EXPLORATIONS™

Activity 6

Objective

- Use the CellSheet™ App to approximate the slope of a line tangent to a curve

The Slope of the Tangent Line (Part 1)

Introduction

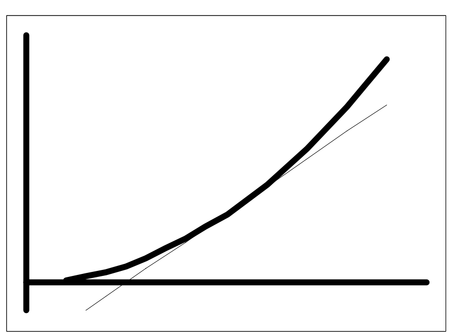
You have learned that the equation $y = mx + b$ is a linear equation, with m being the slope of the line and b the y-intercept. You have also learned how to find the slope using any two sets of coordinate pairs that are on the line, using the formula:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

You found that in a linear equation, the slope (rise over run) is constant. This means that for any change in x (Δx) the change in y (Δy) is constant so that $\frac{\Delta y}{\Delta x} = m$.

The slope of a curve does not have the same equation or the same characteristics. In a parabolic curve, as x changes, the rate by which y changes varies. The value of $\frac{\Delta y}{\Delta x}$ is generally not the same for the two points (x, y) and (x_1, y_1) as it is for the two points (x, y) and (x_2, y_2).

The slope of a curve at any point (x, y) is defined as the slope of the tangent line, a line that intersects a curve at only one point through the point (x, y).



The problem is that you cannot find the slope of a tangent line using the one point at which the line intersects the curve because the formula for calculating slope requires two points.

Instead, you can approximate the slope of a tangent line using a ***secant line***, a line that intersects the curve in two points. If the two points of intersection are fairly close together, that is, if Δx is small, the approximation of the slope is more accurate.

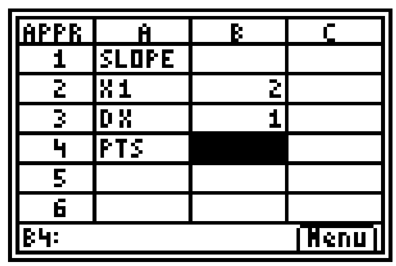
Problem

What is the slope of a line tangent to the curve $y = x^2 + 4x + 5$ at point $x = 2$? You will use a secant line to approximate the slope of the line tangent.

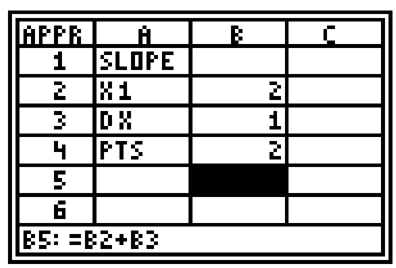
To solve the problem, you are going to use two points on the curve to define a secant line and then use those two points to find the slope of that secant line.

Exploration

1. Create a new spreadsheet in the CellSheet™ App and name it **APPROX**.
2. In cell A1, type **"SLOPE**. In cell A2, type **"X 1**. In cell A3, type **"D X**. In cell A4, type **"PTS**.
3. In cell B2, type **2**. In cell B3, type **1**.

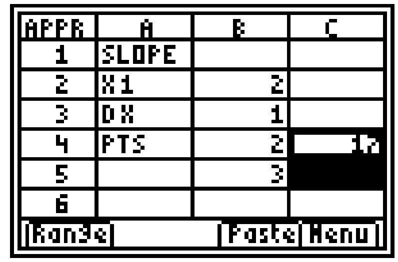
   The first point on the curve is going to be $x_1 = 2$, the point where the line tangent intersects the curve (cell B2). The second point will be defined using $x_1 + \Delta x$ (from cell B3). The coordinates of the two points will be shown in cells B4, B5, C4 and C5.
4. In cell B4, type the formula **=B2**. In cell B5, type the formula **=B2+B3**.

   The values in cells B4 and B5 are the x-values of the two points that will be used to find the slope of the secant line. Because you set the difference between the two points as 1, the 2 x-points are 2 and 3.

   To find the y-values that correspond to these points, you will use the equation for the curve, $y = x^2 + 4x + 5$.

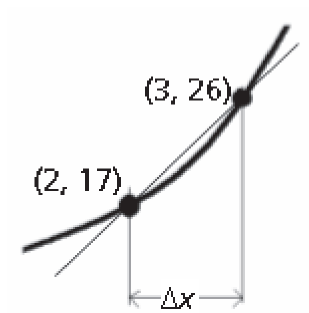
5. In cell C4, type the formula **=B4^2+4*B4+5**. Copy the formula into cell C5.



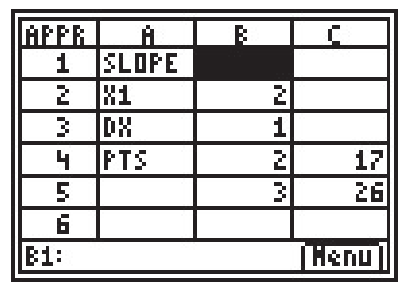
APPS	A	B	C
1	SLOPE		
2	X1	2	
3	DX	1	
4	PTS	2	17
5		3	
6			

You have found two points on the curve $y = x^2 + 4x + 5$. They are (2, 17) and (3, 26).

Now that you have these two points, you can use the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$ to find the slope of the line through those two points.



