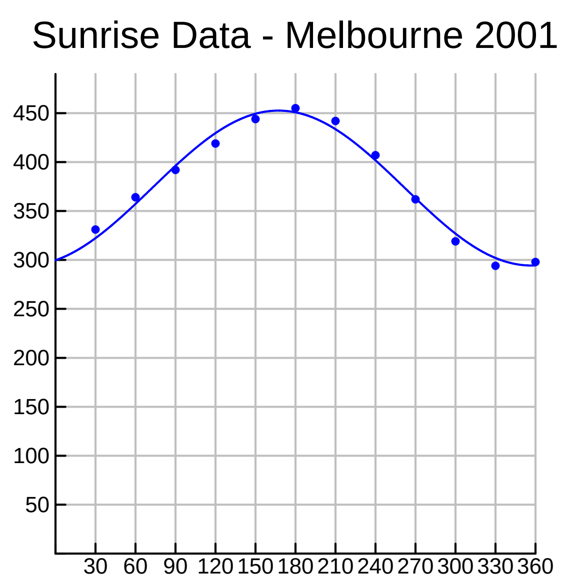
THE LIGHT SIDE OF TRIGONOMETRY

Part 1: SUNRISE DATA - Melbourne 2001

	Sunrise			Sunset			
Day	Hours	Mins	Total	Hours	Mins	Total	Daylight
30	5	31	331	19	34	1174	843
60	6	4	364	18	59	1139	775
90	6	32	392	18	15	1095	703
120	6	59	419	17	34	1054	635
150	7	24	444	17	10	1030	586
180	7	35	455	17	11	1031	576
210	7	22	442	17	30	1050	608
240	6	47	407	17	55	1075	668
270	6	2	362	18	20	1100	738
300	5	19	319	18	38	1118	799
330	4	54	294	19	20	1160	866
360	4	58	298	19	43	1183	885
						Average	723.5

Sunrise - Melbourne 2001
Sinusoidal Regression
regEQ(x) = 79.1246*sin(.016573*x+-1.20082)+373.462
a = 79.1246
b = .016573
c = -1.20082
d = 373.462



Amplitude:

$a\_$

79.1246

Period:

$b\_$

.016573

$$\frac{2\cdot\pi}{b\_}$$

379.118

Horizontal translation:

$$\frac{c\_}{b\_}$$

-72.4557

Vertical translation:

$d\_$

373.462

Hence the model for Melbourne's sunrise is:

$$y(x) := 79.1246\cdot\sin(0.016573\cdot(x - 72.4557)) + 373.462$$

"Done"

To check the accuracy of the rule we can substitute in the day value for October 15th into the general rule.

$y(288)$ 340.432

Calculations:

Hours: $\frac{ans}{60}$ 5.67387

Minutes: $\text{fpart}(ans)\cdot 60$ 40.4325

This is equivalent to 5 hours and 40 minutes from midnight. Hence 5:40 am

The "Sun Cycle" program gave the time of Melbourne's sunrise as 5:35 am.

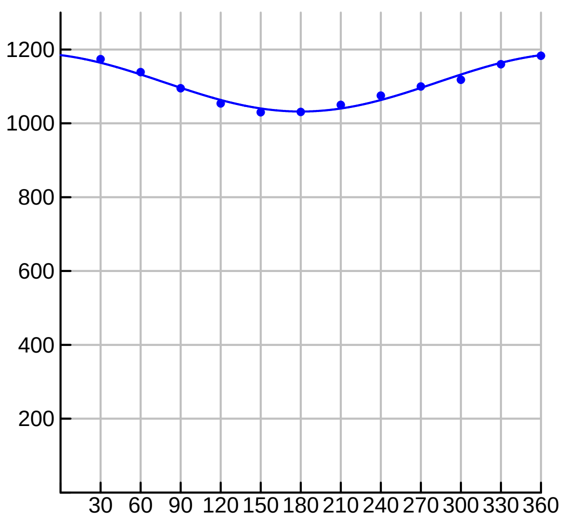
Our result is approximately 5 minutes "out."

