About the Lesson

In this activity, students determine sample spaces for coin flips, find the various probabilities associated with each outcome, and graph the probabilities using the calculator. As a result, students will:

- Calculate the probability of a possible outcome.
- Graph the probability distribution of a discrete random variable given the probability associated with each element in the sample space over which it is defined.
- Use technology to simulate flipping coins and rolling dice.
- Construct and interpret a histogram.
- Analyze and compare the graphs of probability distributions.
- Find and compare experimental and theoretical probabilities.

Vocabulary

- distribution
- histogram
- probability
- sample space

Teacher Preparation and Notes

- Students should already be familiar with the concept of finding sample spaces and writing probabilities of events happening. If review is needed, feel free to have the students determine the sample space for 1 coin being flipped at the same time as one number cube being rolled. Then have them compute the probability of rolling a 6, getting heads and a five, etc.
- The student worksheet is intended to guide students through the main ideas of the activity, while providing more detailed instruction on how they are to perform specific actions using the tools of the TI-84 Plus C graphing calculator. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.
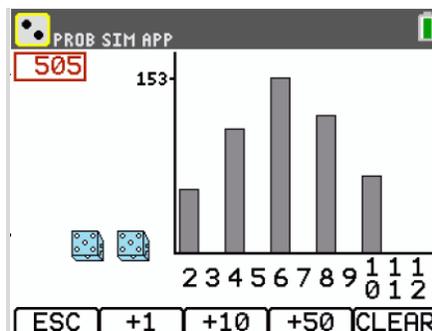
Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- RollTheDice_Student.pdf
- RollTheDice_Student.doc

Three Coin Flips

Have students simulate a scenario in which 3 coins are tossed using the Probability Sim App. They should press APPS, choose Prob Sim, and then select 'Toss Coins'. Press the 'soft key' ZOOM to SET up the toss. Have students choose 3 coins and select OK. Then, select YES to agree to this set up and press WINDOW to begin TOSSing. Have students select +50 to do many tosses at once and observe the pattern.

1. Suppose three coins are flipped. The possible outcomes are listed below. Calculate the probability of each possible outcome.

0 Heads	1 Head	2 Heads	3 Heads
TTT	TTH	HHT	HHH
	THT	HTH	
	HTT	THH	

Answers: P(0 Heads) = $\frac{1}{8}$ = 0.125;

P(1 Head) = $\frac{3}{8}$ = 0.375;

P(2 Heads) = $\frac{3}{8}$ = 0.375;

P(3 Heads) = $\frac{1}{8}$ = 0.125.

P(less than two Heads) = $\frac{1}{2}$ = 0.5;

P(less than four Heads) = 1.0.

Next, have students graph the distribution. To do this, have students enter the number of heads in L1 and their probability in L2. Students can access the lists by pressing STAT ENTER.

Once the data are entered, have students produce a connected scatter plot by pressing 2nd [STAT PLOT] ENTER and have students change the settings as shown at the right. Then, have students press ZOOM and select **ZoomStat** to view the distribution.
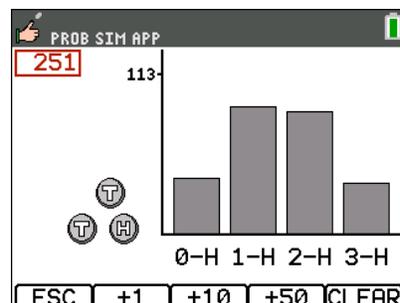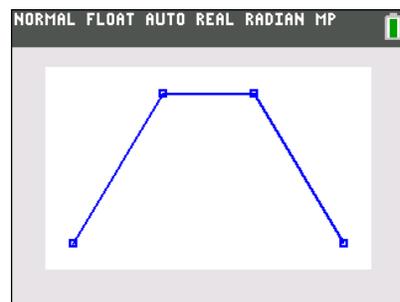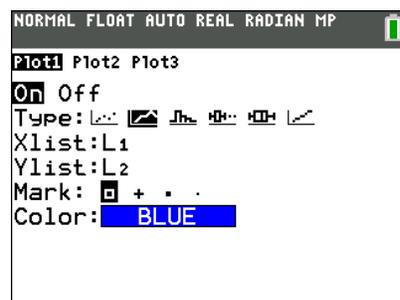
> **Teaching Tip:** A common mistake with the graphing is for students to have equations in the ⟦Y=⟧ screen or to have more than one plot active. Another common mistake students make is not pressing ⟦ZOOM⟧ and selecting **ZoomStat** before graphing to get the window sized correctly. Pedagogically, it is often best to have student think and discuss what a good viewing window is before using **ZoomStat**.

Six Coin Flips

Suppose six coins were flipped. Have students determine the possible outcomes for 3, 4, 5, and 6 heads. Then, have students calculate the probabilities of each outcome listed below and graph the probability distributions.

The completed sample space appears below.

