THE LIGHT SIDE OF TRIGONOMETRY

The earth's movement around the sun is an example of periodic motion. The earth's tilt on its axis and corresponding movement around the sun causes the changing seasons and the corresponding changes in the number of daylight hours. The number of daylight hours is also dependent on the position on the earth's spherical surface. The number of daylight hours on any given day of the year can be modeled by a trigonometric function.

The following diagrams are taken from satellite images: Figure 2 is an image from the moon 368741 kilometres above the earth $19°33'$ S $137°35'$ E at 23:35 UTC on September 12th 2003. Figure 1 is a sequence of photographs taken from a number of satellites. The photographs in figures 1 and 2 were taken at the same time.

Figure 1

The internet site where these photographs were obtained is www.fourmilab.com. Select the Astronomy and Space link under the Science heading and then click on the Earth and Moon viewer link. Select either the **map of the earth** option or the **view from moon** option.

Figure 2

OBJECTIVES

1. To develop a model for:

    a) the time of sunrise in Melbourne for any day of the year.
    b) the time of sunset in Melbourne for any day of the year.
    c) the number of daylight hours in Melbourne for any day of the year.

2. To investigate how the position on the earth's surface affects the equation.

EQUIPMENT REQUIRED

1. Sun Cycle 1.0.8.5 software.

    The software can be downloaded from the site: http://users.pandora.be/suncycle/
    Select the Home and Education category and the Science field. Perform a search for "Sunrise" in order to locate the software quickly.

2. TI-InterActive! software.

COLLECTING DATA

The data to be collected is a combination of sunrise and sunset times. The total number of daylight hours can be determined from this data. Naturally, winter represents the least number of daylight hours and summer represents the greatest number of daylight hours.

Data can be collected at regular intervals or on a specific day each month (such as the first day). The tables below indicate the suggested dates for the standard year or the leap year.

Standard

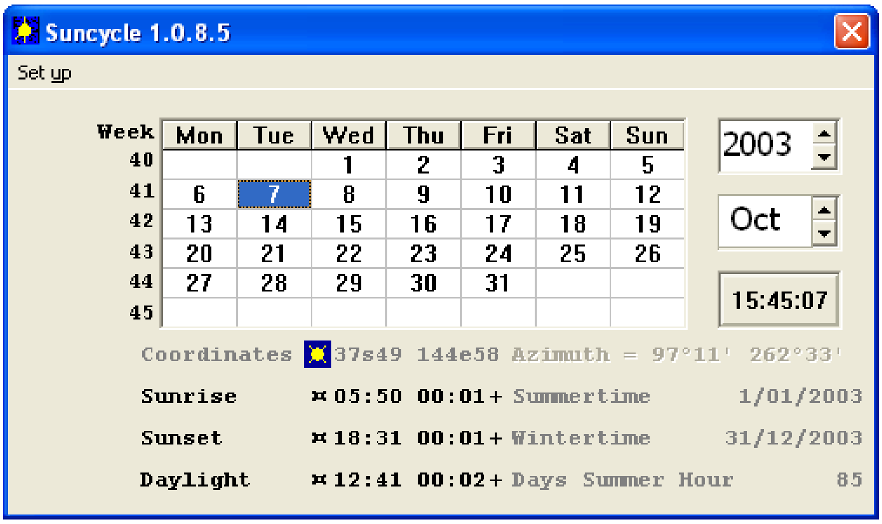
Month	Jan	Mar	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Day	30	1	31	30	30	29	29	28	27	27	26	26
Cumulative	30	60	90	120	150	180	210	240	270	300	330	360
Sunrise												
Sunset												

