Exploring Space Through MATH

Applications in Algebra 2

EDUCATOR EDITION

Lights on the International Space Station

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- analyze data from a simulation;
- model data from a simulation using a graphical and algebraic approach; and
- apply the measure of illumination, $lux = \frac{candela}{m^2}$, to various situations.

Prerequisites

Students should have prior knowledge of scatter plots, modeling functions, and applying formulas using the TI-Nspire handheld. Students should also have prior knowledge of inverse variation and power functions with negative exponents.

Background

This problem is part of a series that applies mathematical principles in NASA's human spaceflight.

According to James C. Maida, author of *An Illumination Modeling System for Human Factors Analyses,* "Seeing is critical to human performance. Lighting is critical for seeing. Therefore, lighting is critical to human performance."

Here on Earth, the sense of sight and the benefits of illumination can easily be taken for granted. While astronauts orbit the Earth, the sun rises and sets approximately every forty-five minutes, causing humans to cope with dynamic lighting conditions. Since contrast conditions of harsh shadowing and glare are also severe, the optimization of lighting conditions is essential for critical operations.

The Lighting Environment Test Facility (LETF), located at NASA Johnson Space Center in Houston, Texas, investigates and evaluates proposed lighting systems for use on space vehicles. Lighting systems enhance the

Key Concepts
Power functions with negative exponents

Problem Duration
80 minutes

Technology
Computer with projector, TI-Nspire™ handheld

Materials
- *Lights on the International Space Station* Student Edition
- Flashlights
- Textbooks
- Tape Measures

Skills
Modeling inverse variation, applying formulas, and analyzing data

NCTM Standards
- Algebra
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Common Core Standards
- Quantities
- Seeing Structure in Expressions
- Reasoning with Equations and Inequalities
- Interpreting Functions
- Linear, Quadratic, and Exponential Models

crews' direct and indirect viewing ability during spaceflight. This effort includes the investigation, measurement, and analysis of:

- artificial lighting systems, such as docking lights, portable lights, and navigation lights;
- reflective characteristics of various materials and the effects of solar lighting;
- transmission characteristics of transparent materials used for visors, displays and windows; and
- camera performance for minimum and maximum illumination.

Figure 1: Astronaut James S. Voss performs tasks in the Destiny Laboratory on the International Space Station.

LETF mission planners determine both the types of lighting (quality of light) and light quantities required for a crewmember to perform certain tasks. Light optimization is required for crew safety, electrical power, and equipment maintainability. Although mission planning is a significant role of LETF, the group is also actively involved in research and development of new and advanced lighting technologies.

NCTM Principles and Standards

Algebra

- Identify essential quantitative relationships in a situation, and determine the class or classes of functions that might model the relationship
- Draw reasonable conclusions about a situation being modeled

Problem Solving

- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems

Reasoning and Proof

- Make and investigate mathematical conjectures

Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

Connections

- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Use representations to model and interpret physical, social, and mathematical phenomena

Common Core Standards

The Real Number System

- Extend the properties of exponents to rational exponents

Quantities

- Reason quantitatively and use units to solve problems

Seeing Structure in Expressions

- Write expressions in equivalent forms to solve problems

Reasoning with Equations and Inequalities

- Understand solving equations as a process and explain the reasoning

Interpreting Functions

- Interpret functions that arise in applications in terms of the context

Linear, Quadratic, and Exponential Models

- Interpret expressions for functions in terms of the situation they model

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

The questions in this activity are embedded in the TI-Nspire document, *Lights-ISS.tns.* Some screenshots have been provided in the solution key to show what the students will see on their handhelds.

1 – Engage (30 minutes)

- With students in small groups of four, ask them to review and discuss the main points of the background section for several minutes to ensure they understand the material. Circulate to facilitate discussion in the small groups. Ask if any group needs clarification.

- Distribute the worksheet, Flashlight Reading Investigation, to each group. (*Lights on the International Space Station* Student Edition, pages 3–4)

- Describe the investigation, and allow students the opportunity to predict the outcomes prior to engaging in the activity.
- Have students select their roles and conduct the Flashlight Reading Investigation.
- After performing the activity, give students the opportunity to discuss the follow-up questions before sharing their observations with the class.

2 – Explore (15 minutes)

- Allow students to remain in their groups.
- Distribute the TI-Nspire document, *Lights-ISS.tns*, to the students' handhelds.
- Have students read the problem set-up and directions, and then answer the questions on pages 1.2–2.5.

