Junction Box

by Mary Bourassa

Activity overview

This activity allows students to explore and solve a typical optimization problem in several ways.

Concepts

- Determining the equation of an optimization problem.
- Determining the minimum of a function numerically, graphically and algebraically.

Teacher preparation

Download the student .tns file.

Classroom management tips

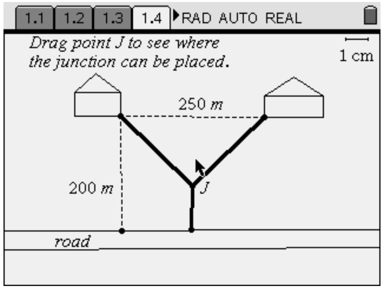
This activity is designed for students who have already solved other optimization problems. Students can work individually or in pairs.

TI-Nspire Applications

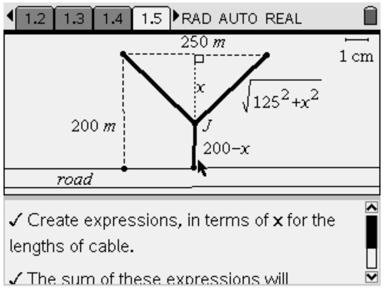
- *Graphs & geometry*
- *Lists & spreadsheet*
- *Notes*
- *Calculator*

Step-by-step directions

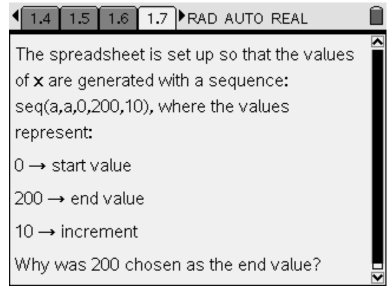
On page 1.4 students can drag point J to get a better sense of the possible locations for the junction box.



On page 1.5 students need to come up with expressions for the two distances (road to junction box and junction box to house) that the cable will run. This page has been scaffolded with *x* already added to the diagram so that their expression for the distance from the junction box to each house is simpler. The labels can be added with a text box (MENU > ACTIONS > TEXT).



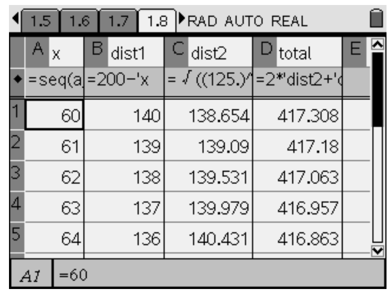
On page 1.7 students are shown the syntax of the sequence command as they will need to edit this command later on. 200 was chosen as the end value as that represents x = 200 or the junction being placed on the road.



On page 1.8 is the spreadsheet. The formulas are:

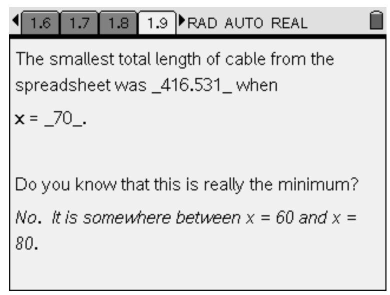
dist1 = 200 – x

dist2 = $\sqrt{125.^2 + x^2}$
Note: if you want approximate results add a decimal point after 125 (as shown).
total = dist1 + 2*dist2

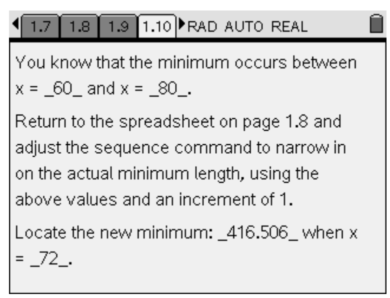


Students should scroll down the **total** column looking for the minimum value and then take note of the corresponding value of x.
On page 1.9 students are asked for the optimal value based on the spreadsheet.

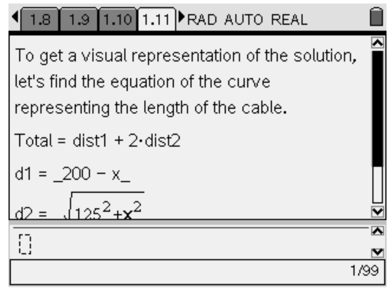
This value does not necessarily represent the true minimum but does tell us that the minimum lies between x = 60 and x = 80.



