

Lesson Overview

This TI-Nspire™ lesson uses interactive number lines to help students investigate mixed numbers as a way to write improper fractions. Division of a whole number by a smaller whole number results in an improper fraction that can be written as a mixed number.



An improper fraction can be converted to the sum of a whole number and a fraction less than one. Converting a mixed number to a fraction can be thought of as fraction addition.

Learning Goals

Students should understand and be able to explain each of the following:

1. A proper fraction is a fraction whose numerator is less than the denominator;
2. An improper fraction is any fraction whose numerator is equal to or greater than the denominator;
3. An improper fraction can be expressed as a mixed number that would be the sum of a whole number and a fraction;
4. Mixed numbers can be added by finding the sum of the whole number parts and adding the sum of the corresponding proper fractional parts;
5. To find the number of whole numbers in an improper fraction, divide the denominator into the numerator.

Prerequisite Knowledge

Mixed Numbers is the sixth lesson in a series of lessons that explore fractions. This lesson builds upon the concepts explored in the previous lessons. Students should be familiar with the terms *equivalent fractions*, *unit fraction*, and *improper fractions* covered in earlier lessons. Prior to working on this lesson students should understand:

- the concept of adding and subtracting fractions.
- that whole numbers can be expressed as fractions.
- how to add and subtract fractions.
- that an improper fraction can be expressed as a whole number and a fraction.

Vocabulary

- **improper fraction:** a fraction whose numerator is equal to or greater than the denominator
- **mixed number:** a whole number and a fraction combined
- **proper fraction:** a fraction whose numerator is less than the denominator

Building Concepts: Mixed Numbers

TEACHER NOTES

Lesson Pacing

This lesson should take 50 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Mixed Numbers_Student.pdf
- Mixed Numbers_Student.doc
- Mixed Numbers.tns
- Mixed Numbers_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



Student Activity Sheet: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.



Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

This TI-Nspire™ activity uses interactive number lines to help students investigate mixed numbers as a way to write improper fractions. Division of a whole number by a smaller whole number results in an improper fraction that can be written as a mixed number. Mixed numbers can be used to find the magnitude of an improper fraction, and division can help find the integer part of the number (i.e., $\frac{243}{12}$

can be thought of as 243 divided by 12, which is 20 and a remainder of 3, $20 + \frac{3}{12}$ or $20\frac{1}{4}$). One

approach to adding and subtracting improper fractions is to add the values as fractions and convert the result to a mixed number. Another approach is to convert each fraction to a mixed number and find the sum by adding the whole parts of the numbers and the fraction parts. A caution here is that the fraction parts may produce another mixed number.

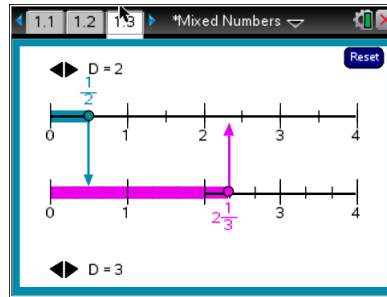
The notation for mixed numbers is confusing to many students: the sum of 2 and $\frac{1}{2}$ is written as $2\frac{1}{2}$

which can be misread as the product of 2 and $\frac{1}{2}$. When a product is intended, the multiplication sign or parentheses should be used to make clear that the operation is not addition.

Part 1, Page 1.3

Focus: Students will explore the relationship between mixed numbers and improper fractions on the number line.

Page 1.3 displays two number lines. The arrows at the top and bottom set the number of equal partitions determined by the denominators with respect to each number line. The top number line displays the number of copies of the unit fraction determined by the denominator. The bottom number line displays the whole number and fractional parts for an improper fraction. Dragging the dots will determine the numerators for each number.



TI-Nspire Technology Tips

Students may find it easier to use the **tab** key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Lead students in a discussion about the relationship between the numerator and the denominator in an improper fraction. Have students name improper fractions. Then have them work together to predict how the improper fraction will be expressed as a mixed number. Use the interactive number lines in the file to show both numbers.

As students discuss the concept of mixed numbers, encourage them to explain their reasoning. Have students work independently, then have them work together to find the solutions using the interactive number lines.

Class Discussion

Have students...

