EXPLORATIONS

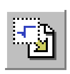
Activity 13

Investigating One Definition of Derivative

Given two points, the slope of the line that passes through these two points is found by computing $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$. One definition of the derivative is given as $\lim_{h \to 0} \frac{f(x + h) - f(x)}{(x + h) - x}$. When $h = 0$, the fraction is undefined. What happens as h gets very close to zero?

Numerical Exploration

1. Open a new TI InterActive! document. Title this document **Definition of Derivative**. Add your name and the date.

2. Select Math Box  and define the function f as $f(x) := x^2 + 1$.

3. Find the slope of the secant line that contains the points $(1, f(1))$, and $(2, f(2))$.

   m = ________________

4. In a math box define h to have a value of 1, by typing **h:=1**.

5. In a math box define *ms* as the slope of the secant line that contains the points $(1, f(1))$, and $(1 + h, f(1 + h))$, by typing $\text{ms} := \frac{f(1 + h) - f(1)}{(1 + h) - 1}$.

6. What is true of $(2, f(2))$. and $(1 + h, f(1 + h))$ when $h = 1$?

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7. Enter the coordinates of the two points on the secant line and the slope, *ms*, in the table below. You can find the value of $f(x + h)$ by typing **f(x + h)** in a math box.

8. Double-click on h: = 1 and change to **h:=.05**. Record the slope, *ms* of the secant line and the coordinates of the two points that contains the points (1, f(1)), and $(1 + h, f(1 + h))$ when $h = 0.5$.

9. Repeat step 8, changing *h* to **0.1, 0.01**, **-1**, **-0.5**, **-0.1**, and **-0.01**. Record this information in the table below.

	Point 1		**Point 2**		
h	*x*	*f(x)*	*x + h*	*f(x + h)*	*ms*
1	1		2		
0.5	1		1.5		
0.1	1				
0.01	1				
⋮					
-0.01	1				
-0.1	1				
-0.5	1				
-1	1				

Numerical Analysis

1. As $x + h$ gets closer to x, what is happening to h?

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2. As $x + h$ gets closer to x, what is happening to the slope?

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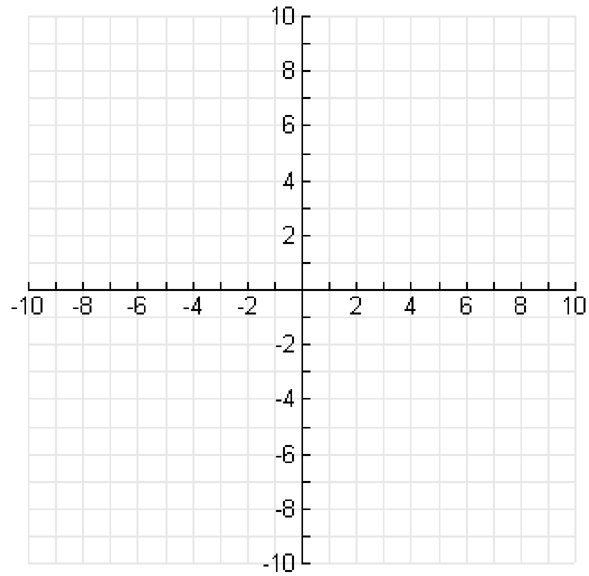
3. As $h \to 0$, the value of the slope appears to be approaching ______________? In a math box, Define *mt* to be this value.

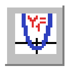
Graphical Exploration

1. Double-click on h: = -1 and redefine h: = 1.

2. In a math box, define the function *g* as the equation of the secant line g(x): = ms * (x − 1) + f(1).

3. In math boxes, store {1, 1 + *h*}→L1 and {f(1), f(1 + *h*)} →L2.



4. Graph *f(x)*, *g(x)*, and the scatter plot L1, L2. Select Graph.  and enter **f(x)** in y1(x): =, **g(x)** in y2(x): =. Click on the Stat Plots tab and enter **L1** and **L2** for the first plot. Close the graph by clicking on Save to Document  .

5. Sketch *f(x)*, *g(x)*, and the scatter plot L1, L2 on the provided grid.

Numerical Analysis

1. Double-click on h: = 1. Change this value to 0.5. How does this change the graph?

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2. Sketch the new *g(x)* and the scatter plot L1, L2 on the same grid.

3. Double-click on the graph. Enter the equation of the line tangent to *f*(*x*) at *x* = 1 by defining y3(x): = mt * (x − 1) + f(1).

4. Repeat step 1 for each of the following *h*-values: 0.1, 0.01, -1, -0.5, -0.1, and -0.01. As *h* → 0, what is happening to the secant line?

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5. How does *t* compare to *g* when *h* is very small?

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6. Save this document as **deriv.tii.** Print a copy of this document.

Additional Exercises

Use the previous steps to investigate each of the following.

1. Use the definition of derivative $\lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$ to approximate the numerical derivative of $f(x) = x^2 + 1$ at $x = 2$. Sketch the function and the secant lines. Determine the slope of the tangent line on the provided grid.

	Point 1		**Point 2**		
h	*x*	*f(x)*	*x + h*	*f(x + h)*	*m*
1	2		3		
0.5	2		2.5		
0.1	2				
0.01	2				
⋮					
-0.01	2				
-0.1	2				
-0.5	2				
-1	2				

mt = ______________________________

2. Use the definition of derivative $\lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$ to approximate the numerical derivative of $f(x) = x^2 + 1$ at $x = -1$. Sketch the function and the secant lines. Determine the slope of the tangent line.

	Point 1		Point 2		
h	*x*	*f(x)*	*x + h*	*f(x + h)*	*m*
1	-1		0		
0.5	-1		-0.5		
0.1	-1				
0.01	-1				
⋮					
-0.01	-1				
-0.1	-1				
-0.5	-1				
-1	-1				

mt = ______________________________

3. Use the definition of derivative $\lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$ to approximate the numerical derivative of $f(x) = \frac{x+1}{x-1}$ at x = -1. Sketch the function and the secant lines. Determine the slope of the tangent line.

	Point 1		**Point 2**		
h	*x*	*f(x)*	*x + h*	*f(x + h)*	*m*
1	-1		0		
0.5	-1		-0.5		
0.1	-1				
0.01	-1				
⋮					
-0.01	-1				
-0.1	-1				
-0.5	-1				
-1	-1				

mt = ______________________________

4. Use the definition of derivative $\lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$ to approximate the numerical derivative of $f(x) = \frac{x-2}{x^2-4}$ at x = -1. Sketch the function and the secant lines. Determine the slope of the tangent line.

	Point 1		Point 2		
h	*x*	*f(x)*	*x + h*	*f(x + h)*	*m*
1	-1		0		
0.5	-1		-0.5		
0.1	-1				
0.01	-1				
⋮					
-0.01	-1				
-0.1	-1				
-0.5	-1				
-1	-1				

mt = ______________________________