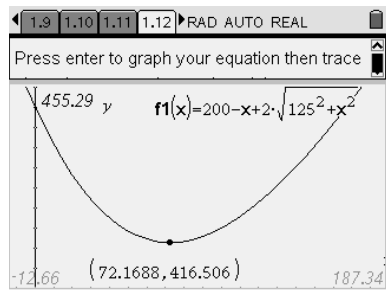
On page 1.10 students are asked to recalculate the distances by editing the sequence command.



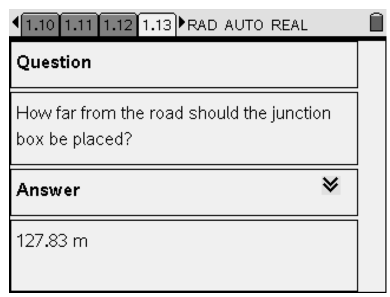
On page 1.11 students will now come up with the equation representing the distance the cable will run using the formulas from the spreadsheet as needed.



Once students have the equation they can graph it on page 1.12. The window has already been set up for the correct graph. Using Trace, students can locate the minimum.

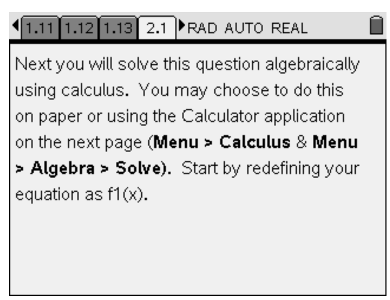


On page 1.13 students will answer the original question. *Note: x is not what the question asked them to find.*



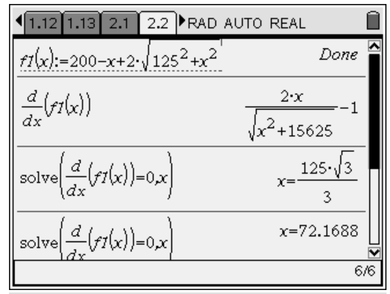
Page 2.1 gives instructions on solving the question algebraically.

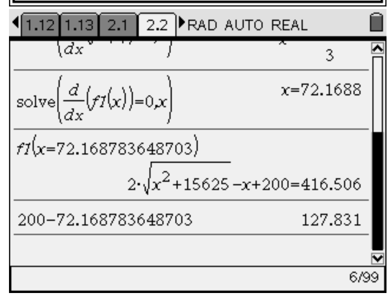
This is set up as a new problem so that the previous values of *x* do not interfere with the variable *x* in the equation to be solved.



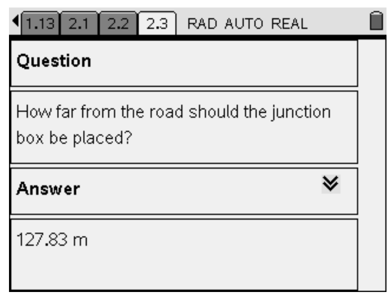
On page 2.2 students can use the CAS functions to

1. take the derivative of their function
2. solve the equation resulting from setting the derivative equal to zero
3. find the optimum value of the function
4. calculate the required distance from the road





On page 2.3 students are again asked for their answer to the problem.



Assessment and evaluation

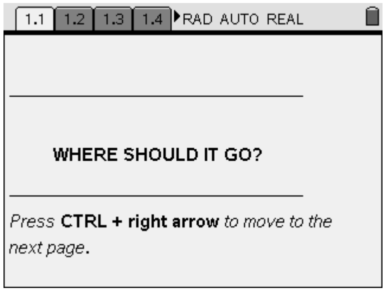
- *This activity is intended as an investigation. Teachers may wish to assess whether students have completed the activity and should debrief at the end.*

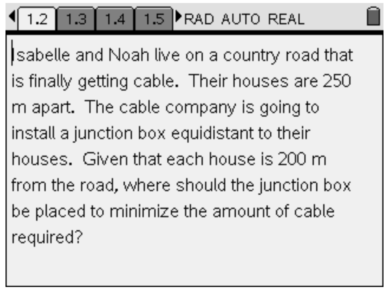
Activity extensions

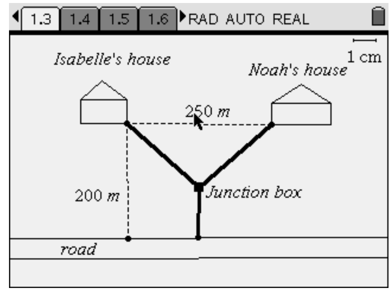
- *The cost of cable could be different going to Isabelle's house and Noah's house. For example it could cost $30/m to one and $25/m to the other. The cost of the cable from the house to the junction box would also have to be provided.*
- *Change the location of the junction box so that it remains on the road between the two houses.*