3 – Explain (15 minutes)

- Allow students to remain in their groups to answer questions on pages 2.6–2.13.
- Call on students to give their answers and discuss.

4 – Extend (10 minutes)

- Still in groups, have students answer the question on page 2.14.
- Encourage student discussion, and ask if there are any questions.

5 – Evaluate (10 minutes)

- Have students work independently to complete questions on pages 3.1–3.3.
- This may be done in class or assigned as homework.

Hands-on Activity: Flashlight Reading Investigation

- To achieve the desired effect, this activity should be done in a room that can be made relatively dark by turning off the lights. If possible, use different types of flashlights and battery strengths. This will help students discover the various factors which may affect the situation.
- Each group will need the following items to perform the investigation: one flashlight, one tape measure, and a textbook.
- Within each group, assign students the following roles: Flashlight Holder, Distance Manager, Book Holder, and Book Reader.
- Students should be directed to investigate the relationship between the flashlight's distance from the book and their ability to read a page of a book.
- After the students have had time to engage in the activity, direct a class discussion of their observations.

Flashlight Reading Investigation

In this exercise, you will be investigating the relationship between the distance of a light source from a book and your ability to read that book. You will divide into groups of four, each with a specific role (outlined below). The materials required for this activity are: a flashlight, a measuring tape, and a text book.

- **Flashlight Holder** Name________________________

  The Flashlight Holder will hold the flashlight at different distances from the book, pointing the flashlight at the book.

- **Distance Manager** Name________________________

  The Distance Manager will measure each distance between the flashlight and the book. The distance between the flashlight and the book should begin at 0.5 meters, and increase in distance at least 0.5 meters each time.

- **Book Holder** Name________________________

  The Book Holder will stand at the designated distance from the flashlight with the book open, and will remain in the same position throughout the investigation.

- **Book Reader** Name________________________

  The Book Reader will stand at a distance from the book which is naturally comfortable to read, and will remain in the same position throughout the investigation. The Book Reader will determine the ability to read the book (as easy, moderate, or difficult).

As a group, you will provide observations, which will be documented in Table 1 (Flashlight Reading Data). A minimum of four readings should be taken. Answer the questions in the prediction section before starting the investigation.

Predictions

1. Will the flashlight's distance from the book affect your ability to read the book? Why?

2. Predict the flashlight's maximum distance from the book (in meters, or m) in which you will still be able to read the book comfortably.

3. Besides reducing your distance from the book, what other things could you do to improve your ability to read the book?

Table 1: Flashlight Reading Data

Flashlight Distance (m)	**Ability to Read the Book** (Easy, Moderate, Difficult)	**Observations**

Questions

1. How well did you predict the outcome?

2. Does the flashlight's distance from the book affect your ability to read? Explain.

3. What is the farthest distance the flashlight can be from the book for you to read comfortably?

4. Is it desirable to have the book close to the flashlight? Explain.

5. Based on your observations from this investigation, what factors would influence an astronaut's ability to perform a task on the International Space Station (ISS)?

Lights on the International Space Station

Solution Key

Open the TI-Nspire document, *Lights-ISS*. Turn to page 1.2 and record your answer to the following question.

1.2 Based on your observations from the Flashlight Reading Investigation, what factors would influence an astronaut's visibility and ability to perform a task on the ISS?

Distance from the light, brightness of the light, age of the light bulb, task that needs to be performed (Answers will vary.)

Problem

As you have discovered, the farther you are from a light source, the more difficult it is to see and perform a task like reading. This is primarily due to illumination. Illumination is the light that allows humans to see their environment in order to live and work.

Sunlight is a source of illumination. However, artificial illumination from lamps, overhead lights, and flashlights provides humans with light to see in school, our homes, and the workplace in both day and night.

The primary measure of illumination is lux.

$$lux = \frac{amount\ of\ light\ produced}{(distance\ from\ light\ source)^2}$$

The amount of light produced is measured in candelas (sometimes called candle power), and the distance from the light source is measured in meters (m).

