Friction and Inclined Planes – ID: 9520

By Peter Fox

Time required
45 minutes

Topic: Force and Motion

- *Describe the effect of friction on the motion of an object.*
- *Measure or calculate the frictional force and the coefficient of friction between two surfaces.*

Activity Overview

In this activity, students explore the relationship between mass, coefficient of friction, and motion down an inclined plane. Students use a dynamic animation to explore these relationships. The animation consists of a block on an inclined plane. Students can change the coefficient of friction, the mass of the block, and the angle of the inclined plane, and then observe the animation to determine whether the block will slide down the plane.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Notes, Graphs & Geometry

Teacher Preparation

Before carrying out this activity, make sure students are familiar with the concepts of frictional forces, the coefficient of friction, gravity, and the normal to a plane.

- *If you wish, you may introduce this activity with a demonstration of the effects of angle on the motion of an object down an inclined plane.*
- *More advanced students may benefit from a challenge to estimate the coefficient of friction for each situation. Also, note that this activity assumes that the coefficient of friction is the same as the kinetic coefficient of friction. You may wish to discuss this with more advanced students, as well.*
- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7 and 8 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9520" in the search box.***

Classroom Management

- *This activity is designed to be* ***teacher-led*** *with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in* ***this*** *document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

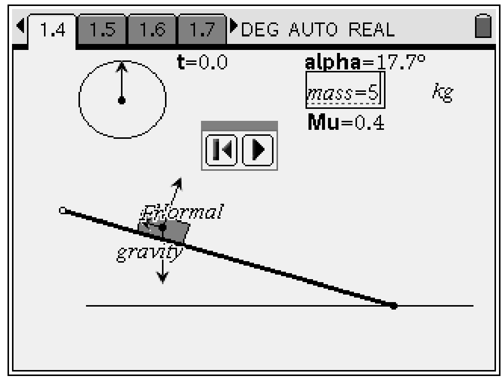
The following questions will guide student exploration in this activity:

- How are the angle of an inclined plane and the coefficient of friction between the plane and an object on the plane related to the motion of the object down the plane?
- How is the mass of an object related to its acceleration down an inclined plane?

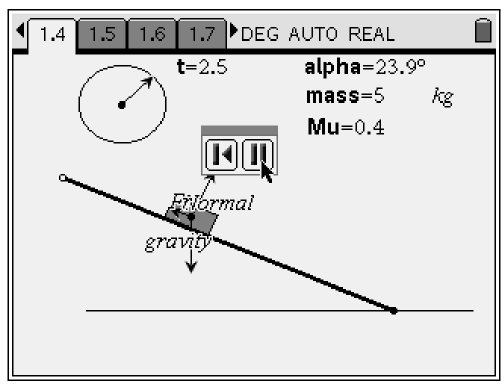
Students will use an animation to explore the relationships between angle, coefficient of friction, mass, and acceleration. Students will then attempt to determine the coefficient of friction required to prevent a mass from sliding down an inclined plane.

Problem 1 – Determining the effects of angle and mass on motion down an inclined plane

Step 1: Students should open the file **PhyAct_9520_FrictionInclinedPlane.tns** and read the first three pages. Page 1.4 contains an animation of a box sitting on an inclined plane. The mass of the box (**mass**) should be set at 5 kg, and the coefficient of friction (**Mu**) should be set at 0.4. The time (**t**) should be set to 0.0. If the time is not set to 0.0, students should reset the animation by pressing the reset (**|◂**) button. If the mass and coefficient of friction are not set correctly, students should change them to the values above by double-clicking on them and typing in the correct values. Note: Make sure students do not alter or delete the *Calculator* or *Lists & Spreadsheet* applications at the end of this problem or the next. These applications contain information required to make the animations function correctly.



Step 2: Next, students should vary the angle of the inclined plane, by dragging the endpoint of the plane, and observe the effects on the box. Each time students change the angle of the plane, they should reset the animation and then play it. If the angle of the plane is sufficiently steep for the box to move, the box will begin to slide down the plane after students press the play button. The box will accelerate as it slides. After students have experimented with the angle of the inclined plane, they should answer questions 1–4.



Q1. What happens to the box when the angle of the inclined plane is small (less than 22°)?

A. *When the angle of the inclined plane is small, the box does not move. Encourage students to discuss why this is true. They should be able to state that the frictional force prevents the box from sliding when the angle is small.*

Q2. What happens to the box when the angle of the inclined plane is large (greater than 22°)?

