

## Hypothesis Testing: Means

ID: 10240

Time required  
40 minutes

## Activity Overview

In this activity, students test a claim about a mean with a large sample size at the five-percent significance level. They find the test statistic and compare it to the critical value. They also find the area under the curve to find the P-value. Then, they see how the result would change if a one-percent significance level were used instead. In the second half of the activity, students suppose that the sample mean and standard deviation came from a smaller sample size and see how that change reflects in the results of the hypothesis test. An optional extension further allows students to study the importance of sample size.

## Topic: Hypothesis Testing

- Use the sampling distribution of the mean  $\bar{x}$  of the population (with known standard deviation  $\sigma$ ) to test the null hypothesis  $H_0: \mu = \mu_0$  against the alternative one-tailed hypothesis  $H_a: \mu < \mu_0$  or the two-tailed hypothesis  $H_a: \mu \neq \mu_0$ .

## Teacher Preparation

- This activity can be used while introducing hypothesis testing, to compare methods of hypothesis testing (critical value or P-value), or to compare testing claims about a mean with large and small sample sizes.

- This activity assumes familiarity with the normal distribution and finding a z-score,

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}, \text{ and the } t\text{-distribution and finding a } t\text{-score, } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}.$$

For large  $n$ ,  $s$  approximates  $\sigma$ .

- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “10240” in the keyword search box.**

## Associated Materials

- HypothesisTestingMeans\_Student.doc
- HypothesisTestingMeans.tns
- HypothesisTestingMeans\_Soln.tns

## Suggested Related Activities

To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.

- Cancer Clusters (TI-Nspire technology) — 9996
- Run Me A Hypothesis Test (TI-84 Plus family with TI-Navigator) — 5135
- Candy Pieces (TI-Nspire technology) — 9997
- Testing Claims About Proportions (TI-Nspire technology) — 10131

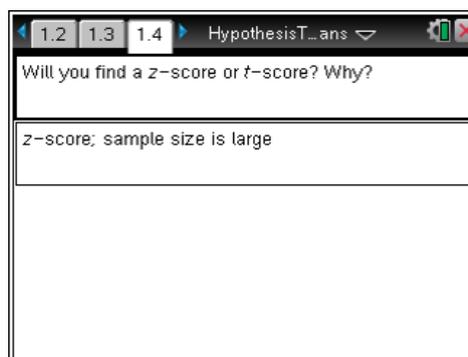
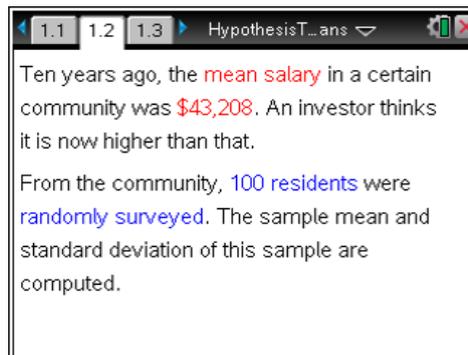
**Problem 1 – Large Sample,  $\alpha = 0.05$**

After introducing the scenario on page 1.2, have students determine the null and alternative hypotheses.

This is a right-tailed test so,  
 $H_0: \mu = \$43,208$ ,  $H_a: \mu > \$43,208$ .

Discuss whether to use a z-score (the normal distribution) or a t-score (the t distribution) for the test statistic.

The results of the survey are shown on page 1.5. Students are to use this information to calculate the test statistic on page 1.6. Because  $n$  is large, the sample standard deviation is a good estimate of the population standard deviation.



**TI-Nspire Navigator Opportunity: Quick Poll**

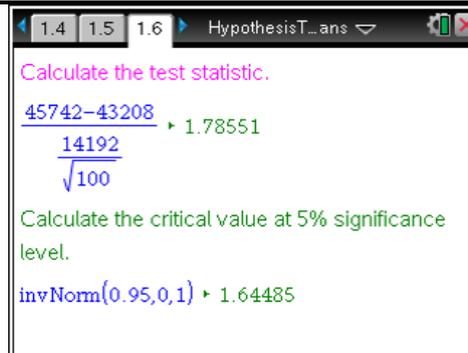
See Note 1 at the end of this lesson.

