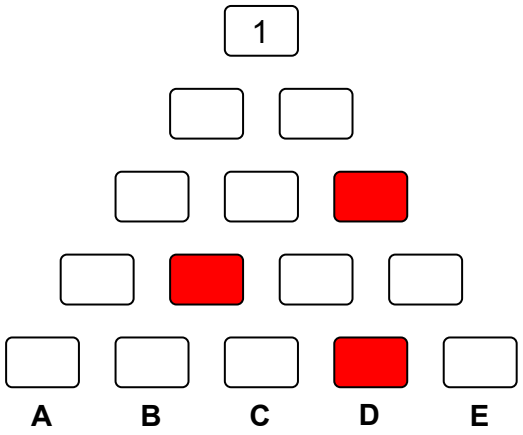
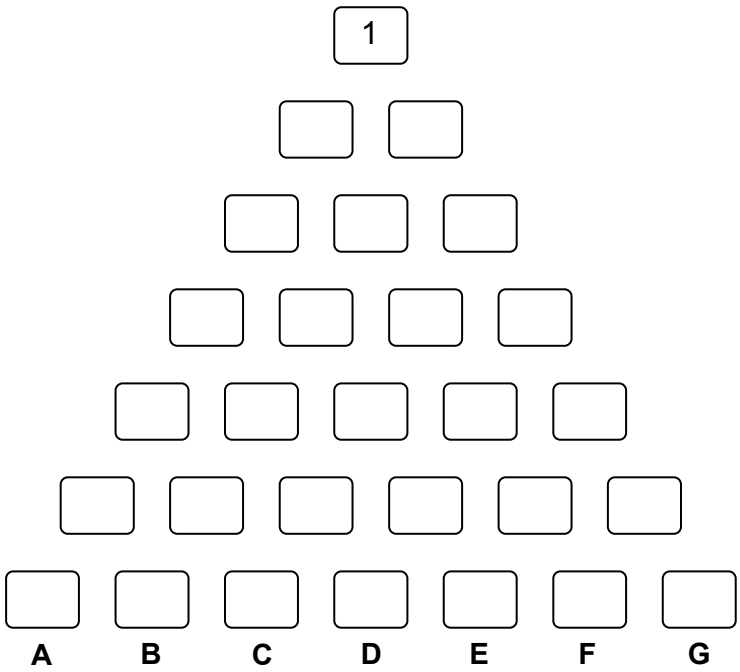
3. The figure below represents a Galton board with some obstructions. Calculate the probabilities for this board.



4. Are there any bins in which the ball can no longer land?

5. On the Galton board below, place four obstructions in any of the boxes. Select one bin as the capture bin (the bin where you want the ball to land). Determine the probability that the ball will land in each bin, and plot the distribution. Compare your results with the results of other students in your class. How do various configurations change the distribution? Is there a particular distribution that makes the probability of a ball landing in bin A more likely than it landing in any other bin?



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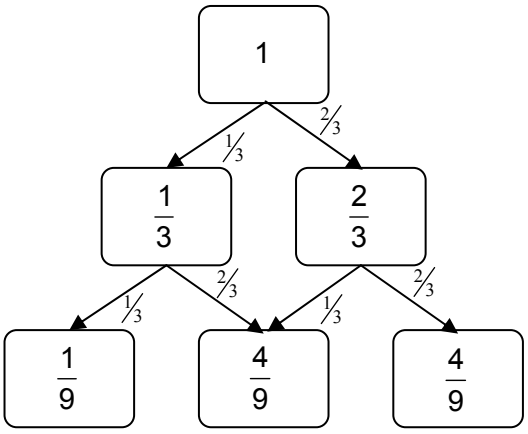
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Another variation of the Galton board is a biased Galton board. This board would favor some outcomes more than others. For example, on the board at the right, it is assumed that the probability of the ball falling to the left is $\frac{1}{3}$, and the probability of the ball falling to the right is $\frac{2}{3}$.



1. Calculate the probabilities for the biased Galton Board above through 5 rows. Then graph the resulting distribution.

2. What do you notice about the symmetry of the distribution compared to an unbiased Galton board?

Additional Resources

- A Web site with an applet that shows a Galton board in action can be found at: **http://teacherlink.org/content/math/interactive/flash/quincunx/quincunx.html**

- This Web site is dedicated to the work of Sir Francis Galton, for whom the Galton board is named: **http://galton.org/**.

- On the game show *The Price is Right*, a game called "Plinko" uses a Galton board to determine what amounts of money a contestant will win. This Web site has a "Plinko" simulation for the TI-83 Plus/TI-84 Plus and TI-89 graphing calculators: **http://mathdemos.gcsu.edu/mathdemos/plinko/**

- A *NUMB3RS* Activity for the season 2 episode "Soft Target" uses the TI-Navigator™ system to simulate a Galton board. This activity can be downloaded by going to **http://education.ti.com/exchange** and searching for "6584."

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