Part 2: SUNSET DATA - Melbourne 2001

Day	Sunrise Hours	Mins	Total	Sunset Hours	Mins	Total	Daylight
30	5	31	331	19	34	1174	843
60	6	4	364	18	59	1139	775
90	6	32	392	18	15	1095	703
120	6	59	419	17	34	1054	635
150	7	24	444	17	10	1030	586
180	7	35	455	17	11	1031	576
210	7	22	442	17	30	1050	608
240	6	47	407	17	55	1075	668
270	6	2	362	18	20	1100	738
300	5	19	319	18	38	1118	799
330	4	54	294	19	20	1160	866
360	4	58	298	19	43	1183	885

Sunrise - Melbourne 2001
Sinusoidal Regression
regEQ(x) = 79.1842*sin(.015335*x+1.95061)+1111.24
a = 79.1842
b = .015335
c = 1.95061
d = 1111.24

Sunset Data - Melbourne 2001



Amplitude:

$a\_$

79.1842

Period:

$b\_$

.015335

$\frac{2\cdot\pi}{b\_}$

409.741

Horizontal translation:

$\frac{c\_}{b\_}$

127.204

Vertical translation:

$d\_$

1111.24

Hence the model for Melbourne's sunset is:

$y(x) := 79.1842\cdot\sin(0.015335\cdot(x + 127.204)) + 1111.24$

"Done"

To check the accuracy of the rule we can substitute in the day value for October 15th into the general rule.

$y(288)$ 1117.88

This is equivalent to 18 hours and 38 minutes from midnight. Hence 6:38 pm

Calculations:

Hours:

$\frac{ans}{60}$ 18.6314

Minutes:

$fpart(ans)\cdot 60$ 37.8811

The "Sun Cycle" program gave the time of Melbourne's sunset as 6:36 pm.

Our result is approximately 2 minutes "out."

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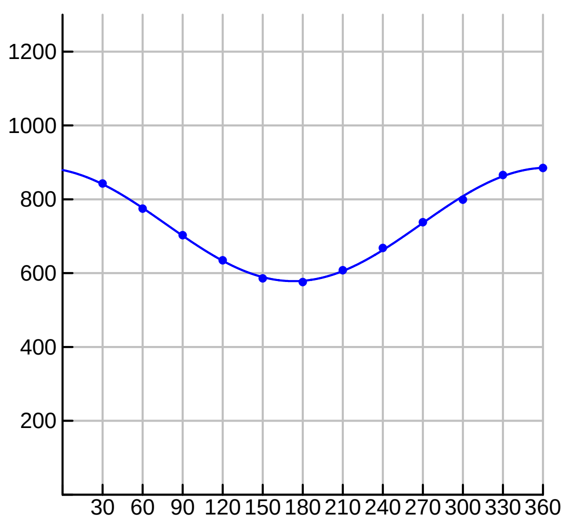
Part 3: DAYLIGHT MINUTES - Melbourne 2001

	Sunrise			Sunset			
Day	Hours	Mins	Total	Hours	Mins	Total	Daylight
30	5	31	331	19	34	1174	843
60	6	4	364	18	59	1139	775
90	6	32	392	18	15	1095	703
120	6	59	419	17	34	1054	635
150	7	24	444	17	10	1030	586
180	7	35	455	17	11	1031	576
210	7	22	442	17	30	1050	608
240	6	47	407	17	55	1075	668
270	6	2	362	18	20	1100	738
300	5	19	319	18	38	1118	799
330	4	54	294	19	20	1160	866
360	4	58	298	19	43	1183	885

Daylight minutes - Melbourne 2001
Sinusoidal Regression
regEQ(x) = 153.647*sin(.016485*x+1.85837)+731.992
a = 153.647
b = .016485
c = 1.85837
d = 731.992

Daylight minutes - Melbourne 2001



Amplitude:

$a\_$

153.647

Period:

$b\_$

.016485

$$\frac{2\cdot\pi}{b\_}$$

381.155

Horizontal translation:

$$\frac{c\_}{b\_}$$

112.733

Vertical translation:

$d\_$

731.992

Hence the model for the number of daylight minutes for Melbourne is:

$$y(x) := 153.647\cdot\sin(0.016485\cdot(x + 112.733)) + 731.992$$

"Done"

To check the accuracy of the rule we can substitute in the day value for October 15th into the general rule.

$y(288)$ 780.747

This is equivalent to 13 hours and 1 minute for the day.

