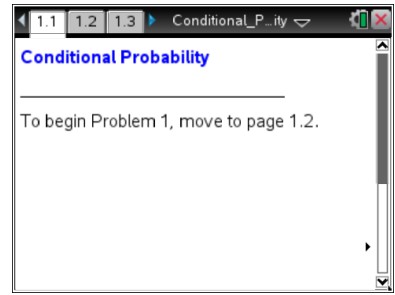## Math Objectives

- Students will identify the difference between the probability of an outcome and the conditional probability of an outcome.
- Students will interpret information presented in a two-way table and in a corresponding bar graph.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practices).

Vocabulary

- conditional probability
- two-way table
- probability

About the Lesson

- This lesson involves thinking about probability when additional information is given. As a result, students will:
    - Investigate probability questions using tabular and graphical information.
    - Explain why a two-way table with two rows and two columns with fixed row and column totals needs only one input to determine the others.
    - Answer questions about the probability of an outcome using data from a two-way table.
    - Use bar graphs separated according to categories to answer probabilities and connect these graphs to information of the form P(A|B).
    - Choose between information presented as the probability of A given B and the probability of B given A (A|B and B|A) to answer specific probability questions.

TI-Nspire™ Navigator™

- Send out the *Conditional_Probability.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes class captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/ calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
- Conditional_Probability_ Student.pdf
- Conditional_Probability_ Student.doc

TI-Nspire document
- Conditional_Probability.tns

Discussion Points and Possible Answers

> **Teacher Tip:** The lesson would be a good precursor to lay the foundation for the formal definition of conditional probability, $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ or work with Baye's Theorem. However, for this lesson, students should at least be familiar with the concept of probability.
>
> This activity lends itself to the use of PublishView, which gives you an opportunity to display the two-way table (Page 1.2) and the two bar graphs (Page 1.4) simultaneously so the students can manipulate the two-way table and see how the two sets of bar graphs change dynamically.

> **Teacher Tip:** The lesson is based on an actual poll reported in the June 1992 *Life* magazine article "If Women Ran America." One possible extension at the end of the lesson might be to have students take a poll and compare the results today from those in 1992.

In 1992, *Life* magazine published an article entitled "If Women Ran America*",* which reported the results of a poll of U.S. men and women asked whether the problem of unequal pay for equal work is a serious one in our country. Suppose the poll was given to 44 people, 20 males and 24 females, and that 26 of the people polled agreed that the problem of unequal pay for equal work is serious.

1. Suppose further that 10 of the males agreed the problem is serious.
    a. Indicate which of the table cells below you could fill in with this information.

    Sample Answers:

Gender/ problem	yes	No	Row Totals
Male	10	10	20
Female	16	8	24
Column Totals	26	18	44

Move to page 1.2.

b. On this Lists & Spreadsheet page, enter 10 into the cell representing the number of males who agree the problem is serious, and compare the resulting table to your answer to part a.

Sample Answers: The worksheet table and the table in the TI-Nspire document should match. Note that one entry in any cell will determine the other cell because the totals for the row and column containing that cell have to be preserved.

Tech Tip: To hide the keyboard after entering a value into the spreadsheet, press the keyboard down button in the lower right corner 🖮.

Tech Tip: To add or modify the data in a spreadsheet cell, double-tap the cell.

Tech Tip: Students might want to use the calculator Scratchpad to find the probabilities in question 2.

2. a. If a person is chosen randomly from the 44 people, what is the probability that the person is a male?

    Sample Answer: 20/44 or 0.4545…

    b. What is the probability that the person is male and the person agrees the problem is serious?

    Sample Answer: $\dfrac{10}{44}$ or 0.227273…

Move to page 1.3.

3. Page 1.3 shows both the table and a graphical representation of the data in the table. Select the bars in the graph.
    a. Based on the probabilities you see, which outcome is most likely, and what is the probability of that outcome?
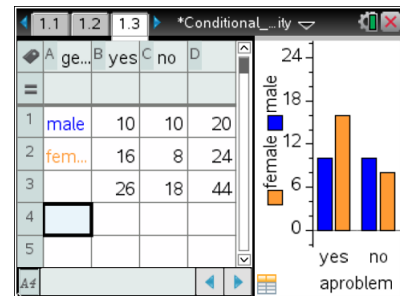
    Sample Answer: The outcome that is most likely (0.615 or 61.5%) is a female who agrees the problem is serious.