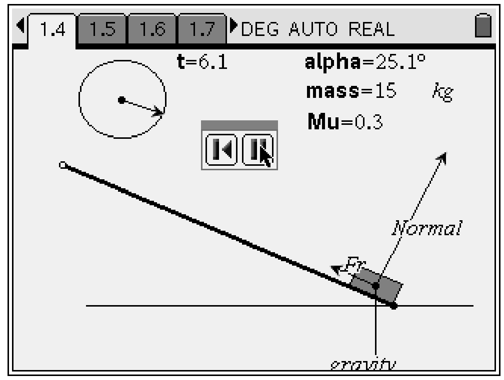
A. *When the angle of the inclined plane is large, the box slides down the plane. Encourage students to describe the motion in detail. They should be able to state that the box accelerates as it slides—that is, its velocity increases as it moves down the plane.*

Q3. How is the acceleration of the box related to the angle of the inclined plane? (Hint: Use the time required for the box to slide the whole way down the plane as a proxy for acceleration.)

A. *The greater the angle of the plane is, the greater the box's acceleration is. Students may have a difficult time making this connection, as acceleration is not measured or displayed anywhere on the screen. Remind them that the box starts at the same point in all cases, that its initial velocity is always zero, and that it travels the same distance (from the starting point to the end of the plane) in each case. Therefore, the time it takes the box to slide down the plane is related to the acceleration of the box. The greater the acceleration is, the higher the box's average speed down the plane will be, and therefore the less time it takes to slide down the inclined plane.*

Q4. Predict how the mass of the box will affect its acceleration if the angle of the inclined plane and the coefficient of friction are constant.

A. *Student answers will vary. Encourage students to explain and justify their answers.*



Step 3: Next, students should read page 1.7, which instructs them to move back to page 1.4 and change **Mu** to 0.3 and **alpha** to 25° (if students cannot drag the plane such that **alpha** is exactly 25°, they should try to get as close as possible). Students will now test their predictions about the effect of mass on acceleration down the plane. Students should vary the mass of the box to test the predictions they made in question 4. After they have varied the mass of the box several times, they should answer questions 5–8.

Q5. Was the prediction you made in question 4 correct? If not, explain any errors in your reasoning.

A. *Student answers will vary. Many students will probably have predicted that a box with a greater mass will accelerate more than a box with a smaller mass, or that if the mass of the box is large enough, it won't slide at all. Encourage students to discuss their predictions and identify their errors in thinking. Help them understand why the mass of the box does not affect its acceleration: The increased gravitational force parallel to the plane offsets the increase in frictional force caused by the increased mass. An increase in the mass produces an increase in the gravitational force. This creates a greater force "down" the plane. An increase in the gravitational force increases the normal force. The frictional force is directly proportional to the normal force. The subsequent increase in the frictional force is traded off against the increase in the force down the plane. This can be shown mathematically. Thus acceleration is independent of mass.*

Q6. Does the mass of the box determine whether the box will slide down the plane?

A. *No; no matter what the box's mass is, it will always slide down the plane when the angle and coefficient of friction are as they are set here.*

Q7. How does the mass of the box affect the size of the frictional force?

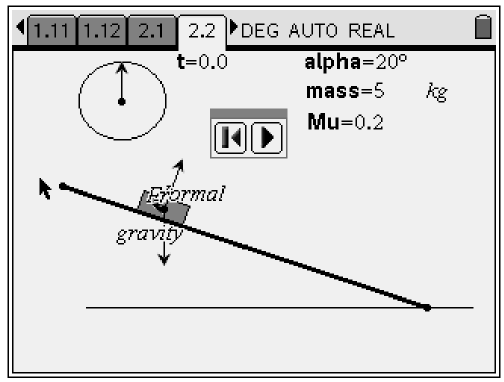
A. *The greater the mass of the box is, the greater the frictional force on the box is. Help students understand why this is. Remind them of the equation relating frictional force (*F_r*) to coefficient of friction (*μ*) and normal force (*N*):* $F_r = \mu N$.

Q8. What other forces on the box are affected by changing its mass?

