Breaking Up Over Model Bridges

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Concepts
- Data collection
- Mean of a data set
- Graphing a data set
- Identifying a non-linear function
- Developing a function from data

Materials
- Balsa wood sticks 1/8" x 1/8"
- Masses (200g. – 1,000g.)
- Fishing line
- Meter stick
- Scissors
- TI-83/84

Technology Goals
- Entering data into a list
- Arithmetic with lists
- Graphing ordered pairs and functions

Overview

Hands on activity to introduce reciprocal functions. Load testing of a balsa wood, model bridge. Use of TI-83/84. Information about craft-stick bridge competition at The Citadel

Introduction

The learning objective of this activity is to introduce the concept of reciprocal functions having the form: xy = k or y = f(x) = k/x, where k is a constant and x and y are variables. In Part I, twelve one inch paper squares arranged in various rectangles illustrate that length x width = 12 square inches. In Part II an experimental load constant is determined for a balsa wood cantilever: mass x failure length = k. The TI-83/84 graphing calculator is used to display the three representations of the functions.

References

"Using Engineering to Understand Reciprocal Functions", by Janet Sharp, Loren Zachary, and Greg Luttenegger, *Mathematics Teaching in the Middle School* Volume 11, Number 8, April 2006

"Strength of Wood Beams: An Engineering Application", by Mary Dengeraud-Au, *Mathematics Teacher* Volume 93, Number 7 October 2000.

Note: the above references are available on-line at: http://my.nctm.org/eresources/

STUDENT WORKSHEET

Part I

1. Arrange the 12 one-inch squares into a rectangle and note that the number of squares for width multiplied by the number of squares for length is 12 square inches.
2. Arrange the squares in as many different ways as you can and record (length, width) as ordered pairs. Note that (4,3) and (3,4) are different rectangles.
3. Enter all of the ordered pairs in ([L1],[L2]) of your TI-83/84. Graph a scatter-plot with window: xMin =0, xMax = 13 and yMin = 0, yMax =13. Sketch and label the graph in your lab notebook.
4. Having discovered the pattern that length x width = 12, you can illustrate this using list arithmetic: [L1]*[L2] →[L3].
5. Note that x * y = 12 can be written as y = 12/x. On the function screen (y=) set Y1= 12/x. Graph the function and note that it traces the ordered pairs. The common name for this family of functions is “reciprocal function”.

Part II

1. Attach a 200g. mass near the end of a balsa wood stick using about a 12 inch piece of the fishing line.
2. Place the stick on your desk with the mass and line near the edge. Gently lower the mass over the edge of the desk while holding the stick flat on the desk.
3. Continue holding the stick flat on the desk and ease the stick out over the edge until the stick breaks. Note: catch the weight or allow it to fall on something soft to avoid injury/damage.

4. Record on the data table the mass in grams and the length from the break to where the fishing line is attached.
5. Repeat the experiment using masses of 250, 300, 350, 400, 450, and 500 grams.
6. Enter the data as ordered pairs in [L1],[L2] of your TI-83/84. Graph a scatter-plot. Sketch and label the graph in your lab notebook.
7. Fill [L3] with the product of [L1] and [L2]: [L1]*[L2]→[L3]. Comment on the conjecture that the product of mass and failure length is a constant.
8. Calculate the mean of [L3]: (2^{nd} LIST MATH – 3: mean[L3]). Note that the mean[L3] is the experimental constant for the reciprocal function: mass x failure length = k.
9. Graph Y1 = (mean[L3])/x. Record comments on goodness of fit in your lab notebook.

Data Table

Mass (g): [L1]	Failure length:[L2]	Product:[L1]*[L2]→[L3]
200 g.		
250 g.		
300 g.		
350 g.		
400 g.		
450 g.		
500 g.		

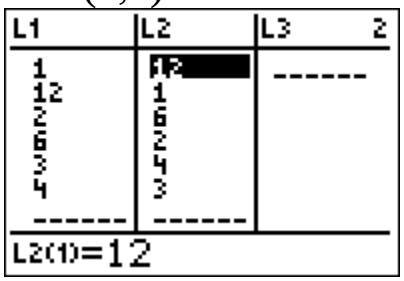
mean[L3] =

LESSON PLAN – TEACHER NOTES

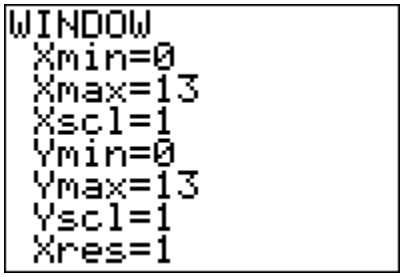
Part I

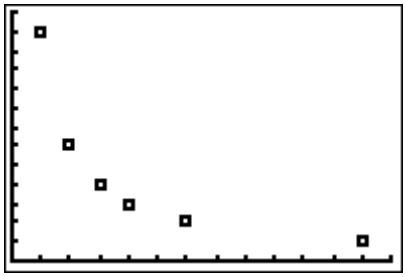
1. Arrange the 12 one-inch squares into a rectangle and note that the number of squares for width multiplied by the number of squares for length is 12 square inches.
   Give each student 12 one-inch paper squares. Use heavy paper such as file folders.