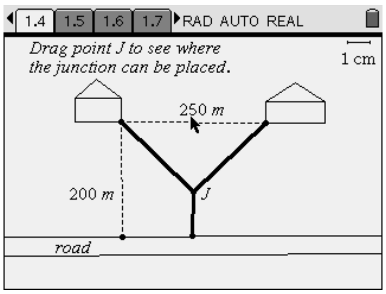
Student TI-Nspire Document:

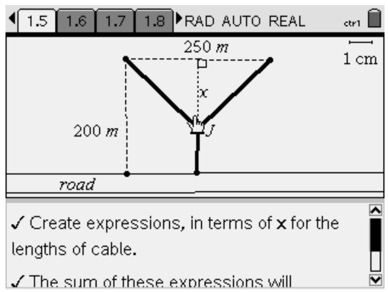
junction box.tns

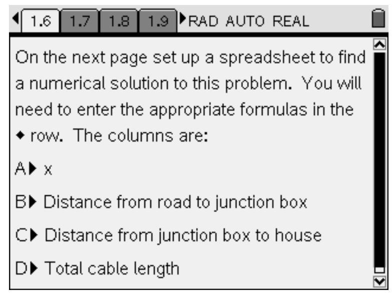

1.1 1.2 1.3 1.4 RAD AUTO REAL
WHERE SHOULD IT GO?
Press CTRL + right arrow to move to the next page.

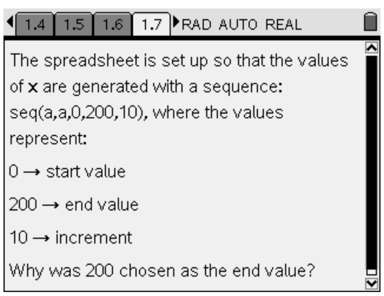

1.2 1.3 1.4 1.5 RAD AUTO REAL
Isabelle and Noah live on a country road that is finally getting cable. Their houses are 250 m apart. The cable company is going to install a junction box equidistant to their houses. Given that each house is 200 m from the road, where should the junction box be placed to minimize the amount of cable required?

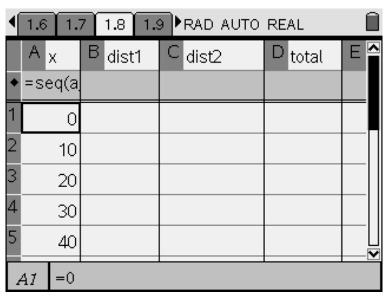

1.3 1.4 1.5 1.6 RAD AUTO REAL
Isabelle's house
Noah's house
1 cm
250 m
200 m
Junction box
road

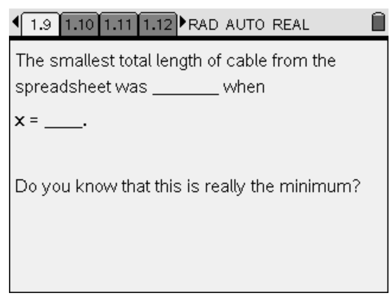

1.4 1.5 1.6 1.7 RAD AUTO REAL
Drag point J to see where the junction can be placed.
1 cm
250 m
200 m
J
road

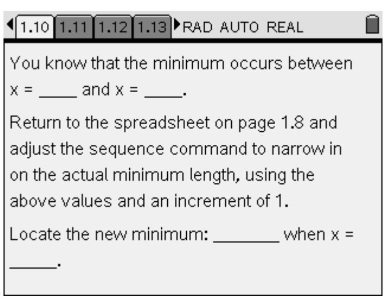

1.5 1.6 1.7 1.8 RAD AUTO REAL
250 m
1 cm
x
200 m
J
road
✓ Create expressions, in terms of x for the lengths of cable.
✓ The sum of these expressions will