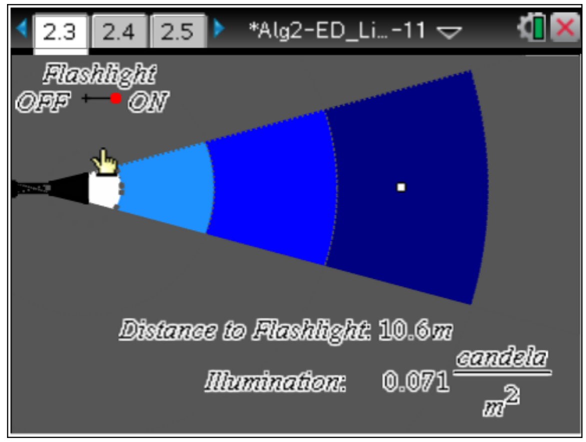
Therefore, $lux = \dfrac{candela}{m^2}$.

On page 2.3, you will find a simulation of the Flashlight Reading Investigation that you performed. The measurements recorded for this demonstration are illumination (lux). On this page, turn on the flashlight by dragging the closed circle to the ON position. Then drag the open point to collect the measurements necessary to complete the table below.

Table 2: Distance vs. Illumination

Distance (m)	Illumination (lux)
0.0-1.0	**0-8.635**
2.0-3.0	**1.912-0.884**
4.0-5.0	**0.507-0.313**
6.0-7.0	**0.221-0.164**
8.0-9.0	**0.123-0.098**
10.0-11.0	**0.080-0.065**
12.0-13.0	**0.055-0.047**

Notice that when the flashlight is in the OFF position, there are no measurements defined for distance and light intensity because it is simulating darkness.



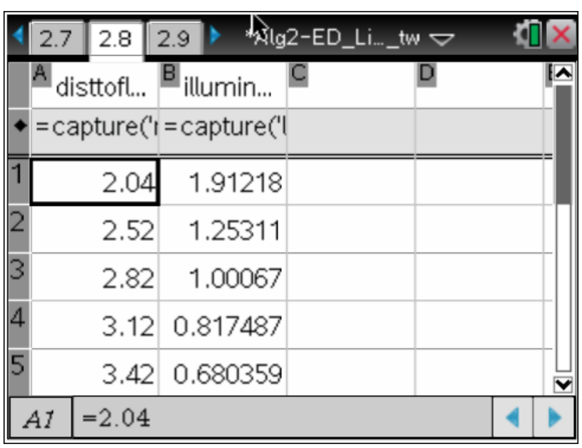
2.5 Does the table support your understanding on how illumination changes with the distance from the light source? Explain.

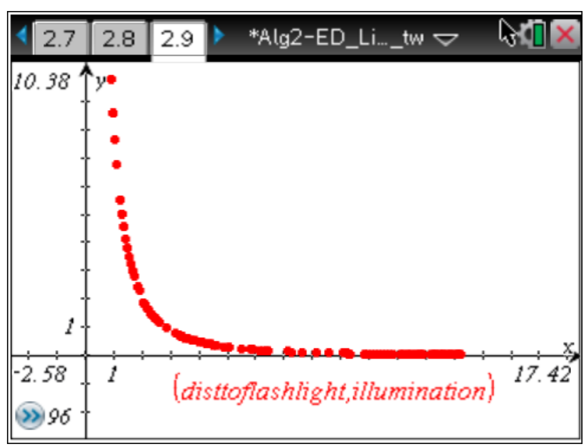
Yes. As the values for the distance from the flashlight increase, the illumination values decrease.

2.6 What mathematical relationship exists between the distance from the light source and the amount of light produced?

There is an inverse variation between distance from the flashlight (or light source) and illumination. This type of relationship is modeled by an equation of the form, $y = \frac{k}{x}$, *for some constant, k. Notice this is also a power function with an exponent of -1.*

On page 2.8, you will see the information recorded in the previous table along with many of the data values that were collected as you dragged the open point. Investigate the scatter plot of the data collected, which is distance from flashlight versus illumination (shown on page 2.8).



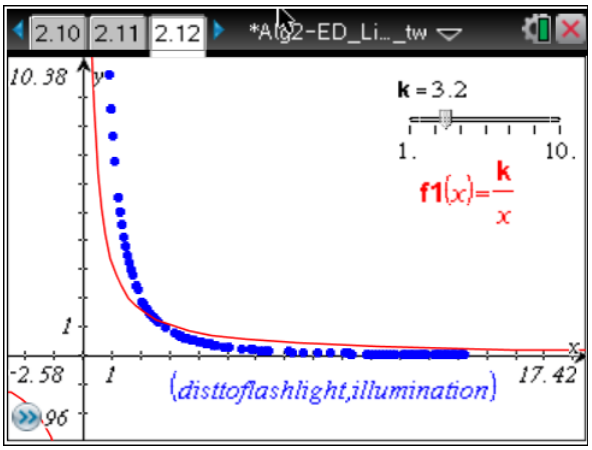


2.10 Is the scatter plot consistent with the behavior of an inverse variation? Explain.

Yes. As the x-coordinate gets larger (independent variable, domain), the y-coordinate gets smaller (dependent variable, range).

