EXPLORATIONS

Activity 5

Number Power!

Teacher Notes

Concepts

- Patterns in exponents
- Rules for exponents
- Positive and negative exponents
- Power of 10

Calculator Skills

- Exponentiation: [^]
- Natural logarithm: [LN]
- Scientific Notation: [2nd] [SCI/ENG], [2nd] [EE]
- Power of 10: [2nd] [10^x]

Materials

- TI-30X IIS
- Student Activity pages (p. 50-52)

Objective

- In this activity, students will use the power of the calculator to explore patterns and rules in dealing with exponents and logarithms. They will evaluate expressions with exponents and logarithms and display them in standard and scientific notation.

Topics Covered

- Exploring the economy, power, and elegance of mathematical notation
- Formulating mathematical definitions and expressing generalizations discovered through investigations
- Finding equivalent expressions

Introduction

When computers were first invented, they filled entire classrooms or office space in buildings. Although very powerful, these computers have since been replaced by some that are so small that you can hold them in your hand! A computer's total memory is measured in kilobytes or megabytes. A kilobyte (K) is 2^{10} bytes. A megabyte (MB) is 2^{20} bytes. In this activity, students will use the power of their calculators to discover the power of exponents and their usefulness in the world.

© 1999 Texas Instruments Incorporated

Investigation

Evaluate 6.5^4 and then display the result with 3 decimal places. Change the result back to floating-point format.

1. Demonstrate how to use the TI-30X IIS to evaluate expressions with exponents.

Press:	The calculator shows:
[CLEAR] 6.5 [^] 4 [ENTER]	6.5 ^4 1785.0625 DEG
[2nd] [FIX] ▶ ▶ ▶ ▶	F 0 1 2 3 4 5 6 7 8 9 DEG
[2nd] [ANS] [ENTER]	Ans 1785.063 DEG
[2nd] [FIX] ◀ ◀ ◀ ◀ [ENTER]	Ans 1785.0625 DEG

2. Revisit the original problem statement in the Introduction. How may bytes are in a kilobyte? *(1024 bytes)* How many bytes are in a megabyte? *(1,048,576 bytes)*

© 1999 TEXAS INSTRUMENTS INCORPORATED

3. Demonstrate how to express numbers in Scientific Notation on the TI-30X IIS.

   Display 85,000 and 258 x 10^4 in Scientific Notation. Then return the calculator to Floating Notation.

Press:	The calculator shows:
CLEAR 2nd [SCI/ENG] ▶	FLO SCI ENG DEG
ENTER	SCI DEG
85000 ENTER	85000 $8.5_{x10}04$ SCI DEG
258 2nd [EE] .4 ENTER	258E4 $2.58_{x10}06$ SCI DEG
258 2nd [10^x] 4) ENTER	25810^(4) $2.58_{x10}06$ SCI DEG
2nd [SCI/ENG] ◀ ENTER	25810^(4) 2580000 DEG

4. Show the students how to calculate common and natural logarithms.

   Common (base 10) logarithms: Calculate log 6^2, log (6 x 6), and log (6) + log (6), and observe that the results the same for all three procedures. What is the result if you calculate 2 x log (6)?
   Same as all the others.

   Natural (base *e*) logarithms: Calculate ln 4 and then calculate e^x of the result. Try this with numbers other than 4, and observe the results.

Press:	The calculator shows:
CLEAR LOG 6 x^2) ENTER	log (6^2) 1.556302501 DEG
LOG 6 x 6) ENTER	log (6 * 6) 1.556302501 DEG
LOG 6) + LOG 6) ENTER	log (6) + log (→ 1.56302501 DEG

© 1999 TEXAS INSTRUMENTS INCORPORATED

Natural (base e) logarithms:

LN 4) ENTER	ln (4) 1.386294361 SCI DEG
2nd [e^x] 2nd [ANS]) ENTER ◀ ENTER	e^(Ans) 4 DEG

Wrap-Up

As the depth and complexity of the mathematics content increases for students, pattern development is essential. You should allow students to examine patterns that result from algebraic manipulation, make conjectures about general algebraic properties based on their observations, and verify their conjectures with numerical substitutions of their own. Logarithms and exponents provide a context to illustrate these properties well.