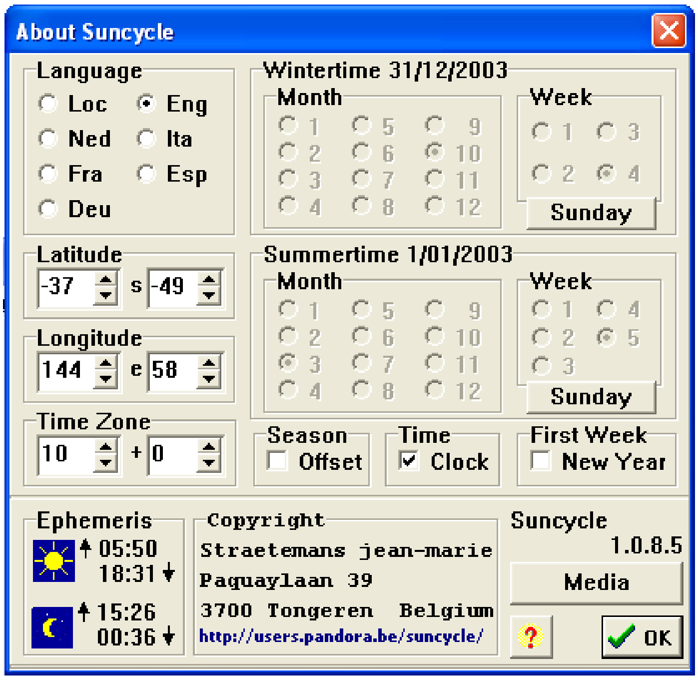
Leap Year

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Day	30	29	30	29	29	28	28	27	26	26	25	25
Cumulative	30	60	90	120	150	180	210	240	270	300	330	360
Sunrise												
Sunset												

COLLECTING DATA

1. Open the “Sun Cycle” program. The calendar, as indicated below, should appear.





2. Right click on the calendar, or click on Set-up, to access the specialized controls.

3. As we are interested in the sunrise and sunset times for Melbourne we need to enter the longitude, latitude and time zone for Melbourne.

   Change each of the following:

   a) Language: to English (Eng)

   b) Latitude: to −37 s −49

   c) Longitude: to 144 e 58

   d) Time Zone: to 10

4. Uncheck the Season Offset box.

5. Click OK to return to the calendar.

6. Use the Up and Down arrows top select the appropriate Month and click on the Day required.

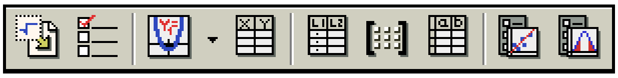
7. Record the data for Sunrise and Sunset in the table provided.

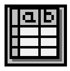
PART 1 – DETERMINING A MODEL FOR THE TIME OF MELBOURNE'S SUNRISE

1. Once the data has been collected it can be transferred to a spreadsheet in TI-InterActive!

2. To speed up the data entry process, open the template file "The Light Side of Trigonometry - Template."

   Otherwise, from the TI-InterActive! toolbar, click on the Spreadsheet icon.



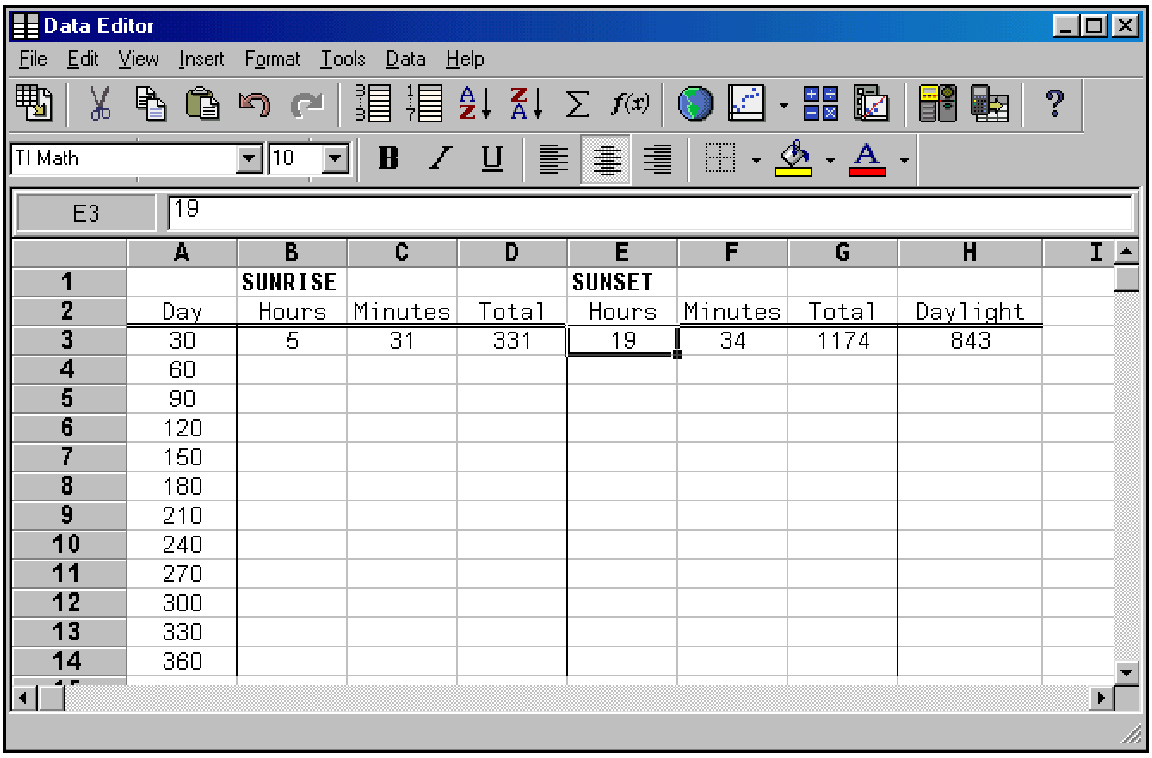


   a) Label the columns as Hours, Minutes and Total as indicated in the diagram below.

   b) Enter an appropriate formula to calculate the values in the Total column.