Teacher Tip: Students might think that the highest bar will always represent the greatest probability of occurring. The next question should address this concern, but it is important to be sure this possible misconception is made clear.

b. The two blue bars are the same height. Do they represent the same probabilities?
Why or why not?

Sample Answer: The frequency of both bars is 10, but they represent different things. The blue bar to the left represents the 10 males of the 26 (38.5%) people who answered yes, while the blue bar to the right represents the 10 males out of the total number, 18, (55.6%) who said no.

4. What is the probability that a person chosen at random from those who agree the problem is serious is a male? Explain how you can get this answer from the table and from the graph.

    Sample Answer: 10 out of 26 or 38.5% are male, given the person agrees the problem is serious. From the table, use the "yes" column total for the denominator, and use the number of males in that column for the numerator. Using the graph, you would go to the two bars over the label "yes," and choose the bar representing the males.

5. Assume the row and column totals remain fixed (that is, there are a total of 44 people in the survey, 24 are female and 26 of the 44 agree that the problem of unequal pay for equal work is serious. A person is chosen at random from the 44 people, and you know that the person chosen agrees that the problem of unequal pay for equal work is serious. Given this information, enter different values in the table to find the following:
    a. The maximum probability that the person is a male. Explain why this is the maximum.

        Sample Answer: The maximum possible value for the cell representing male and agreeing the problem is serious is 20 because the total number of males is 20. There are 26 people all together who answered 'yes' to the survey. Thus, the maximum probability the person is a male is 20/26 or about 0.769231 (76.9%).

    b. The minimum probability that the person is a male. Explain why this is the minimum.

        Sample Answer: The minimum possible value for the cell representing male and agreeing the problem is serious is 2 because there are 24 females and a total of 26 people who answered 'yes' to the survey. Thus, the minimum probability the person is a male is 2/26 or about 0.076923 (7.7%).

Suppose eight men agreed that the problem is serious. Enter 8 in the appropriate cell in the table.

Move to page 1.4.

6. Explain the differences in the two graphs.
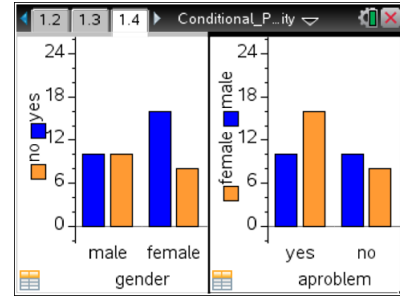
> **Sample Answer:** The graph in the left panel assumes you know whether the person is a male or a female and from that assumption, shows the probability the person thinks the problem of unequal pay for equal work is serious (assumed: gender; probability: yes/no). The graph in the right panel assumes you know whether the person agrees or does not agree that the problem is serious and based on that knowledge, shows the probability the person is male or female (assumed yes/no; probability: gender).

> **Teacher Tip:** This is a good place to have a discussion about what information is considered "given" or "assumed" in each set of bar graphs.

7. Identify the graph you would use to answer each of the following questions, and then answer the question. A newspaper randomly chooses a person to be interviewed from the 44 who were given the poll. What is the probability that:

   a. The person agrees that the problem of unequal pay for equal work is serious, given that the person is male.

   > **Sample Answer:** The panel on the left, split by gender on the horizontal axis; $\frac{8}{20}$ = 40%.

   b. The person is male, given the person does not agree that the problem of unequal pay for equal work is serious.

   > **Sample Answer:** The panel on the right, split by yes or no to whether the problem is serious on the horizontal axis: $\frac{12}{18}$ or 66.7%.

   c. The person does not agree that the problem of unequal pay for equal work is serious.

   > **Sample Answer:** Either graph would work as you would have to read the frequencies from two of the bars to capture both males and females who answered no to the question of whether the problem is serious and then divide by the total number of people in the survey to obtain 18/44=40.9%.

d. The person does not agree that the problem of unequal pay for equal work is serious given that the person is female.

