Domain and Range of Exponential Functions

TEACHER NOTES

MATH NSPIRED

Math Objectives

- Students will determine the domain and range of an exponential function $f(x) = b^x$, with $b > 0$, $b \neq 1$.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- domain
- range
- exponential function

About the Lesson

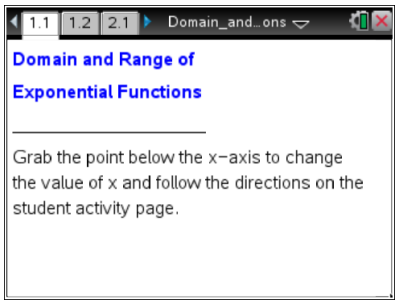
- This lesson involves students moving a point that changes the value of the exponent x in an exponential function $f(x) = b^x$. The corresponding point on the graph is shown, as well as the value of $f(x)$.
- As a result, students will:
  - Compare exponential functions of the form $f(x) = b^x$, where $b > 1$ or $0 < b < 1$.
  - Describe the domain and range of exponential functions in the form $f(x) = b^x$.
  - Compare and contrast the domain and range of exponential functions with a rational base and exponential functions with an integral base.

TI-Nspire™ Navigator™

- Use Class Capture and Quick Polls to determine if students understand how to use the TI-Nspire document and the mathematics involved in the lesson.
- Use TI-Nspire™ Navigator™ Teacher Software to review student documents.

Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity:

- Domain_and_Range_of_Exponential_Functions_Student.pdf
- Domain_and_Range_of_Exponential_Functions_Student.doc

TI-Nspire document

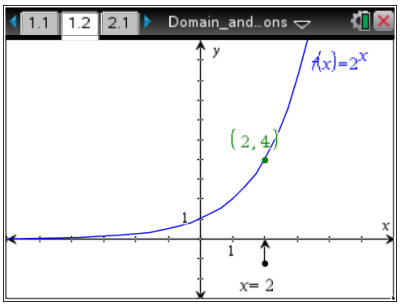
- Domain_and_Range_of_Exponential_Functions_Student.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl ☜ to grab the point and close the hand (✊). When finished moving the point, press esc to release the point.

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. a. How do the values of the function $f(x) = 2^x$ change as the value of x increases? Use the function rule $f(x) = 2^x$ to explain your answer.

Answer: As the value of x increases, the function values also increase. The function value doubles whenever x is increased by one unit (by moving the pointer one unit to the right).

Teacher Tip: Check that students are making the connection to the function rule and noticing that the x-values are actually the powers to which you are raising the base 2. As x increases, substituting x-values for the exponent results in more and more factors of 2. For further exploration after completion of the activity, the base can be changed by clicking on the text, moving the cursor over to the base, deleting the 2, and typing in a new base.

b. How do the values of the function $f(x) = 2^x$ change as the value of x decreases?

Answer: As the value of x decreases, the function values also decrease. The function value is halved whenever x is decreased by one unit (by moving the pointer one unit to the left).

Teacher Tip: This might be the first time students have been introduced to negative exponents. In this exploratory lesson, it is important for students to notice that it is possible to have negative exponents and to

look at patterns to begin making sense of what happens when a number is raised to a negative power. You might or might not decide to investigate negative exponents further at this point.

c. If x is a negative number, is the value of 2^x also negative? Explain why or why not.

Answer: No, the value of 2^x would not be negative. If x is negative, the value of 2^x would be between 0 and 1.

2. Not all function values are being calculated for you because the document was created to display only values for x that are integers. Use the graph to help you estimate what value for x produces a value of $2^x = 6$.

Answer: The value of x should be between 2 and 3. Some students might say $x = 2.5$ because 6 is halfway between 4 (2^2) and 8 (2^3), and 2.5 is halfway between 2 and 3. This reasoning can be misleading. $2^{2.5} = (2^2)(2^{0.5})$ or about 4 • 1.4 or 5.6. However, the number halfway between 4 (2^2) and 8 (2^3) is 6. Another example, $10^{1.5}$ is a bit over 30, which is not halfway between 10^1 (10) and 10^2 (100).

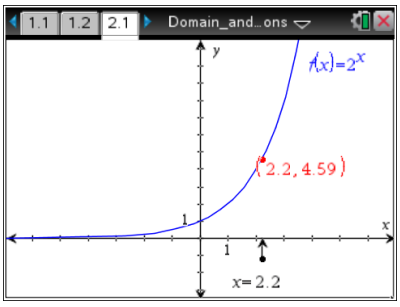
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: This might be the first time students have encountered rational exponents. In this exploratory lesson, it is important for students to notice that it is possible to have rational numbers as exponents. You might or might not decide to investigate rational exponents further at this point.

Move to page 2.1.

3. Use page 2.1 to support your response to question 2 or to revise your estimate.

Answer: The x-values are now changing by tenths. The exponent is some value between 2.5 and 2.6.



Teacher Tip: Advancing in *x*-increments of one tenth, $x = 2.6$ will produce a *y*-value nearest to 6. The class should discuss how using smaller *x* increments such as hundredths or thousandths would produce function values even closer to 6. This is also a good chance for students to share how they made their initial estimates and discuss why it is not exactly 2.5, highlighting the difference between linear and exponential functions.

4. a. What is the set of all values of *x* (the domain) that can be used as inputs in $f(x) = 2^x$?

Answer: The domain is the set of all real numbers.

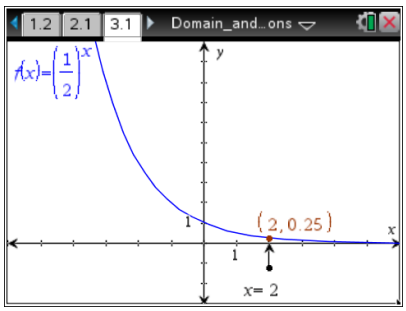
b. What is the set of all outputs (the range) of $f(x) = 2^x$?