Calculations:

Hours:

$\frac{ans}{60}$ 13.0124

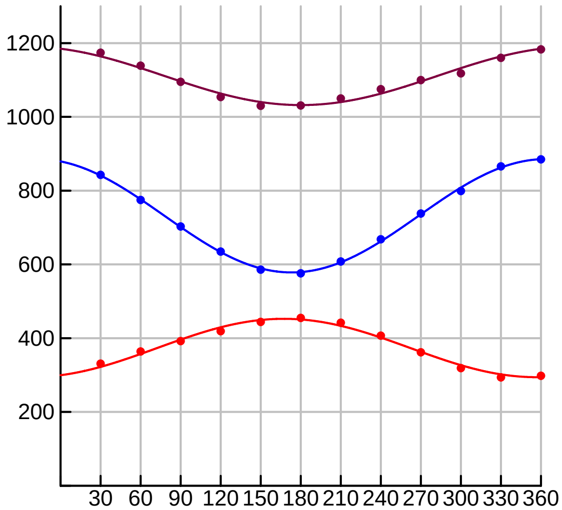
Minutes:

$fpart(ans)\cdot 60$ .746699

The "Sun Cycle" program gave the daylight hours for Melbourne as 13 hours and 1 minute.

Our rule gives exactly the same result!

The graph of sunrise, sunset and daylight minutes on the same set of axes is as follows:



Sunset
Daylight minutes
Sunrise

The solstice marks the longest and shortest days of the year.

Winter Solstice:

The minimum value of daylight hours occurs at (173.128, 578.345)

Day 173 of the year corresponds to June 22nd. The actual value is June 21st - so our result is one day out!

Summer Solstice:

The maximum value of daylight hours occurs at (360., 885.353)

Day 360 of the year corresponds to December 26th. The actual value is December 22nd - so our result is four days out.

Equinox:

The equinox is defined as the point where there are an equal number of daylight and night minutes.

This corresponds to 12 hours or 720 minutes. We can use the equation
$y(x) = 153.647 \cdot \sin(0.016485 \cdot (x + 112.733)) + 731.992$ where $y(x) = 720$ and solve for *x*.

$\text{Solve}(720 = 153.647 \cdot \sin(0.016485 \cdot (x + 112.733)) + 731.992, x) \mid 0 < x \text{ AND } x < 365$

$x = 263.673 \text{ or } x = 82.5792$

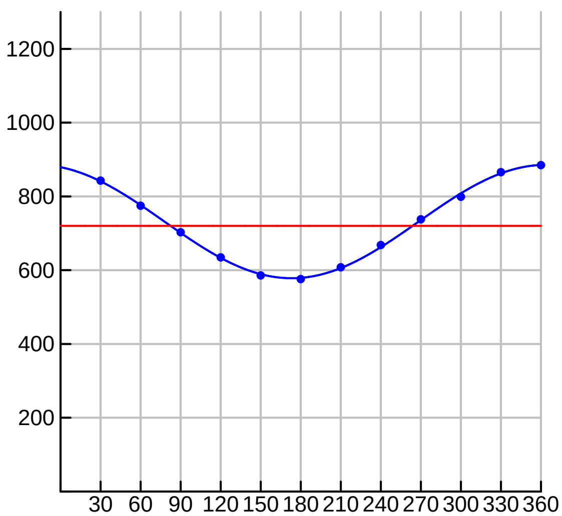
Day 83 corresponds to March 24th. The actual value of the Autumnal equinox is day 80 - March 21st, so our result is out by three days.

Day 263 corresponds to September 20th. The actual value of the Spring equinox is day 264 - September 21st, so our result is out by one day.

Using the graph and finding the point of intersection with y = 720 yields the following points:

Autumnal equinox: (82.5812, 720.) Spring equinox: (263.675, 720.)

Sunrise, Sunset and Daylight minutes



The average number of daylight hours for Melbourne can be determined from the table of daylight minutes.

The spreadsheet in part 1 indicates that the average from the twelve data points taken is 723.5. Hence there are, on average, 12 hours and 3 minutes of daylight per day in Melbourne.

Part 4: GENERALIZING THE EQUATION

The longitude selected was $144°\cdot58'\cdot\text{East}$ with a corresponding time zone of +10 hours.
The pair of latitudes selected were $50°\cdot\text{North and } 50°\cdot\text{South}$.