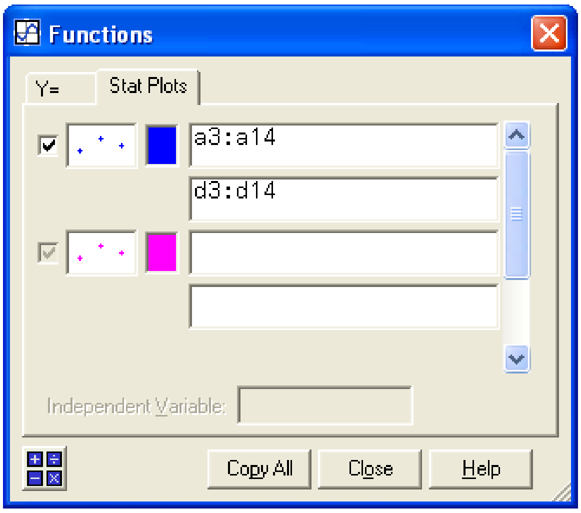
   c) Repeat for the Sunset data.

   d) Enter an appropriate formula to calculate the difference between the time of sunset and the time of sunrise.



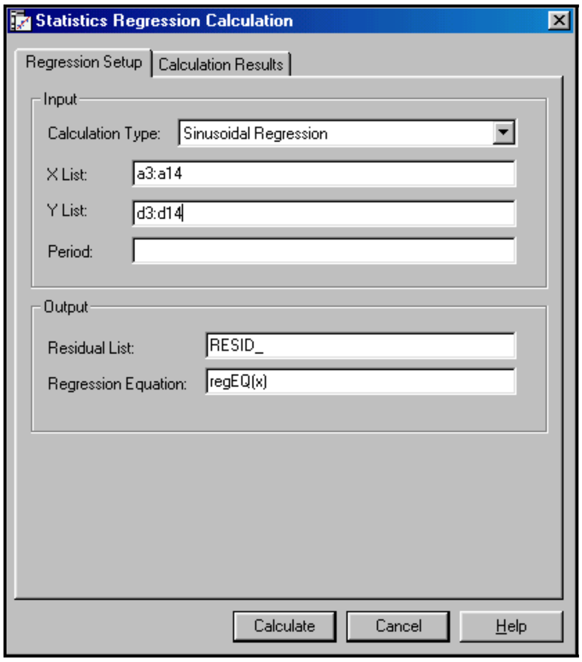
	A	B	C	D	E	F	G	H	I
1		SUNRISE			SUNSET				
2	Day	Hours	Minutes	Total	Hours	Minutes	Total	Daylight	
3	30	5	31	331	19	34	1174	843	
4	60								
5	90								
6	120								
7	150								
8	180								
9	210								
10	240								
11	270								
12	300								
13	330								
14	360								

3. Prepare a Statistical Plot for the Sunrise data using the cells a3:a14 and d3:d14.



4. Calculate a sinusoidal regression to determine a model for the total number of minutes between midnight and sunrise.

Note: the sinusoidal regression model given by TI-InterActive! is of the form $y = a\sin(bx + c) + d$.



5. Copy this function into the $\boxed{\text{Y=}}$ editor. Graph this function on the same set of axes as the statistical plot of the data.

7. Use a Math Box to determine the amplitude, period, horizontal and vertical translations of the function.

 .

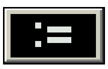
Note: TI-InterActive! uses the notation, $a\_$, to represent the variable *a*.

Remember: Period $= \dfrac{2\pi}{b}$

8. Write an expression for the total number of minutes between midnight and sunrise in the form $y = a\sin b(x + c) + d$.

You will need to use the variable values from the sinusoidal regression in order to calculate the value of *b*.

TI-InterActive uses the notation $y(x) :=$ to define a function.

In particular it is important to use the  button.

9. Check the accuracy of your rule by using it to determine an approximate sunrise time for today.

   Use a Math Box and Math, Number, Fractional Part to determine the number of minutes.

