EXPLORATIONS

Chapter 8

Features Used

[∫], [|], **limit(), Σ,** [Y=], **NewProb, DelVar,** [WINDOW], [GRAPH], [ANS]

Setup

[◆] **1**
NewFold fourier

Fourier Series

This chapter shows how to compute and graph the complex Fourier Series coefficients for a square wave.

Topic 35: Square Wave: Computing the Coefficients

The TI-89 can easily sum the Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j2\pi k f_0 t}$$

and evaluate the complex Fourier coefficients defined by the integral

$$c_k = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) e^{-j2\pi k f_0 t} \quad \text{where } f_0 = \frac{1}{T_0}$$

Suppose x(t) is a square wave as shown in Figure 1.

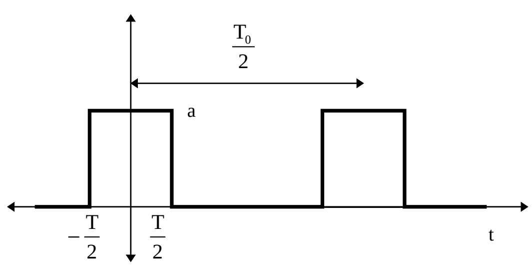

Figure 1. Periodic pulse train

For this example, the complex coefficient becomes

$$c_k = \frac{1}{T_0} \int_{-\frac{T}{2}}^{\frac{T}{2}} a e^{-j2\pi k f_0 t} dt$$

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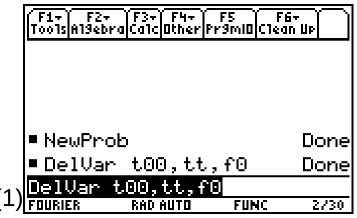
1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

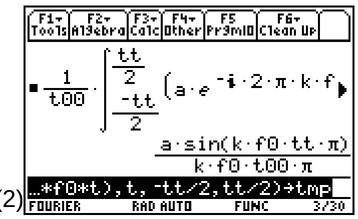
2. Three variable substitutions are needed before entering the expression for the complex coefficient. T_0 is entered as **t00** (**t0** is reserved), T is entered as **tt** (**t** and **T** are the same on the TI-89), and j is [*i*] which is entered as [2nd] [*i*]. Before entering the equation, any previous values of the variables used must be deleted as shown in screen 1.

   [CATALOG] **DelVar t00** [,] **tt** [,] **f0**

(1) 

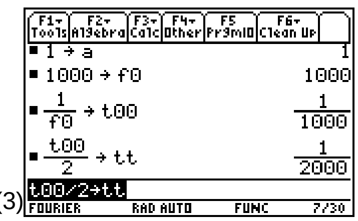
3. Enter the integral (screen 2).

   1 [÷] **t00** [2nd] [∫] **a** [◆] [e^x] [(-)] [2nd] [*i*] **2** [2nd] [π] **k** [×] **f0** [×] **t** [)] [,] **t** [,] [(-)] **tt** [÷] **2** [,] **tt** [÷] **2** [)] [STO▸] **tmp**

(2) 

***Note**: If a Domain Error message appears, try switching to radian angle mode by pressing [MODE] and selecting **RADIAN**.*

4. Next plot the coefficients. To do this, pick values for **a**, **f0** (which sets **t00** also), and **tt**. Try the values shown in screen 3.

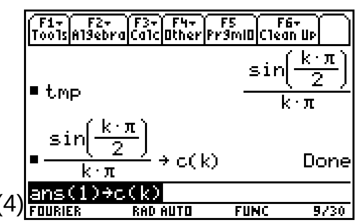
(3) 

5. In this example, **a** is set to 1 and **f0** to 1000 Hz. With **tt** set to **t00/2**, the duty cycle is ½ so the square wave will be "on" half the time. Display the value of **tmp** (top of screen 4).

6. Now save the formula for the coefficient in a function called **c**(**k**) as shown in screen 4.

   [2nd] [ANS] [STO▸] **c** [(] **k** [)]

   Using the answer from the integral ensures that the value stored in **c**(**k**) is the result of the integral, not the integral itself. If the integral is saved, it is reevaluated every time a coefficient is computed. With **c**(**k**) stored as a function, the integral is evaluated once and the resulting formula is used each time a coefficient value is needed.

