Activity Title: Paths of Rectangles

1. In this exploration, you will look at how the sides of rectangles with equal areas are related. Open the file: PathsOfRectangle.tns on your Handheld. In problem 1, on the second page is a graph of a rectangle constructed with a vertex at the origin.

* Moving point A at the top will change the area of rectangle.

* Moving point B on the x-axis will display rectangles in the "family" of rectangles that have the same area.

* Watch the path that point C (it's coordinates are displayed) makes as you move point B.

* Explore at least 4 families of rectangles to determine the general shape of the curve point C makes.

Write your answers to the questions below:

*1. Describe **in words** the curves you found.*

2. Return to the graph and construct the locus of point C as point B moves along the x-axis.

Press (menu), (9), (6) *for Menu, Construction, Locus then choose point C and then point B.*
***Describe in words** the curve generated.*

3. Describe the shape of the curve for large areas? (Move point A)

4. How does the shape change for small areas?

On the third page, we will input the data into a spreadsheet and graph the results as a scatter plot to help us determine the equation of the curve. The spreadsheet & graph windows have been set for you. Go to the shaded area of the spread sheet and in the command

 dxlength:=capture(xlength, 0)

Change the **0 to a 1**.

Do the same to the column **dxwidth**.

*This command will automatically capture the coordinates of point C as you change the shape of the rectangles on the previous page. Each rectangle will have area equal to your set value at A. (**Do not change A** for this portion of the activity.)*

After you make these changes, return to the graph on page 2 and drag point B to collect your data points. When you are satisfied that you have captured sufficient data return to your spreadsheet.

You may need to adjust the scale of your window so that your data points are displayed. Go to the Window Settings and set an appropriate viewing window for your area. Record your settings below:

Area: Window: Xmin: Ymin:

Xmax: Ymax:

XScale: YScale:

Each of these rectangles has the same area, **demonstrate this characteristic in Column C** *by using your data in column A and B.*

*How can you use the information above to form an equation? Find the equation and graph it on your graph with the locus. (Control G will bring up a line for you to type in your equation to graph)

* Describe how you can find equation for the path of any family of rectangles.

* How would the problem change if instead of point B, we were to move the point on the y-axis to change the size of the rectangles?

* Will other polygons demonstrate the same relationship? Explain.

Problem two *introduces the* **same explorations with a triangle**.

> ***The Paths of Triangles:***
>
> *In this exploration, you will look at how triangles with equal areas are related. This time however, we will look at the base and height rather than the sides. (Why?)*
>
> *On the page 2 of problem two, you'll see a graph of a triangle constructed with a vertex at the origin.*
>
> ** Moving point A at the top will change the area of the triangle.*
> ** Moving point B on the x-axis will display triangles in the "family" of triangles that have the same area.*
> ** Again watch the **path that point C** ((it's coordinates are displayed**) makes** as you move point B.*
> ** This time, also move point C. What path can C move on?*

Write your answers to the questions below:

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*1. Describe **in words** the curves you found.*

2. Return to the graph and construct the locus of point C as point B moves along the x-axis.

***Describe in words** the curve you found.*

3. Describe the shape of the curve for large areas? (Move point A)

4. How does the shape change for small areas?

Repeat # 1 - 4, but instead, this time choose point C twice. How is the locus different from above? Why do you think this happened? How can you use the information to find the equation for the path of point C?

As with the rectangle exploration, let's use a spreadsheet and scatterplot to look at our data. This time you will need to set up your own windows.

*1. First insert a new page after the triangle sketch, by pressing (ctrl) (⌂) for the **TOOL** menu, and choosing (4) "**Insert**" and (2) "**Page.**"*

*2. Press (menu) and choose (3) **Add Lists and Spreadsheet.***

3. Scroll to the top of column A and name the column for the x-coordinate of the triangle.

*4. In the gray area, we will need the command to input data. Press (menu), (3), (2), (1) for Menu, Data, Data Capture, Automated Data Capture. "VAR" will be highlighted. Since the x-coordinate for point C has been stored in the variable, **xtricoord**, press the (stoᵛar) key and choose **xtricoord**.*
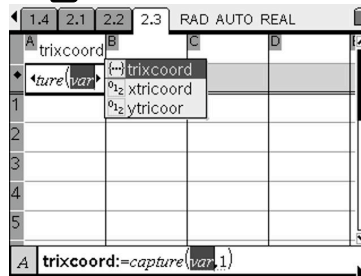
*5. Do the same in Column B for the y-coordinate of point C only this time choose **ytricoor** for the variable.*

*6. To add a window for the scatterplot, use the **TOOL** menu by pressing (ctrl) (⌂) (clear) Choose (5) **PAGE LAYOUT**, and (2) **SELECT LAYOUT** to set your window layout. In the rectangle exploration, we used layout (2)*
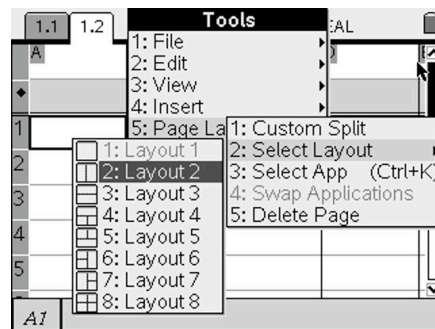
*7. Press (ctrl) (tab) to switch to your new window and pres (menu) and this time choose (2) **Add Graphs & Geometry.***

8. You will need to change the graph to a scatter plot. Press (menu), (3), (4) for Menu, Graph Type, Scatterplot.

9. An x and y drop down menu will appear. Choose your column A name for x and your column B name for y.

10. Now you are set to go back to the graph with the triangle and collect your data as before. Use your data and scatter plot to find an equation that describes the path point C follows when point B is moved.

Questions for discussion after activity is completed:

1. What mathematics is involved in these activities? Where in a curriculum would these activities fit?

2. What would a student need to know to complete this activity?

3. What does the technology add to the activity? How does it detract?

4. Can the activity be completed without the technology? Why or why not?

5. The sketches for the students were constructed for them. If the students were to construct them, how would the mathematics change? What would a student need to know to construct these sketches?

6. Can this activity be extended to other polynomials? How?

Read: Lapp, Douglas, & Denny St. John. John *"Dynamically Connected Representations: A Powerful Tool for the Teaching and Learning of Mathematics."*
Available from:
http://www.cst.cmich.edu/users/lapp1da/Site/Webpage_for_Dr._Douglas_A._Lapp_files/LappStJohnDynamicRe p2009.pdf