<u>TI-Nspire Activity</u>: Paint Can Dimensions
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Activity Overview

Problem 1 explores the relationship between height and volume of a right cylinder, the height and surface area, and the surface area and volume with a set radius measure. Problem 2 explores the relationship between the radius and volume, radius and surface area, and surface area and volume when the height is kept stable. Questions are posed that encourage integration between geometric formulas and algebraic modeling. Students are asked to compare and explore these measures through the interactive functionality of numerical and graphical representations.

Concepts

Volume and surface area measurements
Algebraic modeling

Teacher Preparation

This activity can be used to introduce algebraic modeling or as a review lesson. A challenge question is posed to further work on algebraic manipulation of a formula. Transfer the file *PaintCan.tns* to students' handhelds before starting the activity.

Students will begin with the Paint Can Dimensions worksheet and the PaintCan.tns file. They should be familiar with the basic features of the TI-Nspire such as how to open a file, grab and move objects, enter a function into the Graphs & Geometry, and how to adjust the window setting manually.

The Classroom

This activity is designed to be student-centered - the teacher may act as a facilitator while students work cooperatively to help them with the handheld. The PaintCan_Soln.tns file shows expected results.

List of TI-Nspire and/or TI-Nspire CAS applications used during this activity:

PaintCan.tns file
Graphs & Geometry
Lists & Spreadsheets

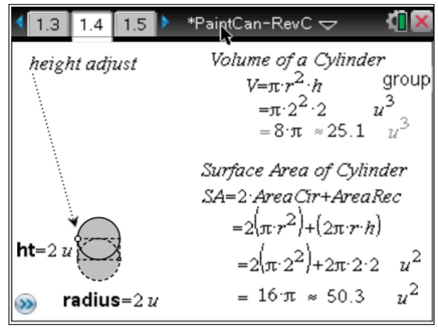
Scenario:

Your company can make paint cans that vary in radius from 1 - 10 inches and in height from 2 - 20 inches. You want to make cans that hold different amounts of paint (volume) but consider the costs of the labels to cover the cans (surface area).

In Problem 1 students are to determine the relationship between height and volume, height and surface area, and surface area and volume when the radius is fixed at 2 units. On page 1.4, a modified view of a right cylinder is given with an open point on the top of the cylinder that adjusts the height. The volume formula, $V = \pi r^2 h$ and the surface area formula, $SA = Area\ of\ Rectangle + 2 \cdot Area\ of\ Circle$, are given and their values calculated for each height measure. On pages 1.5-1.6, a spreadsheet and scatter plot of the captured data will be seen. Students are asked to respond to questions that connect the geometry of a right cylinder to the algebra of the relationship between the measures.

Instruct students to open the file, *PaintCan.tns*, on their TI-Nspire handheld.

Problem 1



1. On page 1.4, grab the open circle (hold for a few seconds to change to) that adjusts the height of the can and practice moving it to different height measures. The radius measure is set at 2 units.

2. Move to a height of 2 units then press ctrl . . The volume of the cylinder is 25.1 units^3 and the surface area is 50.3 units^2.

   Repeat step 2 to move the height to each measure in the table. Be sure to press ctrl . after each move. Record each outcome in Table 1.

Height (units)	Radius (units)	Volume (units^3)	Surface Area (units^2)
2	2	25.133	50.265
4	2	*50.265*	*75.398*
6	2	*75.398*	*100.531*
8	2	*100.531*	*125.664*
10	2	*125.664*	*150.796*
12	2	*150.796*	*175.929*
14	2	*175.929*	*201.062*
16	2	*201.062*	*226.195*
18	2	*226.195*	*251.327*
20	2	*251.327*	*276.460*

Table 1

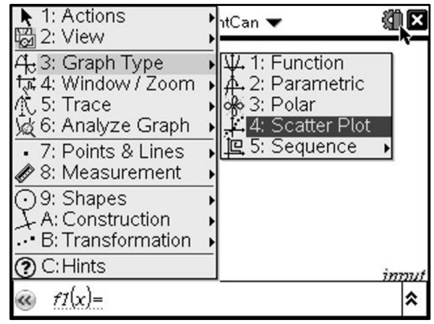
4. From the numerical view of the data, make a conjecture of the relationship between the height and volume, height and surface area, and volume and surface area. Which ones do you think are linear? Are any of them non-linear? Explain your reasoning.

   ANS: Encourage students to write in full sentences. All scatter plots should appear to be linear.

5. Move to page 1.5 (press ctrl ▸) to check your table entries.

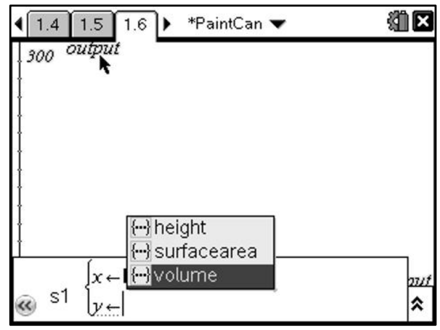
   The data were captured each time ctrl . were pressed.

