Parametric Equations Problems Sampler

Key

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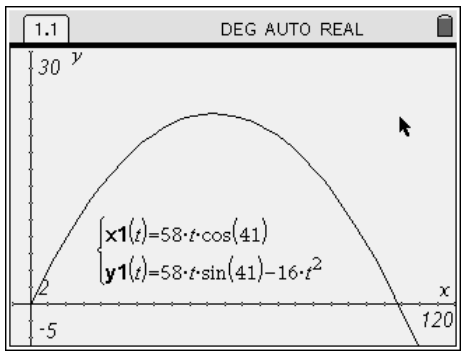
1. A dart is thrown upward with an initial velocity of 58 ft per second at an angle of elevation 41^{o}. Consider the position of the dart at any time t (t = 0 when the dart is thrown.) Neglect air resistance.

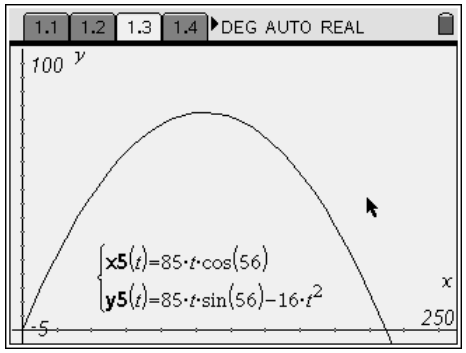


The dart will hit the ground in about **2.4 seconds**.

The maximum height of the dart is about **22.6 ft** and is reached at about **1.2 seconds**.

The ball travels about **104 ft** horizontally.

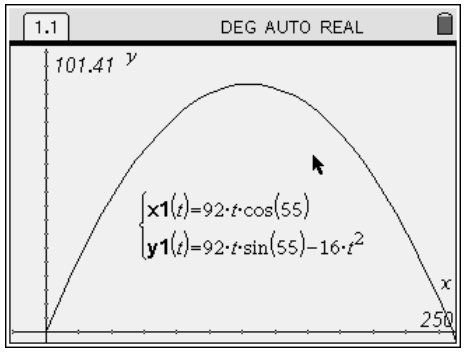
2. An NFL punter at the 15-yard line kicks a football downfield with initial velocity 85 feet per second at an angle of elevation of 56^{o}.



The ball lands about **210 feet, or 70 yards** from where it was kicked.

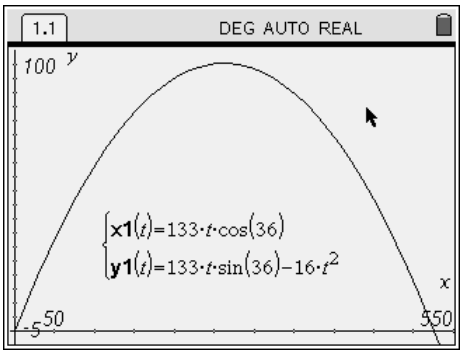
The maximum height reached was about **77.6 ft**.
The hang time was about **4.5 s**.

3. A batter hits a ball with an initial velocity of 92 feet per second and at an angle of 55^{o} from the horizontal. Find the maximum height attained and the total horizontal distance traveled by the ball.



The maximum height reached is about **88.7 ft** and the horizontal distance traveled is about **249 ft**.

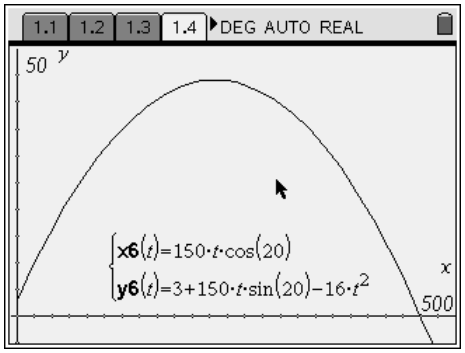
4. A golfer hits a ball with an initial velocity of 133 feet per second and at an angle of 36^{o} from the horizontal. Find when and where the ball will hit the ground. Will the ball clear a fence 9 feet high that is at a distance of 275 feet from the golfer?