50 DEGREES - South

50 Degrees South							
Day	Hours	Minutes	Total	Hours	Minutes	Total	Daylight
30	4	59	299	20	10	1210	911
60	5	52	352	19	15	1155	803
90	6	40	400	18	11	1091	691
120	7	26	446	17	11	1031	585
150	8	6	486	16	32	992	506
180	8	22	502	16	29	989	487
210	7	58	478	16	59	1019	541
240	7	5	425	17	42	1062	637
270	6	0	360	18	26	1106	746
300	4	58	298	19	14	1154	856
330	4	14	254	20	4	1204	950
360	4	11	251	20	33	1233	982

50 Degrees South
Sinusoidal Regression
regEQ(x) = 244.478*sin(.016536*x+1.85846)+737.291
a = 244.478
b = .016536
c = 1.85846
d = 737.291

Amplitude:

$a\_ \rightarrow a50s$ 244.478

Period:

$b\_ \rightarrow b50s$ .016536

$\frac{2\cdot\pi}{b\_} \rightarrow p50s$ 379.961

Horizontal Translation:

$\frac{c\_}{b\_} \rightarrow c50s$ 112.386

Vertical Translation:

$d\_ \rightarrow d50s$ 737.291

Hence, the final equation is:

$S(x) := 244.478 \cdot \sin(0.016536 \cdot (x + 112.386)) + 737.291$ "Done"

Testing this equation for October 15th - day 288.

$S(288)$ 818.267

This is equivalent to 13 hours and 38 minutes for the day.

Calculations:

Hours:

$\frac{ans}{60}$ 13.6378

Minutes:

$\mathrm{fpart}(ans) \cdot 60$ 38.2673

The "Sun Cycle" program gave the daylight hours as 13 hours and 32 minutes.

Our result is approximately 6 minutes "out."

50 DEGREES - North

50 Degrees North								
Day	Hours	Minutes	Total	Hours	Minutes	Total	Daylight	
30	7	59	479	17	12	1032	553	
60	7	6	426	18	3	1083	657	
90	6	1	361	18	51	1131	770	
120	5	1	301	19	38	1178	877	
150	4	19	259	20	20	1220	961	
180	4	15	255	20	35	1235	980	
210	4	46	286	20	9	1209	923	
240	5	30	330	19	15	1155	825	
270	6	15	375	18	10	1090	715	
300	7	2	422	17	8	1028	606	
330	7	51	471	16	27	987	516	
360	8	19	499	16	25	985	486	

50 Degrees North
Sinusoidal Regression
regEQ(x) = 242.442*sin(.016813*x+-1.33212)+730.428
a = 242.442
b = .016813
c = -1.33212
d = 730.428

Amplitude:

$a\_ \rightarrow a50n$

242.442

Period:

$b\_ \rightarrow b50n$

.016813

$\frac{2\cdot\pi}{b\_} \rightarrow p50n$

373.7

Horizontal Translation:

$\frac{c\_}{b\_} \rightarrow c50n$

-79.2294

Vertical Translation:

$d\_ \rightarrow d50n$

730.428

Hence, the final equation is:

$N(x) := 242.442 \cdot \sin(0.016813 \cdot (x - 79.2294)) + 730.428$ "Done"

Testing this equation for October 15th - day 288.

$N(288)$ 643.104

This is equivalent to 10 hours and 43 minutes for the day.

Calculations:

Hours: $\frac{ans}{60}$ 10.7184

Minutes: $\mathrm{fpart}(ans) \cdot 60$ 43.1037

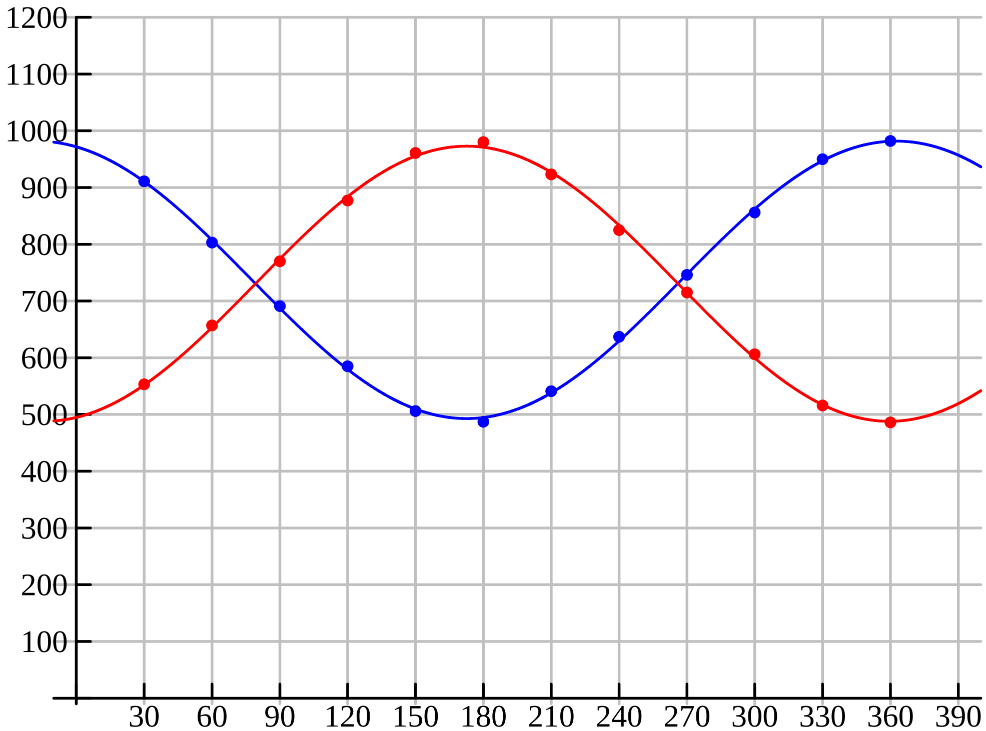
The "Sun Cycle" program gave the daylight hours as 10 hours and 48 minutes.

Our result is approximately 5 minutes "out."

Summary - Daylight minutes: 50 Degrees

50 Degrees		
Day	South	North
30	911	553
60	803	657
90	691	770
120	585	877
150	506	961
180	487	980
210	541	923
240	637	825
270	746	715
300	856	606
330	950	516
360	982	486

50 Degrees - North and South



South

Minimum at (172.589, 492.813)
Maximum at (362.574, 981.769)

North

Minimum at (359.504, 487.986)
Maximum at (172.654, 972.87)

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Comparing the results for the Northern and Southern hemispheres:

The amplitude of each function is very close. The periods are almost identical. The vertical translations are also very close. This should be the case as the latitudes in the Northen hemisphere are a reflection of the latitudes in the Southern hemispheres about the equator.

The major difference is in the horizontal translation. The graphs are "out of phase" with each other by approximately pi radians. The effect of this is the same as reflecting the graph about the vertical translation.

Hence the graph of the Southern hemisphere is a reflection of the graph of the Northern hemisphere about the vertical translation. In a practical sense this corresponds to the Southern summer equating to the Northern winter.

SUMMARY TABLE

		Amplitude	b	Period	Horizontal Translation	Vertical Translation
North	50	242.442	.017	373.700	-79.229	730.428
North	40	167.601	.017	373.233	-79.220	728.642
North	30	110.428	.017	369.896	-80.806	730.391
North	20	71.491	.017	371.479	-79.737	727.612
North	10	34.493	.017	373.545	-79.222	726.783
South	-10	34.773	.017	376.421	109.175	727.409
South	-20	71.944	.017	376.248	109.492	728.380
South	-30	114.502	.017	377.006	110.142	729.656
South	-40	167.978	.017	377.757	110.741	732.185
South	-50	244.478	.017	379.961	112.386	737.291
Average	N	**125.291**	**.017**	**372.371**	**-79.643**	**728.771**
Average	S	**126.735**	**.017**	**377.479**	**110.387**	**730.984**

The significant features of the results are as follows:

Amplitude: the results for the northern and southern hemispheres are almost the same.

b value: the *b* value for both hemispheres is exactly the same, correct to three decimal places.

Period: the period for both hemispheres is almost the same however, the southern hemisphere is slightly longer.

It was anticipated that the period for both hemispheres would have been 365, corresponding to the number of days in the year. This result may have been affected due to the fact that only twelve data points were used in each case over a period of one year. Hence, there was no repetition of the cycle evidenced in the data.