10. Use either the Sun Cycle program or the newspaper to give the actual value. Comment on the accuracy of your rule.

PART 2 – DETERMINING THE EQUATION FOR MELBOURNE'S SUNSET

1. Prepare a statistical plot of the sunset data.
2. Calculate a sinusoidal regression to determine a model for the total number of minutes between midnight and sunset.
3. Graph this function on the same set of axes as the statistical plot of the data.
4. State the amplitude, period, horizontal and vertical translations of the function.
5. Write an expression for the total number of minutes between midnight and sunset in the form $y = a \sin b(x + c) + d$.
6. Check the accuracy of your rule by using it to determine an approximate sunset time for today. Use either the Sun Cycle program or the newspaper to give the actual value. Comment on the accuracy of your rule.
7. Compare the equations for sunrise and sunset. What factors are different and why?

PART 3 – DETERMINING THE EQUATION FOR NUMBER OF DAYLIGHT HOURS

1. Use the same process as outlined for finding the time of sunrise to determine the equation for the number of daylight hours for Melbourne.
2. Graph sunrise, sunset and daylight minutes on the same set of axes.
2. Use your rule to determine the dates of the summer and winter solstice. State the actual values of the summer and winter solstice and compare them with the results obtained from your rule.
3. Use your equation to determine the equinox dates. Comment on the accuracy of your results.
4. What is the average number of daylight hours per day for the whole year in Melbourne?

PART 4 – GENERALIZING THE EQUATION FOR ANY POINT ON THE EARTH

For this part of the investigation each member of the class will use the longitude $140°$ East and the corresponding time zone $+10$ hours.

1. Select **one** of the following pairs of latitudes: $50°\,\text{N}$ and $50°\,\text{S}$; $40°\,\text{N}$ and $40°\,\text{S}$; $30°\,\text{N}$ and $30°\,\text{S}$, $20°\,\text{N}$ and $20°\,\text{S}$ or $10°\,\text{N}$ and $10°\,\text{S}$.

2. Use the Sun Cycle program to compile a table of sunrise, sunset and daylight minutes for the latitudes selected.

3. Determine the equations, in the form $y = a\sin b(x+c)+d$, for the number of daylight minutes for the latitudes selected. State the amplitude, period, horizontal translation and vertical translation in each case. Test your equation and comment on its accuracy.

4. Graph the daylight minutes for the Northern and Southern hemispheres for your selected pair of latitudes on the same set of axes.

5. Compare the equation for the Northern hemisphere with that of the Southern hemispehere. What similarities are there? What differences are there?

6. Complete the table below, which summarises the results for each of the different latitudes. What significant features about the results do you notice?

7. Explain why a different longitude will have no effect on the “daylight minutes” equation.

EXTENSION

1. Using the summary results obtained, propose a model for the equation for daylight minutes for a given latitude.

2. Comment on your results.

THE LIGHT SIDE OF TRIGONOMETRY – SUMMARY TABLE

Longitude: $140°$ East Time Zone: $+10$ hours

Latitude	Amplitude	Period	Horizontal translation	Vertical translation	Equation
$50°$ N					
$40°$ N					
$30°$ N					
$20°$ N					
$10°$ N					
$0°$					
$10°$ S					
$20°$ S					
$30°$ S					
$40°$ S					
$50°$ S					

FLINDERS CHRISTIAN COMMUNITY COLLEGE

MATHEMATICAL METHODS

APPLICATION TASK GRADING SHEET

Student name:	
Date:	

Outcome	**Criteria**		**Marks available**	**Marks given**
Outcome 1 Define and explain key concepts as specified in the content of the areas of study, and apply a range of related mathematical routines and procedures.	1	Appropriate use of mathematical conventions, symbols and terminology	3	
	2	Definition and explanation of key concepts	4	
11 marks	3	Accurate application of mathematical skills and techniques	4	
Outcome 2 Apply mathematical processes in non-routine contexts and analyse and discuss these applications of mathematics.	1	Identification of important information, variables and constraints	3	
	2	Application of mathematical ideas and content from the specified areas of study	5	
14 marks	3	Analysis and interpretation of results	6	
Outcome 3 Select and appropriately use technology to produce results and carry out analysis in situations requiring problem solving, modelling or investigative techniques or approaches.	1	Appropriate selection and effective use of technology	2	
5 marks	2	Application of technology	3	
		TOTAL	30	

Comments

A+	**A**	**B+**	**B**	**C+**	**C**	**D+**	**D**	**E+**	**E**	**UG**
30 - 28	27 – 25	24 – 22	21 - 19	18 - 16	15 - 13	12 - 10	9 - 8	7 - 6	5 - 4	3 - 0