The ball will land about **526 feet** from where it was hit after about **4.8 seconds**.

The ball will **easily clear** the fence.

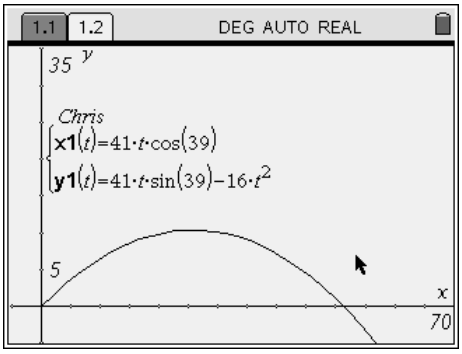
5. It's the bottom of the ninth inning, the Cubs are behind 6-3 and the bases are loaded. Sammy Sosa is at bat. He swings and makes contact with the ball 3 feet above the plate at an angle of 20 degrees from the horizontal at a velocity of 150 feet per second. He hits straight toward center field where there is a fence 400 feet from home plate and 20 feet high. Neglect resistance due to wind. Does Sammy hit a grand slam and win the game?

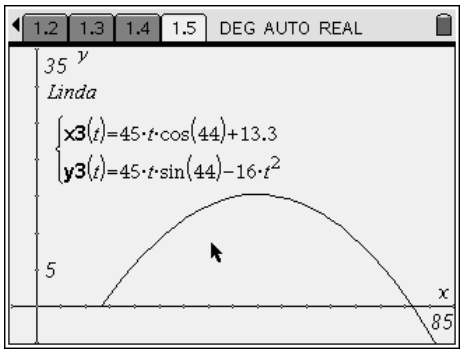


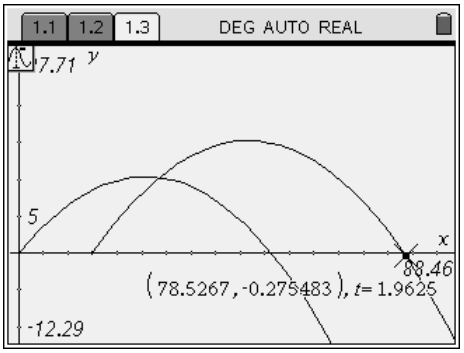
At 400 ft, the ball's height is only 19.8 feet, so it **will not make it** over the fence and Sammy does not get the grand slam.

Velocity is the more important factor for relatively small changes in velocity or angle measure because such changes have a greater impact on the outcome, particularly for horizontal distance. Because the angle is involved in the sine or cosine part of the equation, its impact on the outcome for distance both vertically and horizontally is less dramatic.

6. Chris and Linda are standing 78 feet apart. At the same time, they each throw a softball toward each other. Linda throws her ball with an initial velocity of 45 ft per second with an angle of inclination of 44^{o}. Chris throws her ball with an initial velocity of 41 feet per second with an angle of inclination of 39^{o}.







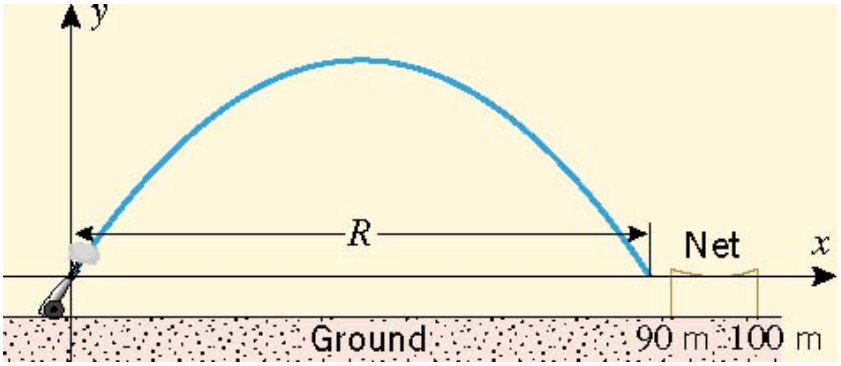
In order to obtain correct time values for Linda's curve, use approximately $1.96 - t$