Here,  $z = 1.78551$ . Have students state what this represents. (It is the number of standard deviations that \$45,742 is from \$43,208.) The question for students to think about is, *Is this a reasonable number of standard deviations for a sample mean to be from zero if the true mean is \$43,208?* The critical value will determine that.

Now students are to find the critical value for  $\alpha = 0.05$ . Remind them that this is a right-tailed test, so 5% is above the critical value, 95% is below.

Students can find the critical value by using the **invNorm** function, which can be typed, selected from the catalog, , or chosen from the menu (**MENU > Calculations > Statistics > Distributions > Inverse Normal**).

Ask students if they should fail to reject or reject the null hypothesis and why. (Because the test statistic is greater than the critical value, they should reject the null hypothesis.)

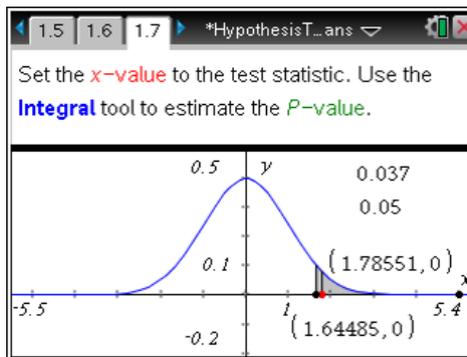
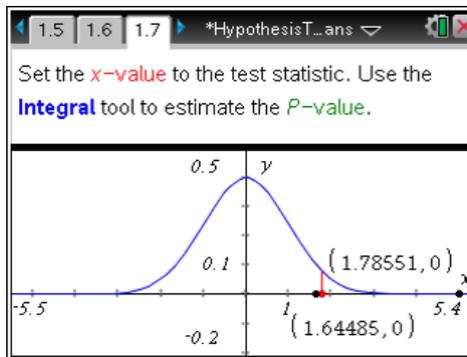


On page 1.7, students are to change the x-value of the plotted point to that of the test statistic.

If desired, you can have students also plot the critical value by using the **Point On** tool to place a point on the x-axis, anywhere other than on a tick mark. Then use the **Coordinates and Equations** tool to display the coordinates and change the x-value to that of the critical value. You can then use this diagram as a tool for discussion of critical values.

The **Integral** tool needs to be used by the students to find the P-value, the area under the curve to the right of the test statistic. To use the **Integral** tool, press **MENU > Analyze Graph > Integral**, select the curve, the leftmost point (1.78551, 0) and then the rightmost point, which for this problem can be at about (5, 0). The area is about 0.037089.

If students plotted the critical value, they can use the **Integral** tool to confirm that the area to the right of that point is 5% of the area, or 0.05. Use this diagram as a tool for discussion of the P-value method.



**TI-Nspire Navigator Opportunity: Live Presenter**

See Note 2 at the end of this lesson.

Discuss with students why the null hypothesis is being rejected, using both the critical value and the P-value.

The screenshot shows a text entry field with the question: "Do you reject or fail to reject  $H_0$ ? Why?". Below the field, the student has entered: "Reject  $H_0$ . z-score > 1.64485 P-value < 0.05".

**Problem 2 – Large Sample,  $\alpha = 0.01$**

Students are to test the hypothesis again, but this time with  $\alpha = 0.01$ .

The test statistic, 1.78551, will remain the same because the sample size did not change; however, a new critical value needs to be found on page 2.1 (2.32635).