2. Arrange the squares in as many different ways as you can and record (length, width) as ordered pairs. Note that (4,3) and (3,4) are different rectangles.



3. Enter all of the ordered pairs in ([L1],[L2]) of your TI-83/84. Graph a scatter-plot with window: xMin =0, xMax = 13 and yMin = 0, yMax =13. Sketch and label the graph in your lab notebook.





4. Having discovered the pattern that length x width = 12, you can illustrate this using list arithmetic: [L1]*[L2] →[L3].

L1	L2	L3 3
1	12	[redacted]
12	1	12
2	6	12
6	2	12
3	4	12
4	3	12
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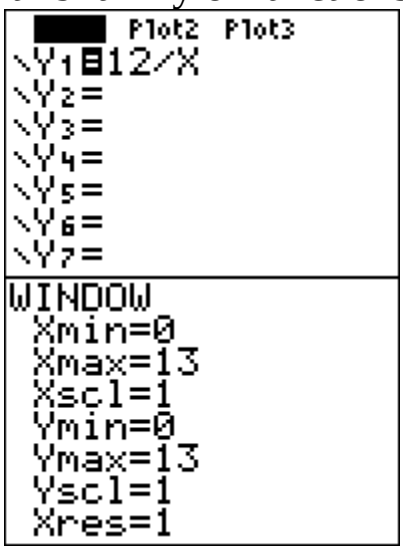
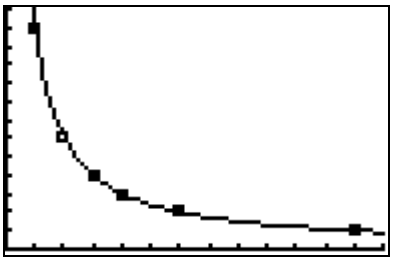
L3(1)=12

5. Note that x * y = 12 can be written as y = 12/x. On the function screen (y=) set Y1= 12/x. Graph the function and note that it traces the ordered pairs. The common name for this family of functions is "reciprocal function".



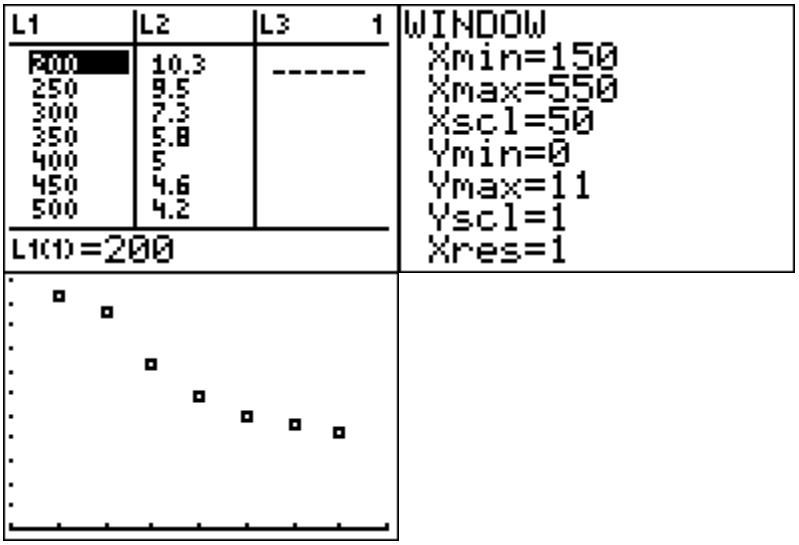


From my Algebra I book: y = kx is direct variation and xy = k is inverse variation where k is the constant of variation.

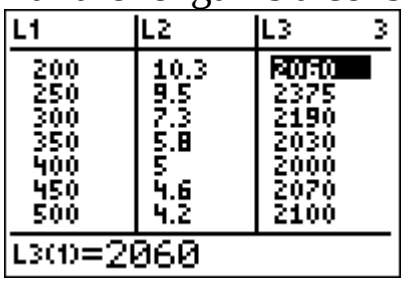
Part II

1. Attach a 200g. mass near the end of a balsa wood stick using about a 12 inch piece of the fishing line.
   I used 10 lb. test monofilament fishing line. It is fairly easy to tie and it does not slip when tied to the balsa wood as it cuts into the wood when the mass is applied.
2. Place the stick on your desk with the mass and line near the edge. Gently lower the mass over the edge of the desk while holding the stick flat on the desk.