Extension

- Suppose you earn 1 cent the first day, 2 cents the second day, 4 cents the third day, 8 cents the fourth day, and so on, doubling the amount you earn each day. What is the total of your earnings after working for 30 days?

  $(2^{31} - 1)(.01) = \$21,474,836.47$

- Some brands of computers have 64 MB of memory. Express 64 MB of memory in bytes.

  $2^{20} \times 64 = 67,108,864$ bytes

© 1999 TEXAS INSTRUMENTS INCORPORATED

Solutions Part 1

Evaluate the following exponential expressions. Round all results to the nearest thousandth.

1. 6^6 *(46656)*

2. 1.2^8 *(4.3)*

3. $(0.43)^{-4}$ *(29.250)*

4. $\frac{1}{(0.43)^4}$ *(29.250)*

5. $3^5 \times 2^{-5}$ *(7.594)*

6. $(4^3 + 6^2)^3$ *(1000000.000)*

7. $\left(\frac{135.24}{142.78}\right)^{28}$ *(0.219)*

8. $\left(\frac{13524}{14278}\right)^{28}$ *(0.219)*

9. $\text{Log}(5^8)$ *(5.592)*

10. $2\text{Log}(5^4)$ *(5.592)*

11. $\frac{10^7(10^3 \times 10^5)}{10^{12}}$ *(1000.000)*

12. $\frac{10^{-4}(10^{-3} \times 10^{-5})}{10^{-15}}$ *(1000.000)*

Did any of the pairs of expressions above produce the same result?

Yes; 3 and 4, 7 and 8, 9 and 10, 11 and 12

Explain.

They are all equivalent mathematical expressions; their representations are different.

Use the calculator to answer these:

13. What is $(-1)^5$? *-1*

14. What is $(-1)^{13}$? *-1*

15. What is $(-1)^8$? *1*

16. What is $(-1)^{97}$? *-1*

17. What is $(-2)^7$? *-128*

18. What is $(-5)^6$? *-15625*

Describe any patterns that you discovered in the exercises above.

Negative numbers raised to odd powers produce negative results; even powers produce positive results.

© 1999 TEXAS INSTRUMENTS INCORPORATED

Use the calculator to evaluate these. Then rewrite the expression in a different way that will produce the same result.

19. $(-2)^{16}$ *65536; $(2)^{16}$*

20. $(-7)^7$ – 823543; $-(7)^7$

21. $(-0.048)^{-3}$ - 9042.24537; $\frac{1}{(-0.048)^3}$

22. $(-3.5)^{-4}$ *0.00666389;* $\frac{1}{(-3.5)^4}$

Use the calculator to determine if the expressions are equivalent:

23. 5^7 and $2^7 + 3^7$ *78125 and 2315; No*

24. 5^7 and $5^3 + 5^4$ *78125 and 750; No*

25. $(4 \times 9)^3$ and $4^3 \times 9^3$ *46656; Yes*

26. $(4 \times 9)^3$ and 4×9^3 *46656 and 2916; No*

27. $(4^5)^3$ and 4^8 *1073741824 and 65536; No*

28. $\left(4^5\right)^3$ and 4^{15} *1073741824; Yes*

© 1999 TEXAS INSTRUMENTS INCORPORATED

Solutions Part 2

Perform the following operations using scientific notation mode on your calculator.

1. Enter 1 ÷ 2.

   5 x 10^{-1} *(answers will vary)*

   Explain the result.

2. Enter 572 [2nd] [EE] 3.

   5.72 x 10^{5} (answers will vary)

   Explain the result.

3. What calculation can you do on your calculator that will have a result of 2 x 10^{02}?

   2 x 100 *(answers will vary)*

4. What calculation can you do on your calculator that will have a result of 4 x 10^{-03}?

   4 x 1 ÷ 1000 *(answers will vary)*

5. What is the largest power of 2 that can be displayed on your calculator screen?

   2^332 = 8.749002899 x 10^{99}

   Did you use standard Floating Notation or Scientific Notation to display your result? *It does not matter—both will work.* Why?