6. On page 1.6, set up a scatter plot for each relationship by following the step:
   a. Select scatter plot by pressing menu – Graph Type – Scatter plot enter.



   b. Change the labels of the axes. Hover the cursor over the label of the horizontal axis then press enter twice to open the text box of the label. Delete the existing text using clear then change the label on the input (horizontal) axis to "Input". Do the same to change the label on the output (vertical) axis to "Output".

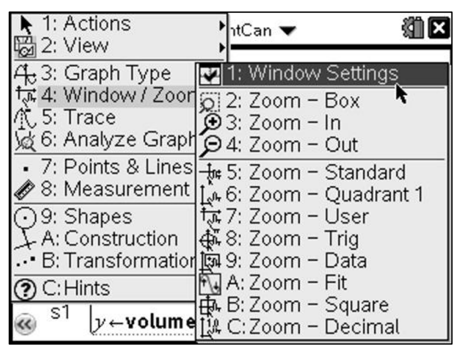
   The use of "Input" and "Output" instead of "x" and "y" are used for accurate interpretation of the graphs when all three plots are placed on the same coordinate plane. Encourage students to verbalize actual names for the variables.



   c. Select the data for the first scatter plot. In s1, press sto→ var to open the pull down menu for "x" and select "height", press tab, then open the pull down menu for "y" and select "volume". Press enter to see the data.

   d. Students may need to be reminded to change the graph viewing window settings to see the entire scatter plot. Requiring them to do this manually reinforces the concepts of domain and range.

   Manually change the window settings to view all of the data.
   What are the lowest and the highest input values needed to see the data?
   ANS: *The smallest height is 2 units and the largest is 20 units.*



   What are the lowest and the highest output values needed to see the data?
   ANS: *The smallest volume is approximately 25 units^3 and the largest is approximately 250 units^3.*

   What are the algebraic names used to refer to these settings?
   ANS: *Domain and range are the algebraic terms. A window setting that shows the first quadrant is sufficient.*

e. Describe the pattern and relationship you observe in the scatter plot. Is the pattern linear or non-linear?
   ANS: *Answers may vary but students should see a linear pattern.*

f. Repeat steps (a)-(e) to create scatter plots for height and surface area and for surface area and volume. Describe the pattern and relationship you observe in the scatter plots.
   ANS: *All patterns are linear.*

7. Enter regression models for the following relationships on page 1.6:
   a. Using the formula $V = \pi r^2 h$, write an equation for volume in terms of height. Enter your equation into f1(x) as an approximation of the data pattern for height and volume. Explain the significance of $\left(\pi r^2\right)$ in this function.

      ANS: *Since the radius is set at 2 units,* $\pi r^2 \to \pi \cdot 2^2 = 4\pi$ *. The input variable is "height" and the output is "volume". The volume formula reduces to* $V = 4\pi h$ *;* $\left(\pi r^2\right)$ *generates the slope of* $f1(x) = 4\pi x$ *.*

   b. Using the formula $SA = 2\pi r^2 + 2\pi rh$, write an equation for surface area in terms of height. Then, on page 1.6, enter your equation into f2(x) as an approximation of the data pattern for height and surface area. Explain the significance of $\left(2\pi r^2\right)$ in this function.

      ANS: *Since the radius is set at 2 units,* $2\pi rh \to 4\pi h$ *and* $2\pi r^2 \to 2\pi \cdot 2^2 = 8\pi$ *. The input variable is "radius" and the output is "surface area". The surface area formula reduces to* $SA = 8\pi + 4\pi h$ *so* $\left(2\pi r^2\right)$ *generates the output intercept of* $f2(x) = 8\pi + 4\pi x$ *.*

   c. Rewrite the formula for volume in terms of surface area. Then, on page 2.4, enter your equation into f3(x) as an approximation of the data pattern for surface area and volume. Explain the significance of (8π) in this function.

      ANS: *The input variable is "surface area" and the output is "volume". The surface area (SA) formula needs to be written as "height" in terms of SA,* $h = \dfrac{SA - 2\pi r^2}{2\pi r}$ *, and used in the volume formula as* $V = SA - 8\pi$ *. Thus* (8π) *is the output intercept of* $f3(x) = x - 8\pi$ *.*

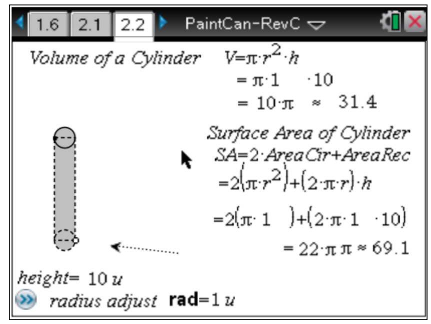
Problem 2

This is similar to Problem 1 with the exception that students are asked to vary the radius with a fixed height of 10 inches.