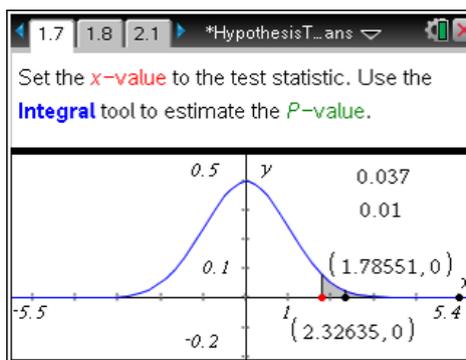
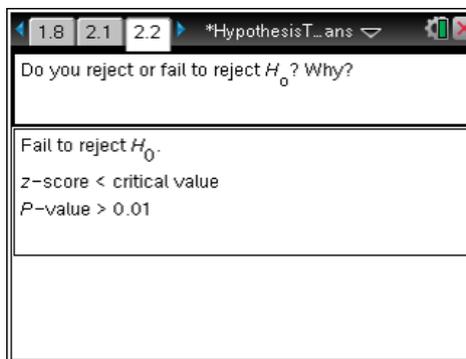
The screenshot shows a text entry field with the instruction: "Calculate the critical value with  $\alpha = 0.01$ ." Below it, the student has entered the command: "invNorm(0.99,0,1) ▶ 2.32635".

Discuss with students if they should fail to reject or reject the null hypothesis and why. They should fail to reject the null hypothesis because the test statistic is not in the critical region, that is, the region beyond the critical value.

Be sure students understand what the critical value stands for. In this case, to reject the null hypothesis, the sample mean would have to be greater than 2.32635 standard deviations above the mean.

If students are confused, have them move back to the diagram on page 1.7 and update the values to see what is happening.

Discuss how the null hypothesis is not rejected with  $\alpha = 0.01$ . A  $P$ -value  $< 0.01$  is more significant than that of one between 0.01 and 0.05, requiring the sample mean to be even more standard deviations away from the claimed mean.



**TI-Nspire Navigator Opportunity: *Live Presenter***

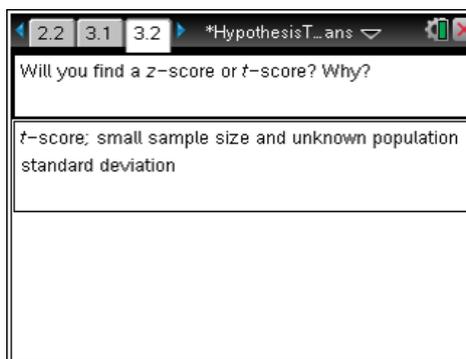
**See Note 3 at the end of this lesson.**

Explain that students have shown what happens with both significance levels for educational purposes. In a real-life application, the significance level is chosen before the hypothesis test is performed. It remains the same throughout the test, and is not changed once the  $P$ -value is known. If the  $P$ -value is close to the significance level, more samples and tests may be employed.

**Problem 3 – Small Sample,  $\alpha = 0.05$**

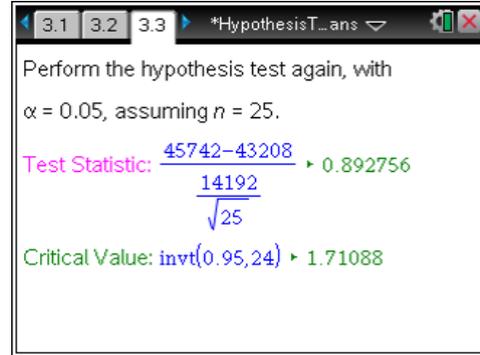
To see how sample size affects the results, students will now suppose that  $n = 25$  rather than  $n = 100$ .

Ask students if they would find a  $z$ -score or  $t$ -score and why. (Because the sample size is small and the population standard deviation is unknown, the  $t$ -distribution is used.) They should assume that the population has a roughly normal distribution.



Students are to calculate the test statistic on page 3.3. ( $t = 0.892756$ )

They may notice that this does not seem very far from the mean, but they'll still need to compare it to the critical value before making judgment. The  $t$ -distribution has different critical values than the normal distribution for small values of  $n$ .