(4) 

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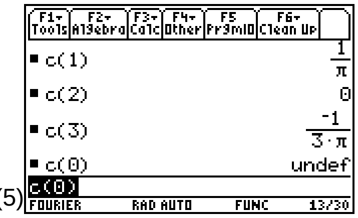
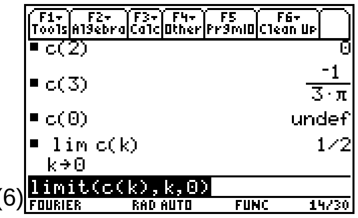
7. Check a few data points as shown in screen 5.

(5) 

8. All the coefficients look fine except for **k**=0. The equation for **c**(**k**) shows that for **k**=0 the result is 0/0. The correct value can be found by using **limit()** (screen 6).

[CATALOG] **limit(c** [(] **k** [)] [,] **k** [,] **0** [)]

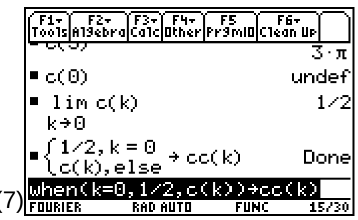
The correct value of **c**(**k**) for **k**=0 is ½. This makes sense since the square wave is turned on half the time with an average value of ½.

(6) 

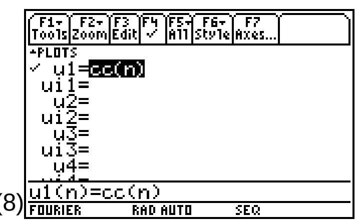
9. Use the **when()** function to define **c**(**k**) so that the **k**=0 case is calculated correctly (screen 7).

[CATALOG] **when(k** [=] **0** [,] **1** [÷] **2** [,] **c** [(] **k** [)] [)] [STO▶] **cc** [(] **k** [)]

The value of **c**(**k**) valid for all **k** is stored in **cc**(**k**). Now the coefficients can be plotted.

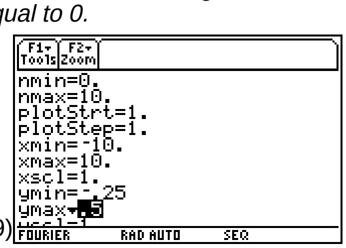
(7) 

10. Set the **Graph** mode to **SEQUENCE** ([MODE] [▶] **4:SEQUENCE** [ENTER]), press [◆] [Y=], and enter **cc**(**n**) as the sequence to be plotted, as shown in screen 8. Note that the sequence plot mode uses the variable **n**.

(8) 

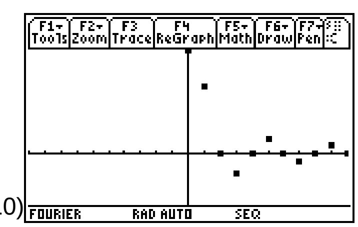
11. Press [◆] [WINDOW] to set the plot range as shown in screen 9.

__Note__: __nmin__ must be greater than or equal to 0.

(9) 

12. Press [◆] [GRAPH] to see the results as shown in screen 10.

The graph also could be done in **Function** graphing mode. **Sequence** graphing mode is chosen to emphasize that the coefficients only appear at integer values.

(10) 

Topic 36: Square Wave: Constructing the Wave from the Coefficients

The original signal can be rebuilt from the coefficients by using

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j2\pi k f_0 t}$$

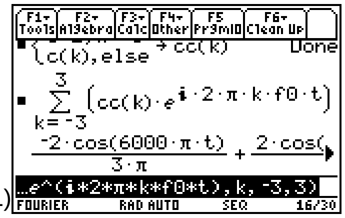
1. To do this, return to the Home screen and enter the expression as shown in screen 11.

   [CATALOG] Σ(**cc** [(] **k** [)] [◆] [e^x] [2nd] [i] **2** [2nd] [π] **k** [×] **f0** [×] **t** [)] [,] **k** [,] [(-)] **3** [,] **3** [)]

   Notice that the TI-89 applied Euler's Identity to terms of the form $e^{i\theta} + e^{-i\theta}$ to get $2\cos(\theta)$.