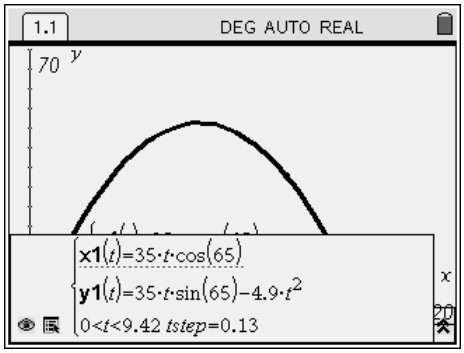
$0 < t < 2$

Thrower	Chris	Linda
Max height	10.4 ft	15.3 ft
Horizontal distance	51.0 ft	64.7 ft
Time to hit ground	1.6 s	2.0 s

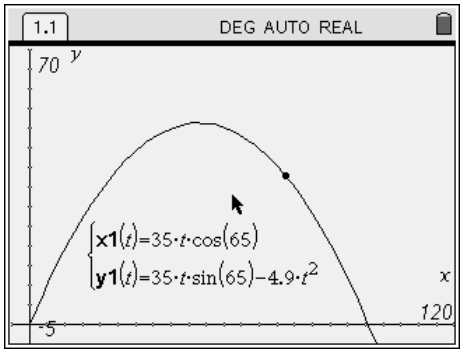
Chris' ball hits the ground first. The two balls paths never meet.

7. Blammo the Human Cannonball will be fired from a cannon and hopes to land in a small net at the opposite end of the circus arena. Your job as Blammo's manager is to do the mathematical calculations that will allow Blammo to perform his death-defying act safely. The methods that you will use are from the field of ballistics (the study of projectile motion). Blammo's cannon has a muzzle velocity of 35 m/s, which means that Blammo will leave the muzzle with that velocity. The muzzle opening will be 5 m above the ground, and Blammo's objective is to land in a net that is also 5 m above the ground and that extends a distance of 10 m between 90 m and 100 m from the cannon opening. Your mathematical problem is to determine the elevation angle of the cannon (the angle from the horizontal to the cannon barrel) that will make Blammo land in the net. Generate Blammo's trajectories, taking elevation angles at increments of 10° from 15° to 85°.





Note that the maximum value for *t* must be increased to view the x-intercept.

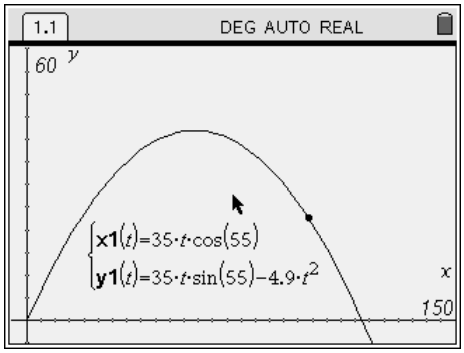


The elevation angle should be set at **65°**. With this setting, Blammo should land at a horizontal distance of 95 feet, right in the middle of the net.

Since Blammo is released and lands 5 feet above the ground, it does not impact the equation to model this situation.

8. Blammo is to be fired from 5 m aboveground level with a muzzle velocity of 35 m/s over a flaming wall that is 20 m high and past a 5-m-high shark pool. To make the feat impressive, the pool will be made as long as

possible. Your job as Blammo's manager is to determine the length of the pool, how far to place the cannon from the wall, and what elevation angle to use to ensure that Blammo clears the pool. Determine the appropriate equation t taking elevation angles at increments of 10° from 15° to 85°.



The angle that maximizes the length of the pool is **55°**.

The flaming wall should be placed **16 meters** from the cannon.

References

Memphis Educational Computer Connectivity Alliance
http://www.mecca.org/~halfacre/MATH/Parametric%20Equations%20Worksheet.pdf

Sosa problem adapted from Demana, Waits, Precalculus, A Graphing Approach, Menlo Park, Addison-Wesley, 1997, pp. 575-576.

Blammo problems adapted from a module by: John Rickert, Rose-Hulman Institute of Technology Howard Anton, Drexel University