- **Set the top number line to represent the fraction $\frac{7}{3}$. Use the bottom number line to identify the number of whole numbers contained in $\frac{7}{3}$.**

Look for/Listen for...

Answer: two whole numbers, one and two, are in $\frac{7}{3}$.



Class Discussion (continued)

- ✓ **Which of the following could be written as the sum of 2 and a proper fraction where the numerator is less than the denominator? Explain your reasoning.**

(Question #1 on the Student Activity sheet.)

a. $\frac{11}{5}$ b. $\frac{7}{2}$ c. $\frac{3}{4} + \frac{9}{5}$

- **Order the following from smallest to largest and explain your reasoning:**

$2\frac{5}{6}, \frac{16}{5}, 2\frac{3}{4}, \frac{15}{6}, \frac{17}{12}$

Describe two strategies for finding the solutions written as mixed numbers for each of the following problems:

- $\frac{7}{3} + \frac{10}{3}$

- $\frac{7}{4} - \frac{5}{4}$

- $2\frac{3}{7} + 1\frac{5}{7}$

- $3\frac{7}{8} + 1\frac{2}{3}$

Answer: a could be written as 2 and $\frac{1}{5}$. In choice c,

$\frac{9}{5}$ is nearly 2 and adding $\frac{3}{4}$ will make the sum greater than 2 but not yet 3,
2 and $\frac{11}{20}$.

Answer: Convert all of the fractions to mixed numbers, then compare the whole numbers first before comparing the fraction part:

$$1 + \frac{5}{12} < 2 + \frac{1}{2} < 2 + \frac{3}{4} < 2 + \frac{5}{6} < 3 + \frac{1}{5}; \text{ thus}$$

$$\frac{17}{12} < \frac{15}{6} < 2\frac{3}{4} < 2\frac{5}{6} < \frac{16}{5}.$$

Answer: Each of the problems can be solved by combining the whole number part of the mixed number and the fraction parts or they can be added as fractions then converted into mixed numbers.

Answer: $2 + \frac{1}{3}$ added to $3 + \frac{1}{3}$ would give $5 + \frac{2}{3}$. Or

by adding the fractions, $\frac{17}{3}$ is also $5 + \frac{2}{3}$; $5\frac{2}{3}$.

Answer: $\frac{2}{4} = \frac{1}{2}$; $1 + \frac{3}{4}$ minus $1 + \frac{1}{4}$ results in $\frac{3}{4} - \frac{1}{4}$
or $\frac{2}{4} = \frac{1}{2}$.

Answer: $3 + \frac{8}{7}$ converts to $4 + \frac{1}{7}$; $\frac{17}{7} + \frac{12}{7}$ can be expressed as $\frac{29}{7} = 4\frac{1}{7}$; $4\frac{1}{7}$.

Answer:

$$4 + \left(\frac{7}{8} + \frac{2}{3}\right) = 4 + \left(\frac{21}{24} + \frac{16}{24}\right) = 4 + \frac{37}{24} = 5 + \frac{13}{24}; 5\frac{13}{24}$$

Class Discussion (continued)

Have students...

Identify the following as true or false. Explain your reasoning.

- *The sum of $\frac{5}{3} + \frac{3}{4}$ is between 2 and 3.*

- ✓ *A fraction whose denominator is 8 will be between 3 and 4 if the numerator is between 18 and 32.*

(Question #2 on the Student Activity sheet.)

- *An improper fraction whose numerator is 5 will be between 1 and 2 if the denominator is 4 or 3.*

Answer each and explain your reasoning.


- ✓ *A recipe calls for $2\frac{1}{3}$ cups of flour.*

Suppose you only have a $\frac{1}{3}$ -cup

measuring cup. How many times will you have to fill the $\frac{1}{3}$ cup?

(Question #3 on the Student Activity sheet.)

- *How much will each person get if 15 apples are divided among 4 people?*

-  *Arnie has traveled $\frac{23}{12}$ of the way towards his destination. The number of miles he has traveled will be between what two whole numbers?*

Look for/Listen for...

Possible answer: True. $\frac{5}{3}$ is $1 + \frac{2}{3}$ and the sum of $\frac{2}{3}$ and $\frac{3}{4}$ is more than one but less than 2 so adding the one will make the sum between 2 and 3.