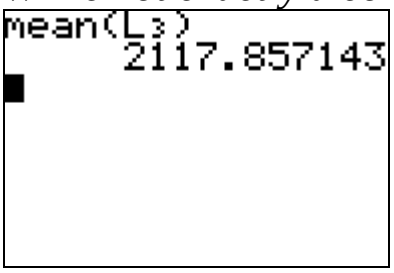
3. Continue holding the stick flat on the desk and ease the stick out over the edge until the stick breaks. Note: catch the weight or allow it to fall on something soft to avoid injury/damage.
4. Record on the data table the mass in grams and the length from the break to where the fishing line is attached.
5. Repeat the experiment using masses of 250, 300, 350, 400, 450, and 500 grams.
6. Enter the data as ordered pairs in [L1],[L2] of your TI-83/84. Graph a scatter-plot. Sketch and label the graph in your lab notebook.



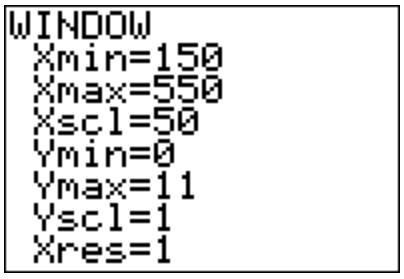
7. Fill [L3] with the product of [L1] and [L2]: [L1]*[L2]→[L3]. Comment on the conjecture that the product of mass and failure length is a constant.

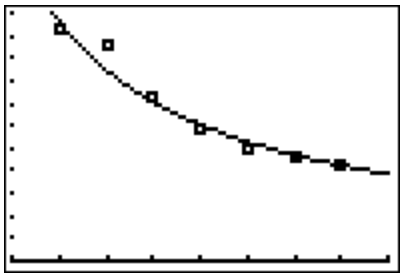


While not exactly a constant, it is close enough to proceed.



8. Calculate the mean of [L3]: (2^{nd} LIST MATH – 3: mean[L3]). Note that the mean[L3] is the experimental constant for the reciprocal function: mass x failure length = k.
   Mass x failure length = 2117.9
9. Graph Y1 = (mean[L3])/x. Record comments on goodness of fit in your lab notebook.





Fit looks good for this data set. One might consider trying the 250 g. mass again as that is the one data point which is somewhat of an outlier.

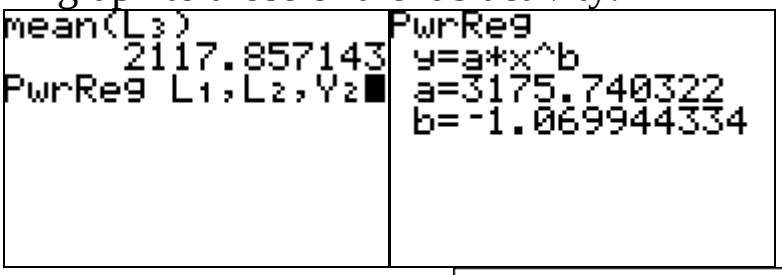
Data Table

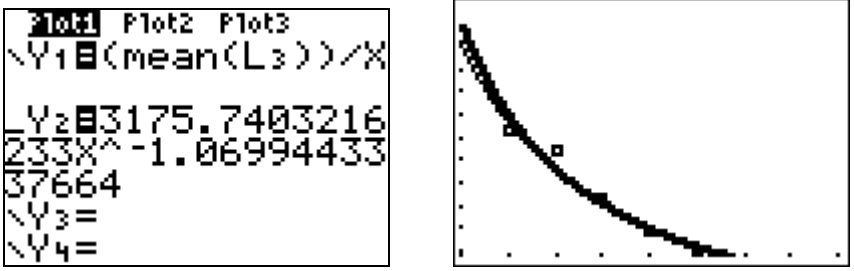
Mass (g): [L1]	Failure length:[L2]	Product:[L1]*[L2]→[L3]
200 g.	10.3	
250 g.	9.5	
300 g.	7.3	
350 g.	5.8	
400 g.	5	
450 g.	4.6	
500 g.	4.2	

mean[L3] =

Extension Activities

1. The STAT CALC PwrReg will calculate a curve of "best fit" using the model y = a*x^b. Perform this calculation with your [L1],[L2] data and compare the resulting function and its graph to those of the lab activity.





A close match results.

2. Design a beam bridge to be constructed with 1/8" balsa wood sticks capable of supporting a maximum mass of 300g. with a total length of 50 cm. Conduct a load test of your design. Note: the test may be conducted on a section of the total length.
3. Visit the website: http://www.citadel.edu/ece/eweek/ and consider entering the Low Country Craft Stick Bridge Contest.

For National Engineering Week activities at The Citadel, Military College of South Carolina: http://www.citadel.edu/ece/eweek/