Answer: The range is the set of all positive real numbers $\{y: y > 0\}$.

Teacher Tip: Students should be familiar with the terms *domain* and *range,* but might need prompting to consider the answer for question 4a as it relates to domain and for question 4b as it relates to the range. The real point of these questions is for students to recognize that the domain is the set of inputs that are possible and the range is the set of outputs for those given inputs.

Move to page 3.1.

5. a. How does the value of the function $f(x) = \left(\frac{1}{2}\right)^x$ change as the value of *x* increases? Use the function rule $f(x) = \left(\frac{1}{2}\right)^x$ to explain your answer.



Answer: The function values decrease as the *x*-values increase. Multiplying a positive number by $\frac{1}{2}$ will result in a smaller number.

Teacher Tip: Make sure students recognize that as *x* gets greater, there are more factors of $\frac{1}{2}$ being multiplied.

b. How does the value of the function $f(x) = \left(\frac{1}{2}\right)^x$ change as the value of x decreases?

Answer: As the value of x decreases, the function values increase.

6. a. What is the domain of the function $f(x) = \left(\frac{1}{2}\right)^x$?

Answer: The domain is the set of all real numbers.

b. What is the range of the function $f(x) = \left(\frac{1}{2}\right)^x$?

Answer: The range is the set of all positive real numbers.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

7. Compare the graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = 2^x$.

a. What do these two graphs have in common?

Answer: The graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = 2^x$ both have similar shapes. One is the reflection of the other in the y-axis. The graph goes up sharply on one side and gets close to zero on the other side. Both graphs are above the x-axis. The domains and ranges are the same. Both functions have a domain of all real numbers and a range of all positive real numbers. Both functions have the same y-intercept, (0, 1).

Teacher Tip: Encourage students to share answers and record a list of similarities as a class. It is also important to point out that the functions $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = 2^x$ still have the same domain and range. Any values can be substituted for x, but both functions will yield only positive results.

b. What is different about the two graphs?

Answer: The graph of $f(x) = \left(\frac{1}{2}\right)^x$ decreases or goes down as the value of x increases, but the graph of $f(x) = 2^x$ increases or goes up as the value of x increases.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 4 at the end of this lesson.

8. a. Would the graph of $f(x) = \left(\frac{3}{2}\right)^x$ look more like the graph of $f(x) = \left(\frac{1}{2}\right)^x$ or the graph of $f(x) = 2^x$? Why do you think so?

Answer: The graph of $f(x) = \left(\frac{3}{2}\right)^x$ would look more like the graph of $f(x) = 2^x$ because $\frac{3}{2}$ is greater than 1. Increasing x by 1 results in the function value being multiplied by $\frac{3}{2}$, so the graph will increase.

Teacher Tip: The goal is for students to make the following connection: Exponential functions with a base greater than 1 will have graphs that increase, while exponential functions with a base between 0 and 1 will have graphs that decrease. Watch for students who mistakenly think this graph will look like $f(x) = \left(\frac{1}{2}\right)^x$ simply because the base is a fraction.

b. What is the domain of $f(x) = \left(\frac{3}{2}\right)^x$? What is the range of $f(x) = \left(\frac{3}{2}\right)^x$? Explain.

Answer: The domain is the set of all real numbers. The range is the set of all positive real numbers. A number can be raised to any power. A positive number raised to any power is always a positive number.

TI-Nspire Navigator Opportunity: *Quick Poll* or *Class Capture*
See Note 5 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The domain of an exponential function $f(x) = b^x$, with $b > 0$, $b \neq 1$, and why those values are possible.
- The range of an exponential function $f(x) = b^x$, with $b > 0$, $b \neq 1$, and why those values are possible.

TI-Nspire Navigator

Note 1

Question 1, Class Capture and Live Presenter

Use Class Capture and Live Presenter to monitor and discuss students' work as they proceed through the activity. As you browse, look for students who are having difficulty and help them accordingly. It can also be helpful to pair up students, assigning students of different abilities to the same group. You might also want to use Live Presenter to demonstrate how to use the TI-Nspire document.

Note 2

Question 2, Quick Poll

Send out a *Quick Poll* open response question for question 2. Compare different answers.

After students have completed question 3, they can revise their estimates. Have students enter their current estimate from question 3 into a *Quick Poll*. Discuss the *Quick Poll* results. Let students share how they made their initial estimates, and discuss why the value of x is not exactly 2.5. Students enter their answer to question 4 into a *Quick Poll*. View and discuss the results, focusing on any misunderstandings.

Note 3

Question 6, Quick Poll

Students might enter the domain and range of the function in question 6 into two separate *Quick Polls*. Discuss any misunderstandings.

Note 4

Question 7, Class Capture

Have students add a Notes page (3.2) to the TI-Nspire document by pressing ctrl I. Encourage students to compare and contrast the two functions in question 7 and record their answers on page 3.2.

Use Class Capture to discuss students' responses. (Students can review pages 2.1 and 3.1 to answer question 7, and/or students can go to page 3.1 to press ctrl G for the function entry line. For **f2,** have them type "2^x" and then press enter to compare the functions.)

Note 5

Question 8, Quick Poll or Class Capture

Have students enter their answers to question 8 into two separate Quick Polls or use Class Capture for student answers on a new page 3.3, and discuss the responses. If students need additional information, have students go to page 2.1 and click ctrl G to see the function entry line.

Graph **f2**(x) = $\left(\frac{3}{2}\right)^x$ by typing "$\left(\frac{3}{2}\right)^x$" and pressing enter. Repeat to graph **f3**(x) = $\left(\frac{1}{2}\right)^x$.