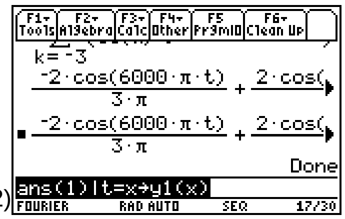
   The complete output is

   $$\frac{-2\cos(6000\pi t)}{3\pi} + \frac{2\cos(2000\pi t)}{\pi} + 1/2$$

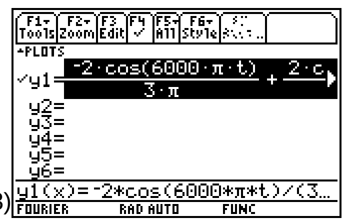
(11)


2. Save the result as **y1**(**x**), screen 12.

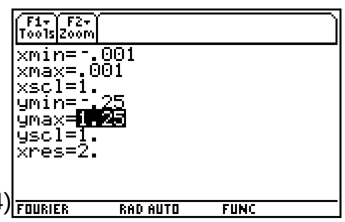
   [2nd] [ANS] [|] **t** [=] **x** [STO▸] **y1** [(] **x** [)]

(12)


3. Switch the **Graph** mode to **FUNCTION** by pressing [MODE] [▸] **1:FUNCTION** [ENTER]. Then press [◆] [Y=] to verify that the equation is entered in the Y= Editor (screen 13).

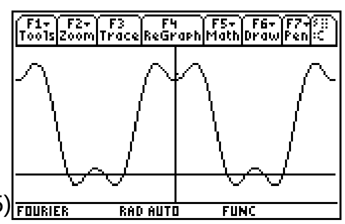
(13)


4. Press [◆] [WINDOW] and set the plot range as shown in screen 14.

(14)


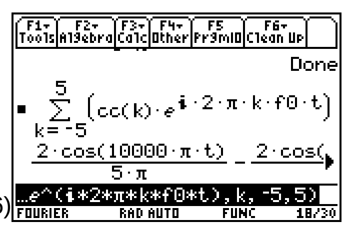
5. Press [◆] [GRAPH] to see the graph of **x**(**t**) as shown in screen 15.

   It's not quite a square wave, but it's not too bad for using only 5 non-zero coefficients. Recall that **c**(-3), **c**(-1), **c**(0), **c**(1) and **c**(3) are non-zero; **c**(-2)=**c**(2)=0.

(15)


6. To get a more accurate representation, include more coefficients of the series. On the Home screen, change the summation range to -5 to 5 as shown in screen 16.

   [CATALOG] Σ(**cc** [(] **k** [)] [◆] [e^x] [2nd] [i] **2** [2nd] [π] **k** [×] **f0** [×] **t** [)] [,] **k** [,] [(-)] **5** [,] **5** [)]

(16)


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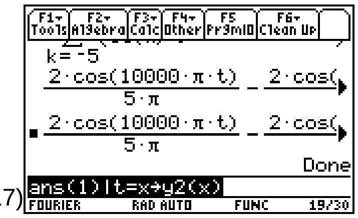
The output for the 5-term series is

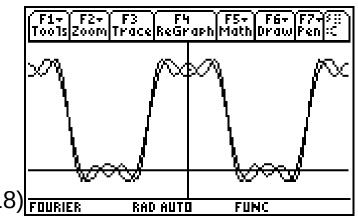
$$\frac{2\cos(10000\pi t)}{5\pi} - \frac{2\cos(6000\pi t)}{3\pi} + \frac{2\cos(2000\pi t)}{\pi} + 1/2.$$

(17)


7. Save the result in **y2**(**x**) as shown in screen 17.

[2nd] [ANS] [|] **t** [=] **x** [STO▸] **y2** [(] **x** [)]

8. Press [◆] [GRAPH]. Screen 18 compares the resulting graph of the sum for **k**=-5 to 5 with the original graph for the sum of **k**=-3 to 3. The new result more closely represents a square wave because the sum more closely represents a square wave as the number of terms increases. With an infinite number of terms, the sum exactly represents the square wave.

(18)


Tips and Generalizations

The TI-89 can easily find the Fourier Series coefficients in closed form for many periodic signals. In this chapter, **c**(**k**) could be expressed as a simple equation. More complex signals may not have a closed form solution. In these cases, use numeric integration (**nInt**) to find each of the coefficients.

Chapter 9 adds a new dimension by showing how the TI-89 can manipulate vectors.

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