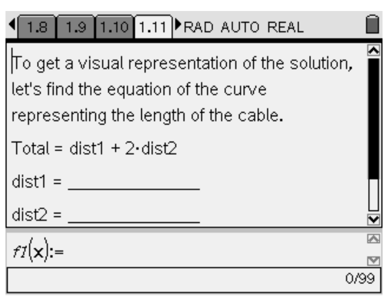

1.6 1.7 1.8 1.9 RAD AUTO REAL
On the next page set up a spreadsheet to find a numerical solution to this problem. You will need to enter the appropriate formulas in the ◆ row. The columns are:
A▸ x
B▸ Distance from road to junction box
C▸ Distance from junction box to house
D▸ Total cable length

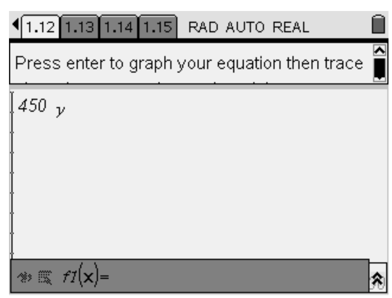

1.4 1.5 1.6 1.7 RAD AUTO REAL
The spreadsheet is set up so that the values of x are generated with a sequence: seq(a,a,0,200,10), where the values represent:
0 → start value
200 → end value
10 → increment
Why was 200 chosen as the end value?

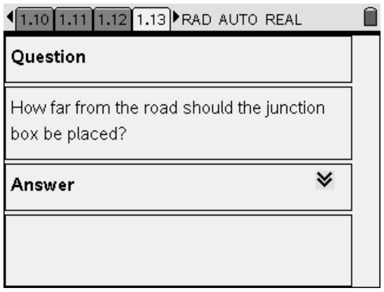

1.6 1.7 1.8 1.9 RAD AUTO REAL
A x
B dist1
C dist2
D total
E
◆ =seq(a
1 0
2 10
3 20
4 30
5 40
A1 =0

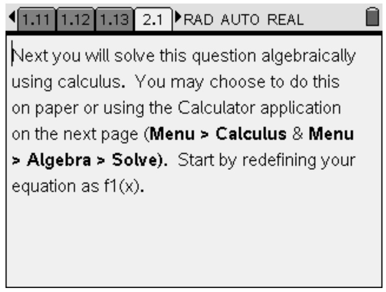

1.9 1.10 1.11 1.12 RAD AUTO REAL
The smallest total length of cable from the spreadsheet was _______ when
x = ____.
Do you know that this is really the minimum?

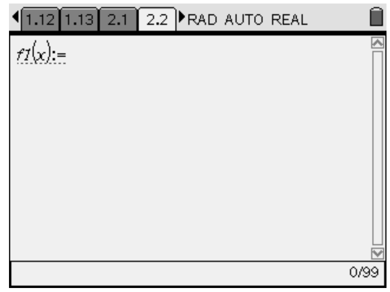

1.10 1.11 1.12 1.13 RAD AUTO REAL
You know that the minimum occurs between x = ____ and x = ____.
Return to the spreadsheet on page 1.8 and adjust the sequence command to narrow in on the actual minimum length, using the above values and an increment of 1.
Locate the new minimum: _______ when x = _____.

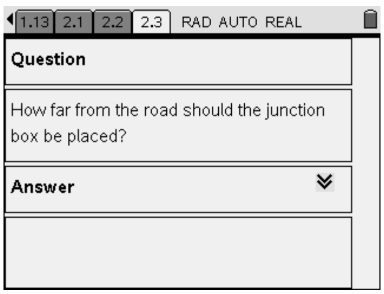

1.8 1.9 1.10 1.11 RAD AUTO REAL
To get a visual representation of the solution, let's find the equation of the curve representing the length of the cable.
Total = dist1 + 2·dist2
dist1 = _____________
dist2 = _____________
f1(x):=
0/99


1.12 1.13 1.14 1.15 RAD AUTO REAL
Press enter to graph your equation then trace
450 y
f1(x)=


1.10 1.11 1.12 1.13 RAD AUTO REAL
Question
How far from the road should the junction box be placed?
Answer


1.11 1.12 1.13 2.1 RAD AUTO REAL
Next you will solve this question algebraically using calculus. You may choose to do this on paper or using the Calculator application on the next page (Menu > Calculus & Menu > Algebra > Solve). Start by redefining your equation as f1(x).


1.12 1.13 2.1 2.2 RAD AUTO REAL
f1(x):=
0/99


1.13 2.1 2.2 2.3 RAD AUTO REAL
Question
How far from the road should the junction box be placed?
Answer