2.11 On page 2.12, the inverse variation function, $f(x) = \frac{k}{x}$ is graphed for *k* = 2, where *k* is the constant of inverse variation. Use the slider to change the value of *k* to get a better model for the collected data. What is the best *k* value? Is this a good model? Explain.

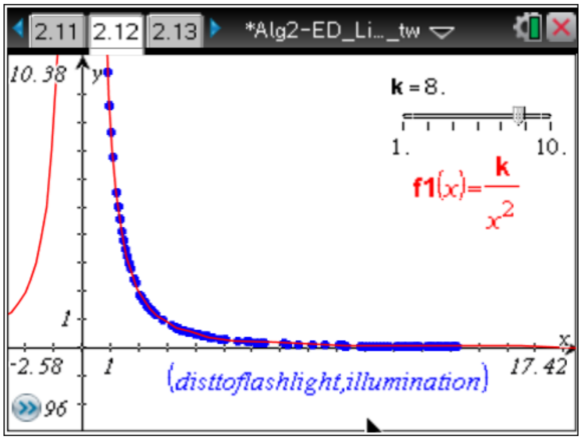
Answers will vary. All students should conclude that this is not a very good model.



2.13 On page 2.12, return the value of *k* to 2. Edit the **f1** function to represent an inverse squared variation function by double clicking on the equation and editing the text within, i.e. $f1(x) = \frac{k}{x^2}$. Use the slider to change the value of *k* to get a better model for the collected data. What is the best *k* value? Is this a good model? Explain.

The value of k is approximately 8.0. (Student answers may vary slightly.) All students should conclude that the model appears to be very good.

Note: If students comment on the graph appearing for values of x less than 0, then a discussion of the domain for the Flashlight Reading Investigation should be pursued.



The relationship between illumination and distance from a light source is modeled by the formula, $lux = \frac{candela}{m^2}$. The constant of inverse variation in this formula is candela, which is the amount of light produced from the source.

2.15 In the front seat of a car, there is a street address written on a piece of paper which requires reading. The size of the writing is small, and it is night time. The overhead map light is 0.5 meters from the paper. If 300 lux of illumination is needed to read the address, how many candelas of light would be required?

$$lux = \frac{candela}{m^2}$$

$$300 = \frac{candela}{(0.5m)^2}$$

$$candela = (0.25\,m^2)(300\,lux)$$

$$candela = 75$$

75 candelas

Directions: Complete the questions on pages 3.2–3.3 independently. Round all answers to the nearest thousandth, and label them with the appropriate units.

The Columbus Laboratory is an ISS module designed for scientific research. The module is cylindrical in shape with a diameter of 4.5 meters and a length of 6.9 meters. Each fluorescent light on the ISS produces 170 candelas.

Figure 2: Astronaut working in the ISS Columbus Laboratory Module

3.2 How many lights are needed to provide the astronauts with 300 lux if the lights are located 1.5 meters overhead?

$$lux = \frac{170\,candelas}{1.5\,m^2} = 75.556$$

$$\frac{300\,lux}{75.556\,lux} = 3.971\,lights$$

At least four lights are required to perform the task.

3.3 The robotic arm on the ISS is being used to repair a loose wire. In order to repair the wire, the robotic arm's video camera must provide a good visual image to the astronauts. To record a good image, the video camera requires 5 lux of illumination. The astronauts prefer to use a powerful 2500-candela spotlight to illuminate the wire. What is the maximum distance the spotlight can be away from the wire, yet still provide enough light to obtain a good image? Explain.

$$lux = \frac{candela}{m^2}$$

$$5\,lux = \frac{2500\,candelas}{m^2}$$

$$m = \sqrt{\frac{2500\,candelas}{5\,lux}}$$

$$m = 22.361\,meters$$

To get a good image, the spotlight must be no further than 22.361 meters away from the wire.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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