Sample Answer: The panel on the left, split by gender: $\dfrac{6}{24}$ or 25%.

e. The person is male and the person agrees that the problem of unequal pay for equal work is serious.

Sample Answers: Either panel, looking for both male and the answer yes to whether the problem is serious. But the denominator is 44, which has to come from the total of all the people surveyed; $\dfrac{8}{44}$ = 18.2%.

f. The person is female or the person agrees that the problem of unequal pay for equal work is serious.

Sample Answer: Either panel can give the answer by adding up the number who are female and who answered yes to whether the problem is serious. Students should be careful not to count the overlap between females and those who answered yes; $\dfrac{32}{44}$ = 72.7%.

8. Some probability questions ask for the probability of a single outcome out of the total number possible. Some ask for the probability of a single outcome with a given condition. Others ask for the probability of an outcome that has compound (satisfies a and/or b) conditions. Categorize the situations from Question 7 into the table below according to the nature of the question.

Sample Answers:

Question	Single outcome out of total possible	Single outcome with a given condition	Outcome with compound conditions
a		x	
b		x	
c	x		
d		x	
e			x
f			x

> **Teacher Tip:** You might want to spend more time exploring P(A and B) and P(A or B) using the data in the table at a later time. The focus of this activity is just on conditional probability.

> **Teacher Tip:** This activity does not use the formal notation: *P*(A|B). If you
> choose to introduce the notation it might be better to wait until the end of
> the activity in order to make sure the concept is well understood before
> students have to decipher the notation. In many situations, the words
> should enable students to understand when a conditional probability is
> involved.

9. a. Describe how you would identify a probability question where the denominator is not the total
        number of outcomes.

    Sample Answer: When the question is based on knowing some given information, you use
    the subset of the total number of outcomes that specifies that given set; for example, when the
    problem says, given A, find the probability of B.

    b. If A and B are two distinct outcomes, what is the difference between the probability of A given B
        and the probability of B given A?

    Sample Answer: The probability of A given B says you only look for As in the set of all
    possible Bs. The probability of B given A says you look for Bs in the set of all possible As. The
    numerator in both cases will be the same, the intersection of A and B, but the denominator in
    the first case will be the number of outcomes that are B and, in the second case, the number
    of outcomes that are A.

    > **Teacher Tip:** You might want to put the two-way table on the board and
    > have the students display the bar graphs on their handhelds. Make sure
    > they understand how the bar graphs connect with the table. In one case
    > when you are given the gender, you are considering the row totals. In the
    > other case, when you are given the knowledge "if a person agrees the
    > problem is serious" then you are considering the column totals.

Extension

The actual results of the survey were that $\frac{1}{2}$ of the men agreed the problem of unequal pay for equal

work was serious, while $\frac{2}{3}$ of the women did. Refer to the Math Nspired Statistics activity *Two-way*

Tables and Association to decide whether men and women really have different opinions about whether
equal pay for equal work is a problem.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The difference between probability of an outcome and the conditional probability of an outcome.
- How to interpret values in a two-way table.
- How to compute probabilities and conditional probabilities from graphs and tables.

Assessment

Identify each of the following as sometimes true, always true, or never true. Be prepared to justify your reasoning.

1. The probability of A given B is the same as the probability of B given A.

    Answer: Sometimes true. It is true if the number of outcomes for A and for B is the same number.

2. If the row and column totals of a two-way table with two rows and two columns are constant, knowing one of the cells is sufficient to complete all of the cells in the table.

    Answer: Always true.

3. Answer the questions below given the following conditions: the row and column totals remain constant; the entry in each cell is different.

	C	D	
A			35
B			42
	20	57	77

    a. The probability of A given C is 20/35.

       Answer: Never true. Given C means the denominator has to be 20.

    b. The probability of C given B will be the value in the cell for B and C (row B, column C) divided by 42.

       Answer: Always true.

TI-Nspire Navigator

Quick Poll can be used to assess student understanding. The questions above could be used to do this. Another approach is to display a two-way table or bar graphs similar to the ones in the exercise and use Quick Poll to collect students' responses to probability questions.