Horizontal translation: the translation in the northern hemisphere was approximately 79 degrees in a negative direction and 110 degrees for the southern hemisphere.

It was anticipated that the difference in translation would be equivalent to 180 degrees so that the graphs would be an exact reflection of each other about the vertical translation. The actual result however, was 189 degrees.

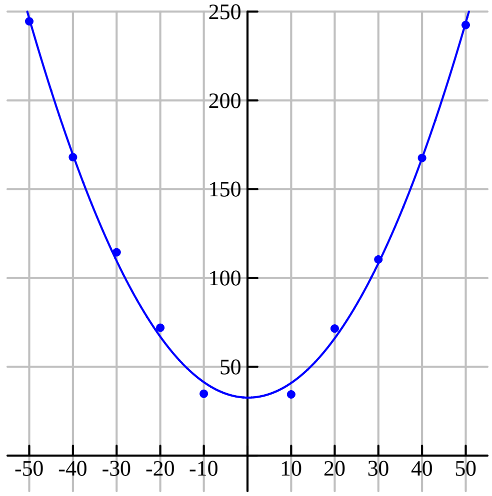
Vertical translation: the vertical translations for both hemisphere were about the same. It was anticipated that the vertical translation would be 720, corresponding to the number of minutes in half a day.

The reason why a different longitude will not affect the daylight minutes equation is because all places on a given longitude will be exposed to the sun in the same way throughout the course of a day. The only influence on the number of daylight minutes is the displacement from the equator. This is due to the fact that the earth rotates about a north-south axis which is inclined at an angle of 23.5 degrees relative to the sun. The orbit of the earth around the sun and the tilt on the axis causes the periodic variations in the amount of sunlight that hits the earth at a given place.

Extension

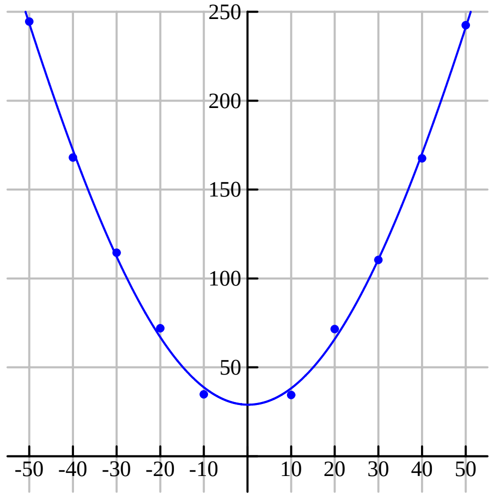
In order to generalise the result of the amplitude it is necessary to determine a model for the data. A quadratic and a sinusoidal model have been calculated as the data seems to most closely resemble these functions.

Latitude and Amplitude - Quadratic



Latitude and Amplitude
Quadratic Regression
regEQ(x) = .084869x^2 + -.022813x + 32.6569
a = .084869
b = -.022813
c = 32.6569

Latitude and Amplitude - Sinusoidal



Latitude and Amplitude
Sinusoidal Regression
regEQ(x) = 355.851*sin(.023172*x+-1.5741)+384.839
a = 355.851
b = .023172
c = -1.5741
d = 384.839

While both models appear to fit the data quite well we have decided to use the sinusoidal model.

General model for any point on the Earth:

1. Substitute the LATITUDE into the equation

$A(x) := a\_\cdot\sin(b\_\cdot x + c\_) + d\_$

"Done"

$A(40) \rightarrow Amp$

170.266

2. For the Southern Hemisphere, substitute into the general equation

$SD(x) := Amp \cdot \sin(0.017 \cdot (x + 110.387)) + 730.984$

"Done"

For the Northern Hemisphere, substitute into the general equation

$ND(x) := Amp \cdot \sin(0.017 \cdot (x - 79.643)) + 728.771$

"Done"

3. Substitute the DAY value required.

$SD(30)$

847.667

$ND(30)$

601.538

Comparing these result to the original data:

Sun Cycle data for day 30 at the 40th latitude

Northern Hemisphere: 606 minutes daylight
Southern Hemisphere: 853 minutes daylight

This result is reasonable considering the fact that the general equation includes an amplitude that is determined from a regression equation and values of *b, c* and *d* derived from averages of a series of regression equations themselves.