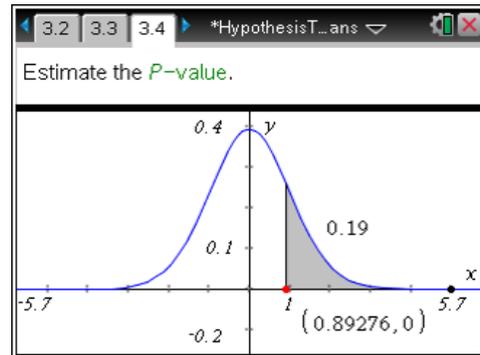


Students will also calculate the critical value on page 3.3. Remind them that there are  $n - 1$  degrees of freedom and that the test is right-tailed.

The **invt** command can be typed, chosen from the catalog, or selected from the menu (**MENU > Calculations > Statistics > Distributions > Inverse t**).

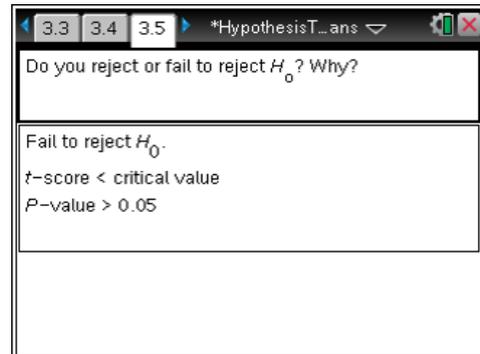
The test statistic is not greater than the critical value. The evidence is not strong enough to reject the null hypothesis, so students should fail to reject.

On the next page, students are to find the  $P$ -value. As should be expected, the  $P$ -value is much greater than 0.05.



Students are to discuss why they fail to reject the null hypothesis.

Remind them that when  $n = 100$ , we did reject the null hypothesis at this same significance level. Ask them why the sample size would change this result. You may wish for them to complete the extension before answering this.



**Problem 4 – Extension**

Students can use the test commands in the *Calculator* application (**MENU > Statistics > Stat Tests > z Test or t Test**) to find the test statistic and *P*-value for sample sizes between 25 and 100. Discuss how these values change and why.

3.4 3.5 4.1 \*HypothesisT...ans

Extension

Use the commands in the **Stat Tests** menu on the next page to find the test statistic and *P*-value for various sample sizes between 25 and 100.

Explain what happens to these values and why.

As *n* increases, the test statistic increases and the *P*-value decreases. As the sample size increases, sample means are closer to the true mean (Central Limit Theorem), and the evidence does not have to be as extreme as with smaller samples to warrant a rejection of the null hypothesis.

3.5 4.1 4.2 \*HypothesisT...ans

tTest 43208,45742,14912,30,1: stat.results

"Title"	"t Test"
"Alternate Hyp"	" $\mu > \mu_0$ "
"t"	0.930746
"PVal"	0.179834
"df"	29.
" $\bar{x}$ "	45742.
"sx := $s_{n-1}x$ "	14912.
"n"	30.

2/2

3.5 4.1 4.2 \*HypothesisT...ans

tTest 43208,45742,14192,50,1: stat.results

"Title"	"t Test"
"Alternate Hyp"	" $\mu > \mu_0$ "
"t"	1.26255
"PVal"	0.106364
"df"	49.
" $\bar{x}$ "	45742.
"sx := $s_{n-1}x$ "	14192.
"n"	50.

1/2

**TI-Nspire Navigator Opportunities**

**Note 1**

**Problem 1, Quick Poll**

Consider sending a *Quick Poll* to gather student answers to pages 1.3, 1.4, 1.6 and 1.7.

**Note 2**

**Problem 1, Live Presenter**

Consider using *Live Presenter* to discuss the *fail to reject/reject* region and how the graph tells us whether to fail to reject or reject the null hypothesis.

**Note 3**

**Problem 3, Quick Poll**

Consider using *Live Presenter* to Discuss how the null hypotheses is not rejected with  $\alpha = 0.01$ . A *P*-value < 0.01 is more significant than that of one between 0.01 and 0.05, requiring the sample mean to be even more standard deviations away from the claimed mean.