Applications of Critical Points

TI-Nspire CX Family

Math Objectives

Students will be able to:

- Identify critical points and local extrema for applications.
- Recognize that the sign of the derivative changes at a local minimum or maximum.

Vocabulary

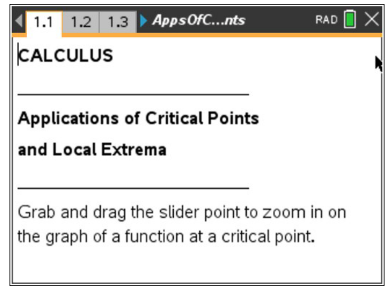
- critical point
- local maximum, minimum, extrema

About the Lesson

- This lesson is a follow-up lesson to the Calculus activity *Critical Points and Local Extrema* and is designed to help students visualize the connections between critical points and local extrema.
- This lesson is meant to provide motivation for the **first derivative test** as a means to identify local extrema. Students will examine the slope of the tangent line as it approaches a critical point and connect this to an understanding of local extrema.
- This activity builds on students' familiarity with the concept of the derivative at a point as the local slope of the function graph at that point.

Related Lessons

- Prior to this lesson: Critical Points and Local Extrema
- After this lesson: First Derivative Test



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Move a slider bar

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- When on a page having multiple screens in its layout, (ctrl) (tab) will move from screen to screen. If the pointer arrow cursor is available, you can simply move to another screen pane using it, but you must click once inside the pane to make it active.
- An active screen pane has a bold outline around it.

Lesson Materials:

Student Activity

AppsOfCriticalPoints_Student.pdf

AppsOfCriticalPoints_Student.doc

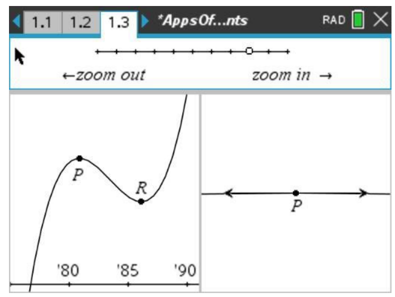
TI-Nspire document

AppsOfCriticalPoints.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand ( ) getting ready to grab the object. Also, be sure that the word point appears. Then press ctrl to grab the object and close the hand ( ). When finished moving the object, press esc to release.

Move to page 1.3.



1. Grab and drag the slider point to zoom in on point *P*.
   a. When you zoom in, what do you notice about the shape of the graph at point *P*?

      Answer: The graph flattens out and becomes a straight line. The curve and the tangent line appear to be the same, and they appear to have the same slope.

> **Teacher Tip:** This is a good time to remind students that if a function is differentiable at a point, it will appear to be linear as you zoom in on the point. The derivative can be found by finding the slope of the tangent line at that point.

   b. What does point *P* represent in the context of this problem?

      Answer: Point *P* is a local maximum, where the number of births reached a peak and started to decline for a number of years.

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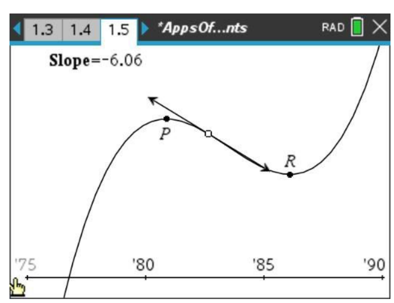


2. The graph from page 1.3 is repeated with a tangent line to a point on the function. The slope of the tangent line has been calculated for you. Grab and drag the point along the graph.
   a. What do you notice about the slope of the tangent line to the left of point *P*? To the right of point *P*?

      Answer: The slope of the tangent line is positive to the left of point *P* and negative to the right of point *P*.

Teacher Tip: This is leading to the **first derivative test**. This is a good time to have students discuss why the derivative would change from positive to negative at that point.