8. On page 2.1, grab the circle that adjusts the radius of the paint can and practice moving it to different radius measures. The height measure is locked at 10 units.

9. Move to a radius of 1 unit then press ctrl . .
   The volume of the cylinder is 31.4 units^3 and the surface area is 16.3 units^2.

10. Repeat step 9 to move the radius to each measure in the table. Be sure to press ctrl . after each move. Record each outcome in Table 2.



Height (units)	Radius (units)	Volume (units^3)	Surface Area (units^2)
10	1	31.416	69.115
10	2	*125.664*	*150.796*
10	3	*282.743*	*245.004*
10	4	*502.655*	*351.858*
10	5	*785.398*	*471.239*
10	6	*1130.973*	*603.186*
10	7	*1539.380*	*747.699*
10	8	*2010.619*	*904.779*
10	9	*2544.690*	*1074.425*
10	10	*3141.593*	*1253.637*

Table 2

11. From the numerical view of the data, make a conjecture of the relationship between the radius and volume, radius and surface area, and volume and surface area.
    a. Which ones do you think are linear? Are any of them non-linear? Explain your reasoning.

       ANS: *None are linear. A look at the first differences can verify this.*

    b. Which measure affects the volume more – height or radius? Is this the same for surface area? Give a plausible explanation.

       ANS: *Answers will vary but students should note that since the radius is squared it will affect volume and surface area more.*

13. On page 2.3, create scatter plots for each relationship.
    a. Select scatter plot.

    b. Open the text box of the label then delete the existing text. Change the labels on the input (horizontal) axis to “Input” and the label on the output (vertical) axis to “Output”.

    c. Select the data for the first scatter plot. In s1 select “radius” for “x” and “volume” for “y”.

    d. Manually change the window settings to view all of the data.
       What are the lowest and the highest input values needed to see the data?
       ANS: *Radius varies from 1-10 units.*

What are the lowest and the highest output values needed to see the data?
ANS: *Volume varies from approximately 30-3100 units^3.*

What are the algebraic names used to refer to these settings?
ANS: *Domain and range are the algebraic terms used.*

e. Describe the pattern and relationship you observe in the scatter plot. Is the pattern linear or non-linear?

ANS: *Answers will vary but students should note that they are non-linear; students may recognize the quadratic nature of the data.*

f. Repeat steps (a)-(e) to create scatter plots for radius and surface area and for surface area and volume. Describe the pattern and relationship you observe in the scatter plots.

14. Enter regression models for the following relationships on page 2.3:

a. Using the formula, $V = \pi r^2 h$, write an equation for volume in terms of radius. Enter your equation into f1(x) as an approximation of the data pattern for radius and volume. Explain the significance of (10π) in this function.

ANS: *Since the height is set at 10 units, $\pi h \to 10\pi$. The input variable is "radius" and the output is "volume". The volume formula reduces to $V = 10\pi r^2$; (10π) determines the shape of parabola $f1(x) = 10\pi x^2$.*

b. Using the formula, $SA = 2\pi rh + 2\pi r^2$, write an equation for surface area in terms of radius. Enter your equation into f2(x) as an approximation of the data pattern for radius and surface area. Explain the significance of (2π) and (20π) in this function.

ANS: *Since the height is set at 10 units, $2\pi rh \to 20\pi r$. The input variable is "radius" and the output is "surface area". The surface area formula reduces to $SA = 20\pi r + 2\pi r^2$ so (20π) determines the output intercept and (2π) determines the shape of the parabola $f2(x) = 20\pi x + 2\pi x^2$.*

c. Challenge: Rewrite the formula for volume in terms of surface area. Enter your equation into f3(x) as an approximation of the data pattern for surface area and volume. Change the window settings to view the scatter plot and function.

ANS: *This is a challenge since the surface area formula must be solved for the radius. Direct students to use the quadratic formula to determine:*

$$SA = 2\pi rh + 2\pi r^2$$

$$2\pi r^2 + 2\pi rh - SA = 0$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 - 4(2\pi)(-SA)}}{2(2\pi)} = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi(SA)}}{2\pi}$$

With the height set at 10 units, $r = \dfrac{-10\pi \pm \sqrt{100\pi^2 + 2\pi(SA)}}{2\pi}$

Replacing this expression into the volume formula yields:

$$V = 10\pi\left(\frac{-10\pi \pm \sqrt{100\pi^2 + 2\pi\left(SA\right)}}{2\pi}\right)^2 = 5\left[100\pi - 10\sqrt{100\pi^2 + 2\pi(SA)} + SA\right]$$

Conclusion
Your company can make paint cans that vary in radius from 1 - 10 inches and in height from 2 - 20 inches. You want to make cans that hold different amounts of paint (volume) but consider the costs of the labels to cover the cans (surface area).

Which measure, height or radius, changes the volume and surface area the most? Why? Explain your reasoning.

ANS: *Answers will vary but students should recognize that volume and surface area increase more with a change in radius than in height because the radius is squared in both formulas.*