6. What formula can you enter in B1 to find the slope? ____________________

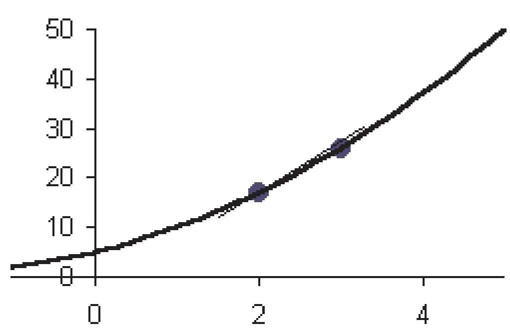


APPS	A	B	C
1	SLOPE		
2	X1	2	
3	DX	1	
4	PTS	2	17
5		3	26
6			

7. What is the slope of the secant line through the points (2, 17) and (3, 26)?

____

Remember that the slope of the secant line is only an approximation of the line tangent to the curve. The approximation can be improved if the difference in the x-values (Δx) is smaller.



8. To make the change in *x* smaller, change the value in cell B3. Try **0.5**.

What two points are now being used to calculate the slope of the line tangent?

__________ and __________

APPR	A	B	C
1	SLOPE	8.5	
2	X1	2	
3	DX	.5	
4	PTS	2	17
5		2.5	21.25
6			
B3: .5			Menu

9. What is the slope of this secant line? ___________

10. Continue to decrease Δx and record the slope. What pattern do you notice?

APPR	A	B	C
1	SLOPE	8.01	
2	X1	2	
3	DX	.01	
4	PTS	2	17
5		2.01	17.08
6			
B3: .01			Menu

Based on the data collected, what is your best guess for the slope of the tangent line through $x = 2$? ___________________

Student Worksheet

Name ______________________

Date ______________________

Reviewing Concepts

1. It is difficult to find the slope of a curve because the slope __________ between any two points. However, the slope of a ___________ tangent to a curve at any point is a good way to measure how the curve is changing shape.

2. The graph of the curve $y = x^2 + 4x + 5$ is _______________.

3. A secant line crosses the curve at __________ points. The slope of the secant line can approximate the slope of the __________ line if the difference between the two x-values is small.

Solving the Problem

4. Using a Δx of 0.01, the best approximation for the slope of a line tangent to the curve $y = x^2 + 4x + 5$ at the point $x_1 = 2$ is _____________.

Analyzing the Data

5. Complete the table.

	Δx	x	y	Slope of the Secant Line
Initial point				
First point	1			
Second point	0.5			
Third point	0.1			
Fourth point	0.01			

6. What pattern can you notice between Δx and the slope of the secant line?

____

Extending the Activity

1. Using the same procedure as above, find the slope of the curve $y = x^2 + x - 6$ at the point $x = 3$.

Teacher Notes

Activity 6

The Slope of the Tangent Line (Part 1)

Objective

- Use the CellSheet™ App to approximate the slope of a line tangent to a curve

Materials

- TI-84 Plus/TI-83 Plus

Time

- 60 minutes

Preparation

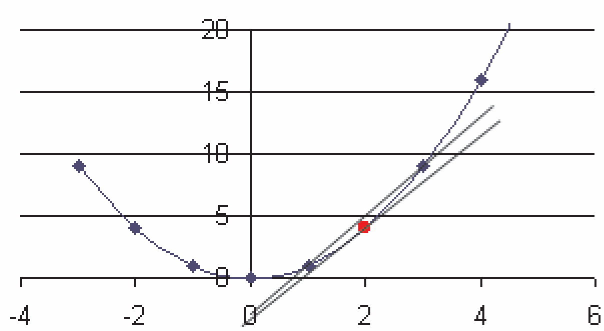
Review with students linear equations, checking that students can define slope and can calculate the slope of a line given two sets of points on the line. Talk about the graphing of a quadratic equation. Ask what shape a quadratic equation will represent on the graph. Have students explain why a quadratic equation is represented as parabolic.

Elicit Questions

Begin with a simple quadratic equation: $y = x^2$. Show a graph of the equation and a line tangent to the curve. Ask students to calculate the slope of the curve at $x = 2$. Make sure students realize that two points are needed to find the slope

$\left(m = \dfrac{y_1 - y_2}{x_1 - x_2}\right)$.

Draw a secant line through (1, 1) and (3, 9).



Management

Have students work in pairs to determine the slope of the secant line with the given points. Then have them calculate the slope of the secant line with points (1, 1) and (2, 4). They should compare the two slopes. What observations can they make?

Answers to Exploration Questions

6. **=(C4 – C5)/(B4 – B5)**
7. Slope = 9
8. (2, 17) and (2.5, 21.25)
9. Slope = 8.5
10. Answers will vary, but students should note that the Δx is the same change in the slope.

Answers to the Student Worksheet

Reviewing Concepts

1. Changes; line
2. Parabolic
3. 2; tangent

Solving the Problem

4. 8.01

Analyzing the Data

5.

	Δx	x	y	Slope of the Secant Line
Initial point		*2*	*17*	
First point	1	*3*	*26*	*9*
Second point	0.5	*2.5*	*21.25*	*8.5*
Third point	0.1	*2.1*	*17.81*	*8.1*
Fourth point	0.01	*2.01*	*17.08*	*8.01*

6. The change in the Δx is the same as the change in the slope.

Extending the Activity

1.

	Δx	x	y	**Slope of the Secant Line**
Initial point		*2*	*0*	
First point	1	*3*	*6*	*6*
Second point	0.5	*2.5*	*2.75*	*5.5*
Third point	0.1	*2.1*	*0.51*	*5.1*
Fourth point	0.01	*2.01*	*0.0501*	*5.01*