Possible answer: False. $\frac{19}{8}$ is $2 + \frac{3}{8}$, which is less than 3.

Possible answer: True because $\frac{5}{4}$ and $\frac{5}{3}$ are both greater than 1 but less than 2.

Possible answer: There are three $\frac{1}{3}$ cups in each whole, so there would be six $\frac{1}{3}$ cups in 2 plus the extra $\frac{1}{3}$ cup. You would fill the $\frac{1}{3}$ cup 7 times.

Possible answer: Each person would get three whole apples and then they each would get $\frac{3}{4}$ of another apple. You could cut three of the apples into half and then half again and give everyone three of the 12 pieces.

Answer: $\frac{23}{12} = 1 + \frac{11}{12}$ so it is between 1 and 2 miles.



Class Discussion (continued)

- Explain why $\frac{10}{6}$ is the same as $1\frac{2}{3}$.

Possible answer: $\frac{10}{6}$ is $1 + \frac{4}{6}$ and $\frac{4}{6}$ is equivalent to $\frac{2}{3}$.

Which is greater and why?

- $\frac{12}{5}$ or $\frac{7}{2}$

Answer: $\frac{7}{2}$ is $3 + \frac{1}{2}$, which is greater than $\frac{12}{5}$, which is $2 + \frac{2}{5}$.

- $\frac{9}{4} - \frac{8}{7}$ or $\frac{9}{4} - \frac{11}{12}$

Answer: $\frac{9}{4} - \frac{11}{12}$ has the greater value because $\frac{11}{12}$ is less than one and $\frac{8}{7}$ is more than one, so you are taking less away from $\frac{9}{4}$ when you subtract $\frac{11}{12}$.

- $\frac{15}{12} + \frac{4}{3}$ or $\frac{12}{10} + \frac{19}{9}$

Answer: $\frac{12}{10} + \frac{19}{9}$ is the greater value because $\frac{12}{10}$ is 1 plus a proper fraction and $\frac{19}{9}$ is 2 plus a proper fraction so the total is more than 3. In the first sum, each fraction is 1+ a proper fraction, but the two proper fractions are $\frac{1}{4}$ and $\frac{1}{3}$, and their sum is less than 1 making the total sum less than 3.

Solve each. Express the answer as a mixed number.

- $2 + \frac{15}{11}$

Answer: $3\frac{4}{11}$.

- $7 - \frac{25}{7}$

Answer: $3\frac{3}{7}$.

- $\frac{31}{8} - 2$

Answer: $1\frac{7}{8}$.



Class Discussion (continued)

Without solving, identify the statement as true or false. Explain your thinking in each case.

- The sum of $1\frac{1}{5}$ and $3\frac{4}{9}$ is less than 5.**

Answer: True because $\frac{1}{5} + \frac{4}{9}$ is less than one and the whole number part only adds to 4.
- The sum of $3\frac{5}{9}$ and $2\frac{7}{12}$ is less than 6.**

Answer: False. The sum of the fraction parts is more than 1 and the whole number parts add to 5 so the total is more than 6.
- The sum of $\frac{7}{3}$ and $\frac{18}{5}$ is more than 5.**

Answer: True. The whole number parts are 2 and 3 so the sum will be more than 5 when you add in the proper fraction parts.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

- Create two fractions, with numerators greater than denominators, equivalent to $3\frac{1}{4}$.

Possible answer: $\frac{26}{8}; \frac{39}{12}$.

- Order the following from smallest to largest.

$$\frac{10}{3}, \frac{7}{4}, \frac{10}{8}, \frac{10}{4}, \frac{7}{3}, \frac{8}{4}$$

Answer: Thinking in mixed numbers is easier than fractions.

$$1 + \frac{1}{4} < 1 + \frac{3}{4} < 2 < 2 + \frac{1}{2} < 3 + \frac{1}{3} \text{ so } \frac{5}{4} \left(\frac{10}{8} \right) < \frac{7}{4} < \frac{8}{4} < \frac{7}{3} < \frac{10}{4} < \frac{10}{3}.$$

- Edwin is making chili. The recipe calls for $3\frac{1}{4}$ cups of chopped onions. When he measures the onions he chopped, he has only $2\frac{3}{4}$ cups. How many more cups of onions does he need?