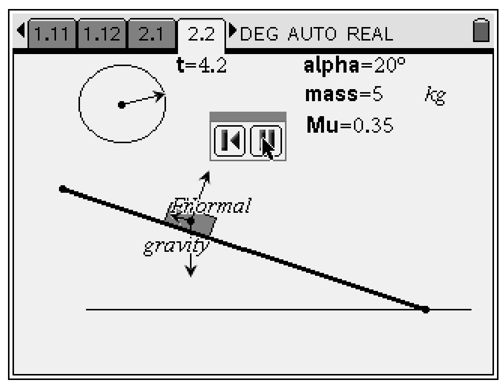
A. *the normal force on the box and the gravitational force on the box*

Problem 2 – The effect of coefficient of friction on motion down an inclined plane

Step 1: Next, students should move to page 2.1 and read the text there. They should then move to page 2.2, which contains an animation similar to that on page 1.4. The mass of the box (**mass**) should be set at 5 kg, and the angle of the inclined plane (**alpha**) should be set at 20°. The time (**t**) should be set to 0.0. If the time is not set to 0.0, students should reset the animation. If the mass and angle are not set correctly, students should change them to the values above.



Step 2: Next, students should vary the coefficient of friction (**Mu**) incrementally and observe the effects on the motion of the box. Each time students change the coefficient of friction, they should reset the animation and then play it. If the coefficient of friction is sufficiently small for the box to move, it will begin to slide down the plane after students press the play button. The box will accelerate as it slides. After students have experimentally determined the minimum coefficient of friction required to prevent the box from sliding, they should answer questions 9–12.



Q9. What was the minimum coefficient of friction required to keep the box from sliding down the inclined plane?

A. *approximately 0.36*

Q10. Describe the motion of the box when it slid down the plane. How did the coefficient of friction affect the box's acceleration?

A. *As the box slid down the plane, it accelerated. The smaller the coefficient of friction was, the more the box accelerated.*

Q11. Calculate to three decimal places the minimum coefficient of friction required to keep the box from sliding down the inclined plane.

A. *0.364; students may struggle with this calculation. Remind them that at the "equilibrium" value of* μ*—the minimum value required to keep the box from sliding, the frictional force is exactly equal to the component of the gravitational force that is parallel to the inclined plane. If* θ *is the angle of the inclined plane,* m *is the mass of the box, and* g *is the acceleration due to gravity near Earth's surface, then the component of the gravitational force that is parallel to the inclined plane is equal to mg(sin θ). The frictional force is equal to* μN, *where* N *is the normal force. The normal force, in turn, is equal to mg(cos θ). Therefore, setting the frictional force equal to the parallel component of the gravitational force yields the following:*

$$mg(\sin\theta) = \mu mg(\cos\theta)$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Therefore, the value of μ *is tan(20º) = 0.364.*

Q12. Calculate the frictional force on the box for the coefficient of friction you calculated in question 11.

A. *16.76 N; the frictional force (*F_r*) is equal to* μN, *or μmg(cos θ). Substituting the given values yields the following:*

$$F_r = (0.364)(5\text{ kg})(9.8\text{ m/s}^2)(\cos 20^\circ) = 16.76\text{ N}$$

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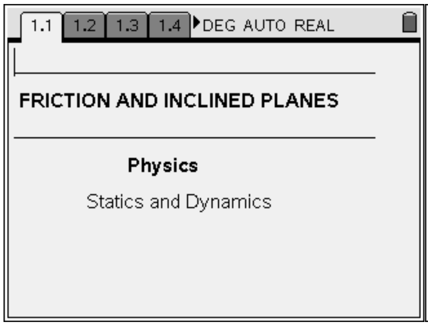
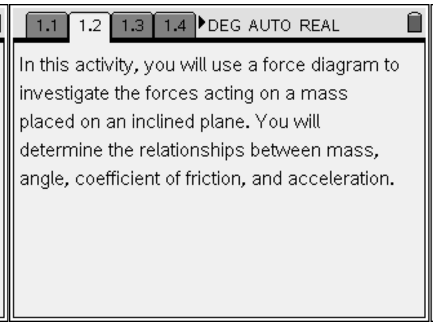
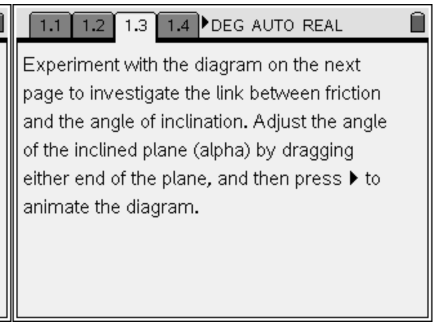
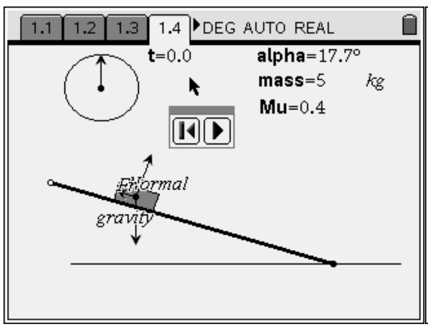
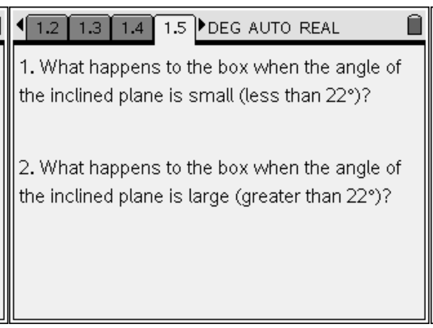
(Student)TI-Nspire File: *PhyAct_9520_FrictionInclinedPlane.tns*