0 Heads	1 Head	2 Heads	3 Heads	4 Heads	5 Heads	6 Heads
TTTTTT	HTTTTT	HHTTTT	HHHTTT	HHHHTT	HHHHHT	HHHHHH
	THTTTT	HTHTTT	HHTHTT	HHHTHT	HHHHTH	
	TTHTTT	HTTHTT	HHTTTH	HHTHHT	HHHTHH	
	TTTHTT	HTTTHT	HTHTTH	HTHHHT	HHTHHH	
	TTTTHT	HTTTTH	HTHHTT	THHHHT	HTHHHH	
	TTTTTH	THHTTT	HTHTHT	HHHTTH	THHHHH	
		THTHTT	HTTHTH	HHTHTH		
		THTTHT	HTTHHT	HTHHTH		
		THTTTH	HHTTHT	THHHTH		
		TTHHTT	HTTTHH	HHTTHH		
		TTHTHT	THHHTT	HTHTHH		
		TTHTTH	THHTHT	THHTHH		
		TTTHHT	THHTTH	HTTHHH		
		TTTHTH	THTHTH	THTHHH		
		TTTTHH	THTTHH	TTHHHH		
			THTHHT			
			TTHHHT			
			TTHHTH			
			TTHTHH			
			TTTHHH			

To help students quickly complete the table, remind them that the number of outcomes for 6 tails = 0 heads, and vice versa (switch the T's for H's.) This same principle can be applied to 5 and 4 heads. This just leaves computing the number of 3 heads there are. To help with that calculation, remind the students that there are a total of 64 different outcomes here ($2 \times 2 \times 2 \times 2 \times 2 \times 2$). This leaves just 20 ways to obtain 3 heads.

2. Compute the probabilities:

   P(0 heads) = _____ P(1 head) = _____ P(2 heads) = _____ P(3 heads) = _____

   P(4 heads) = _____ P(5 heads) = _____ P(6 heads) = _____

   Answer:

   P(0 heads) = 0.016; P(1 head) = 0.094; P(2 heads) = 0.234; P(3 heads) = 0.313;

    P(4 heads) = 0.234; P(5 heads) = 0.094; P(6 heads) = 0.016.

Next, students will graph the probability distribution following the same process that they did with the three coin toss simulation.
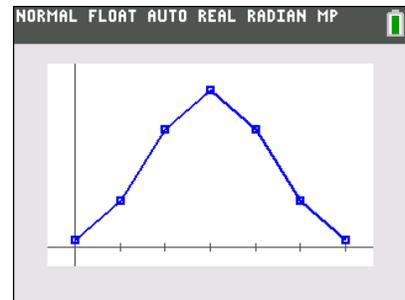
3. Two of the six coins are replaced by unfair coins. The unfair coins have both sides marked heads. Recalculate the possible outcomes and probabilities. How is the graph different? (Be sure to mention how the center and shape of the graph changes.)

Answer: If the last two coins were to change to heads, then:

- Getting 0 heads or 1 head would be impossible, and their probabilities would be 0.

- Number of ways to get 2 heads would be 4, with a probability of 0.0625.

- Number of ways to get 3 heads would be 16, with a probability of 0.25.

- Number of ways to get 4 heads would be 24, with a probability of 0.375.

- Number of ways to get 5 heads would be 16, with a probability of 0.25.

- Number of ways to get 6 heads would be 4, with a probability of 0.0625.
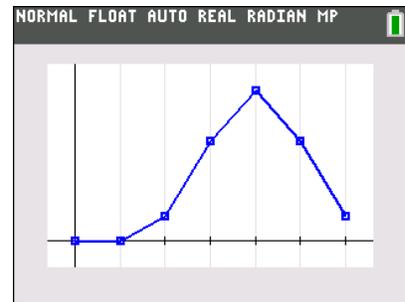
The students should note that the graph is no longer centered, and it is "heavier" on the right. This is called being skewed left.

Funny Dice

Two very unusual number cubes are rolled. They are both labeled 1, 1, 3, 3, 5, 5.

4. Find all the possible outcomes if the sums of the numbers rolled were computed.

Answer:

Sum = 2	Sum = 4	Sum = 6	Sum = 8	Sum = 10
1, 1	1, 3	5, 1	5, 3	5, 5
	3, 1	3, 3	3, 5	
		1, 5		

5. Find the following probabilities. Use an area model to help show the fractional probabilities:

$P(2) = $ _____ $P(4) = $ _____ $P(6) = $ _____ $P(8) = $ _____ $P(10) = $ _____

	1	1	3	3	5	5
1	2	2	4	4	6	6
1	2	2	4	4	6	6
3	4	4	6	6	8	8
3	4	4	6	6	8	8
5	6	6	8	8	10	10
5	6	6	8	8	10	10

	1 1	3 3	5 5
1 1	1/9	1/9	1/9
3 3	1/9	1/9	1/9
5 5	1/9	1/9	1/9

Answer:

$$P(2) = \frac{1}{9} = 0.\overline{111} \; ; \; P(4) = (2)\left(\frac{1}{9}\right) = 0.\overline{222}; \; P(6) = (3)\left(\frac{1}{9}\right) = 0.\overline{333};$$

$$P(8) = (2)\left(\frac{1}{9}\right) = 0.\overline{222} \; ; \; P(10) = \frac{1}{9} = 0.\overline{111}$$

6. Display the probability distributions on the calculator. Sketch your graph below. Use Prob Sim App to confirm your probability distribution.

Answer:
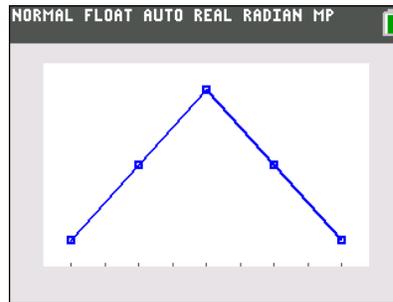
Tech Tip: From the Prob Sim App, select 'Roll Dice'. SET it for 2 dice, and select the ADVanced settings to change the weight as shown below.