Adapted from Ohio, 2006, grade 6

Answer: $2\frac{3}{4}$ is $\frac{1}{4}$ below 3 and $3\frac{1}{4}$ is $\frac{1}{4}$ over 3, so he needs two one fourth cups or one half cup more.



Building Concepts: Mixed Numbers

TEACHER NOTES

4. Every day Suze collects aluminum for a project. The table shows how much she collected during one five-day period.
- Estimate the number of pounds of aluminum she collected during that time. Explain how you made your estimate. **Possible answer: Monday and Tuesday add up to $\frac{16}{8}$ or 2; adding in Wednesday and Thursday make it about 5 plus Friday gives about 6. Another strategy: adding the 8ths gives $\frac{22}{8}$, which is $2 + \frac{6}{8}$ or $2 + \frac{3}{4}$. Add in the whole numbers 2 and 1 and get about 6.**

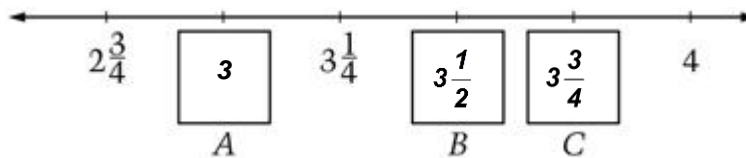
Day	Monday	Tuesday	Wednesday	Thursday	Friday
Pounds	$\frac{13}{8}$	$\frac{3}{8}$	2	$1\frac{1}{8}$	$\frac{5}{8}$

- Suze recorded the amount she collected on Tuesday incorrectly. It should have been $\frac{1}{2}$. How does that change your estimate?

Adapted from Ohio, 2006, grade 6

Possible answer: The total is still about 6 because it was close to 6 and the difference between $\frac{1}{2}$ or $\frac{4}{8}$ and $\frac{3}{8}$ is only $\frac{1}{8}$, so it won't get much larger.

5. Jorge left some numbers off the number line below. Fill in the numbers that should go in A, B, and C.



NAEP grade 4, 2009

Student Activity solutions
Vocabulary
improper fraction:

a fraction whose numerator is equal to or greater than the denominator

mixed number:

a whole number and a fraction combined

proper fraction:

a fraction whose numerator is less than the denominator

In this activity, you will solve problems involving improper fractions and mixed numbers.

1. Which of the following could be written as the sum of 2 and a proper fraction where the numerator is less than the denominator? Explain your reasoning.

a. $\frac{11}{5}$ b. $\frac{7}{2}$ c. $\frac{3}{4} + \frac{9}{5}$

Answer: a could be written as 2 and $\frac{1}{5}$. In choice c, $\frac{9}{5}$ is

nearly 2 and adding $\frac{3}{4}$ will make the sum greater than 2 but

not yet 3, 2 and $\frac{11}{20}$.

2. Identify the following statement as true or false: "A fraction whose denominator is 8 will be between 3 and 4 if the numerator is between 18 and 32." Explain your reasoning.

Possible answer: False. c is $2 + \frac{3}{8}$, which is less than 3.

3. A recipe calls for $2\frac{1}{3}$ cups of flour. Suppose you only have a $\frac{1}{3}$ -cup measuring cup. How many times will you have to fill the $\frac{1}{3}$ -cup? Explain your reasoning.


Possible answer: There are three $\frac{1}{3}$ cups in each whole, so

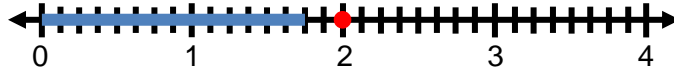
there would be six $\frac{1}{3}$ cups in 2 plus the extra $\frac{1}{3}$ cup. You

would fill the $\frac{1}{3}$ cup 7 times.

Building Concepts: Mixed Numbers

TEACHER NOTES

4.  Arnie hiked $\frac{14}{8}$ miles of a 4-mile hiking trail. Has Arnie hiked more or less than half of the total length of the trail? Complete the number line to show your thinking. Explain your reasoning.



Answer: Arnie hiked less than half the distance. $\frac{14}{8}$ is equal to $1\frac{6}{8}$ or $1\frac{3}{4}$ miles. The half-way point is at 2 miles and $1\frac{3}{4} < 2$.