   When numbers are this large, the calculator defaults to scientific notation.

6. Try to find the missing numbers with just one guess. Then check with your calculator.

   ?^{7} = 823,543 *7*

   ?^{6} = 1,771,561 *11*

   ?^{5} = 371,293 *13*

7. Scientists tell us that human hair grows at a rate of 6 inches per year. How fast does hair grow in miles per hour? Give your answer in scientific notation.

   1.096029742 X 10^{-8} miles per hour

8. A micron is a unit of measurement 10^{-6} meters long. A white blood cell is 0.000007 m to 0.000012 m wide. Give this range of measurement in microns.

   7 x 10^{-12} to 1.2 x 10^{-11}

© 1999 TEXAS INSTRUMENTS INCORPORATED

Student Activity 5

Name _____

Date _____

Number Power!

Objective: *In this activity, you will use the power of the calculator to explore patterns and rules in dealing with exponents and logarithms. You will evaluate expressions with exponents and logarithms and display them in standard and scientific notation.*

Part 1: Practice with Exponents

Evaluate the following exponential expressions. Round all results to the nearest thousandth.

1. 6^6
2. 1.2^8
3. $(0.43)^{-4}$
4. $\frac{1}{(0.43)^4}$
5. $3^5 \times 2^{-5}$
6. $(4^3 + 6^2)^3$
7. $\left(\frac{135.24}{142.78}\right)^{28}$
8. $\left(\frac{13524}{14278}\right)^{28}$
9. $\text{Log}(5^8)$
10. $2\text{Log}(5^4)$
11. $\frac{10^7(10^3 \times 10^5)}{10^{12}}$
12. $\frac{10^{-4}(10^{-3} \times 10^{-5})}{10^{-15}}$

Did any of the pairs of expressions above produce the same result?

Explain.

© 1999 TEXAS INSTRUMENTS INCORPORATED

Use the calculator to answer these:

13. What is $(-1)^5$?

14. What is $(-1)^{13}$?

15. What is $(-1)^8$?

16. What is $(-1)^{97}$?

17. What is $(-2)^7$?

18. What is $(-5)^6$?

Describe any patterns that you discovered in the exercises above.

Use the calculator to evaluate these. Then rewrite the expression in a different way that will produce the same result.

19. $(-2)^{16}$

20. $(-7)^7$

21. $(-0.048)^{-3}$

22. $(-3.5)^{-4}$

Use the calculator to determine if the expressions are equivalent**:**

23. 5^7 and $2^7 + 3^7$

24. 5^7 and $5^3 + 5^4$

25. $(4 \times 9)^3$ and $4^3 \times 9^3$

26. $(4 \times 9)^3$ and 4×9^3

27. $(4^5)^3$ and 4^8

28. $\left(4^5\right)^3$ and 4^{15}

© 1999 TEXAS INSTRUMENTS INCORPORATED

Part 2: Using Scientific Notation and Exponents

Perform the following operations using scientific notation mode on your calculator.

1. Enter 1 ÷ 2. Explain the result.

2. Enter 572 [2nd] [EE] 3. Explain the result.

3. What calculation can you do on your calculator that will have a result of 2×10^{02}?

4. What calculation can you do on your calculator that will have a result of 4×10^{-03}?

5. What is the largest power of 2 that can be displayed on your calculator screen?

   Did you use standard Floating Notation or Scientific Notation to display your result? Why?

6. Try to find the missing numbers with just one guess. Then check with your calculator.

   $?^{7} = 823{,}543$

   $?^{6} = 1{,}771{,}561$

   $?^{5} = 371{,}293$

7. Scientists tell us that human hair grows at a rate of 6 inches per year. How fast does hair grow in miles per hour? Give your answer in scientific notation.

8. A micron is a unit of measurement 10^{-6} meters (m) long. A white blood cell is 0.000007 m to 0.000012 m wide. Give this range of measurement in microns.

© 1999 TEXAS INSTRUMENTS INCORPORATED