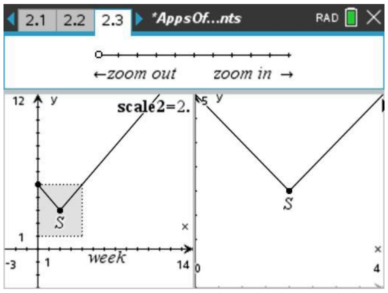
b. What is the derivative of the function at point *P*?

Answer: The derivative of the function at point *P* is zero because the slope of the horizontal tangent line is zero.

c. When is the derivative of the function positive? Negative? Explain.

Answer: The derivative of the function (slope of the tangent line) is positive to the left of point *P* and to the right of point *R*, when births are increasing. The derivative is negative between points *P* and *R,* when the function is decreasing and there are fewer births.

Move to page 2.3.



3. The graph on page 2.3 shows Gina's income, in thousands of dollars, for each week in business. Grab and drag the slider point to zoom in on the graph of the function at the critical point.

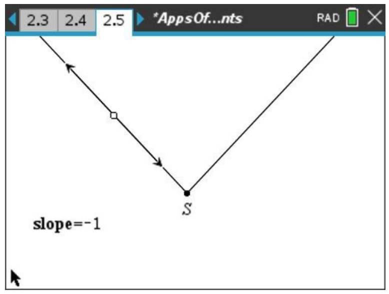
a. What type of critical point is point *S*? What is the meaning of this point in the context of Gina's business?

Answer: Point *S* is a local minimum. It is the point when Gina had the least amount of money just before she started earning income.

b. What do you notice about the shape of the graph when you zoom in on the critical point?

Answer: The shape of the graph remains the same shape regardless of how far it is zoomed in.

Move to page 2.5.



4. The graph on page 2.5 is a magnified view of the graph from page 2.3. A tangent line to a point on the function is graphed, and the slope is calculated. Grab and drag the point along the graph.

   a. What is the slope of the tangent line to the left of the critical point? To the right of the critical point?

      Answer: The slope to the left of the critical point is –1. The slope to the right of the critical point is 1.

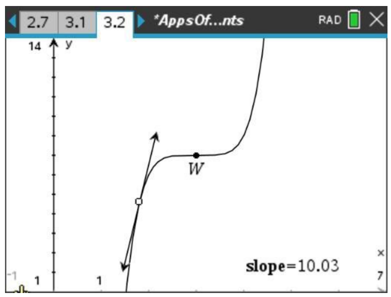
   b. What does the slope mean in the context of this problem?

      Answer: Gina either gained or lost $1,000 per week. When the slope is negative, Gina lost money. When the slope is positive, she earned contract money.

   c. What is the slope of the tangent line to the function at the critical point? Why?

      Answer: The function is not differentiable at the critical point, because the graph does not become linear as you zoom in. Essentially, two different lines with two different slopes meet at that point.

Move to page 3.1.



5. Grab and drag the point along the graph to find the slope to the left and to the right of point *W*.

   a. What do you notice about the derivative of the function to the left of point *W*? To the right of point *W*?

      Answer: The slope of the tangent line remains positive to the left and to the right of point *W*.

b. Is point *W* a critical point? Why or why not?

Answer: Point *W* is a critical point because the derivative is zero at this point.

c. Is point *W* a local maximum or minimum? Why or why not?

Answer: Point *W* is not a local maximum or minimum.

Sample responses: The function continues to increase after the point. The curve does not “turn around” to go another direction. The slope of the tangent line does not change from positive to negative or negative to positive at this point.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students understand:

- How to identify the critical points of a function given its graph.
- Why the local minima or maxima of a function occur at its critical points.
- Why not every critical point is a local minima or maxima.

Note: The assumption of continuity is another idea that should be addressed in the lesson wrap-up. All of the functions presented in this lesson are continuous, a condition for the first derivative test. Although most functions students will encounter will be continuous, this is an important condition to note. Students could be challenged to consider what might happen if this condition were relaxed. Is it then possible for f to have a critical point a such that $f(a)$ is a local minimum, but the derivative $f'(x) > 0$ for $x < a$ and $f'(x) < 0$ for $x > a$? If so, what might it look like?