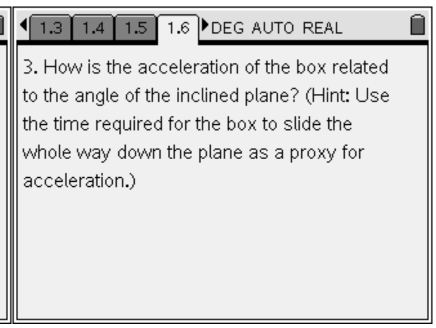


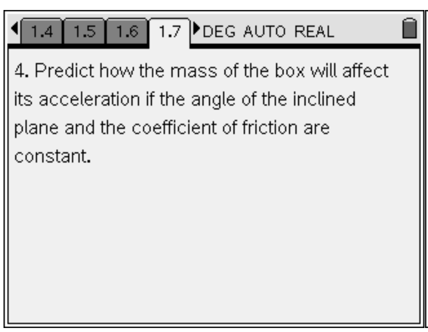


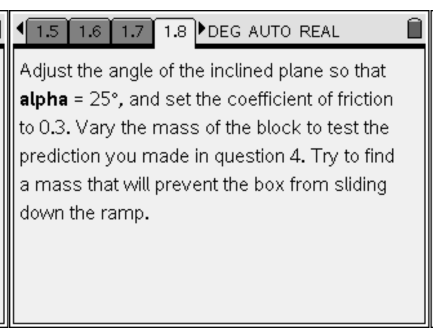


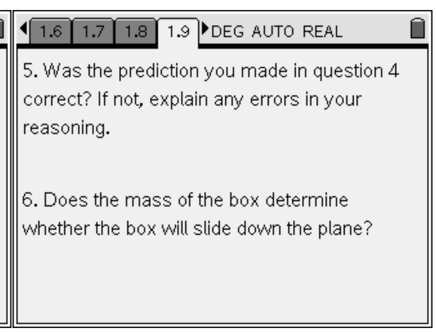


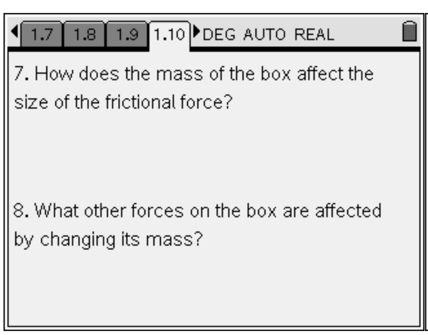


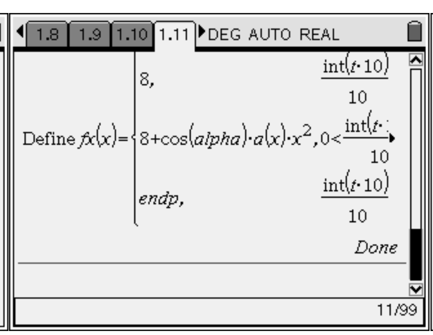


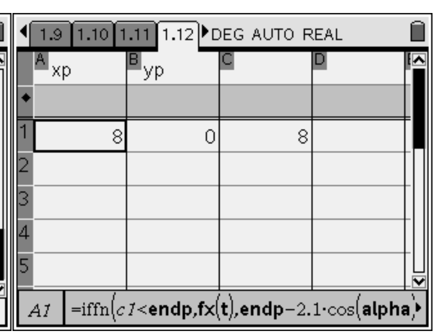


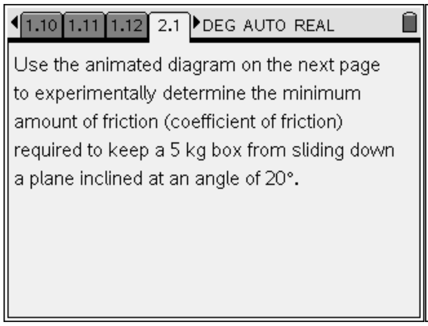


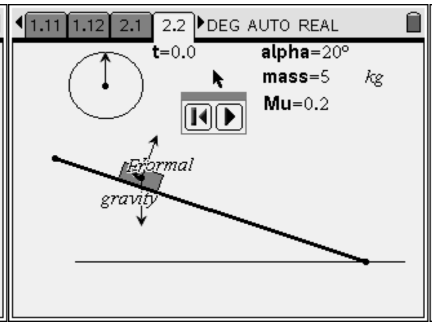


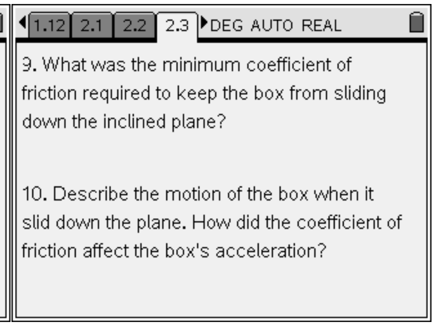


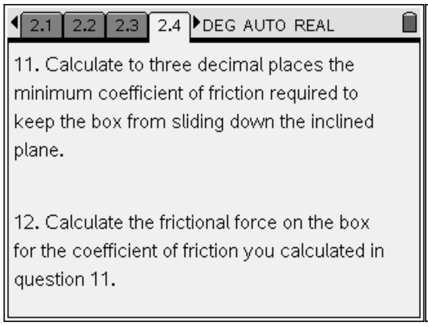


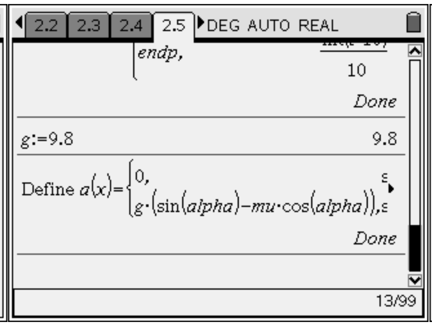


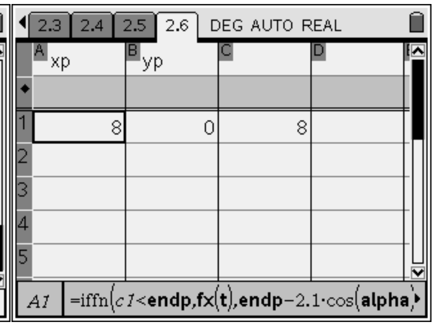

1.10 1.11 1.12 2.1 DEG AUTO REAL
Use the animated diagram on the next page to experimentally determine the minimum amount of friction (coefficient of friction) required to keep a 5 kg box from sliding down a plane inclined at an angle of 20°.


1.11 1.12 2.1 2.2 DEG AUTO REAL
t=0.0
alpha=20°
mass=5 kg
Mu=0.2
Fnormal
gravity


1.12 2.1 2.2 2.3 DEG AUTO REAL
9. What was the minimum coefficient of friction required to keep the box from sliding down the inclined plane?
10. Describe the motion of the box when it slid down the plane. How did the coefficient of friction affect the box's acceleration?


2.1 2.2 2.3 2.4 DEG AUTO REAL
11. Calculate to three decimal places the minimum coefficient of friction required to keep the box from sliding down the inclined plane.
12. Calculate the frictional force on the box for the coefficient of friction you calculated in question 11.


2.2 2.3 2.4 2.5 DEG AUTO REAL
endp,
10
Done
g:=9.8 9.8
Define a(x)= 0, g·(sin(alpha)−mu·cos(alpha)),
Done
13/99


2.3 2.4 2.5 2.6 DEG AUTO REAL
A xp
B yp
C
D
8
0
8
A1 =iffn(c1<endp,fx(t),endp−2.1·cos(alpha)

