What's the Trig ? - An Introduction to Right Triangle Trigonometry

Geometry Major Topics: Right Triangle Trigonometry

NCTM Principles and Standards:
   Content Standards

Geometry:

   Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

   Analyze properties and determine attributes of two- and three-dimensional objects.

   Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

   Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

Objectives: This lesson introduces students to right triangle trigonometry. Using The Geometer's Sketchpad Application for the TI-89 Calculator, students construct a right triangle and calculate the sine, the cosine, and the tangent of various acute angles in terms of their geometric definitions. This lesson can be used as a teacher directed activity or can be given to the students as a discovery activity. If the students are familiar with The Geometer's Sketchpad Application for the TI-89, this activity should take one 50 minute class period.

Materials: TI-89 Graphing Calculator, The Geometer's Sketchpad Application

Prerequisites: Students should have a working knowledge of The Geometer's Sketchpad Application for the TI-89 and should have previously studied similar and right triangles.

Procedure:

Open The Geometer's Sketchpad application on the TI-89 by pressing the APPS button and choosing Sketchpad. Press F1 and choose **A:Preferences**. Press F2:Units and set the **Angle Units** to degrees, the **Angle Precision** to **3:1.00**, the **Distance Precision** to **3:1.00,** and the **Other Precision** to **3:1.00**. This will round all angle measures to the nearest hundredth of a degree, all distance measures to the nearest hundredth, and any calculations to the same precision.

Press F8 and choose the **straightedge** tool. Using the **ray** tool, construct a horizontal ray AB near the bottom of the screen. Construct a second ray AC using endpoint A .
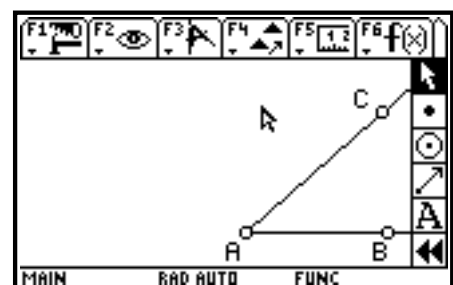
Press F8 and choose the **point** tool. Construct a point D on the
ray AC. Press F8 and choose the **pointer** tool. Select the point D <u>and</u>
the ray A. Press F3 and choose **8:Perpendicular Line**. This will
construct a line perpendicular to ray AC through point D.
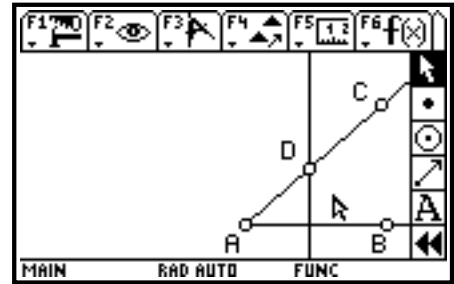
Press F8 and choose the **pointer** tool. Select the perpendicular line and the ray AC. Press F3 and choose
3:Intersection. This will construct the intersection point of the perpendicular line and the ray AC. Press F8
and choose the **label** tool. Label the intersection point E. Press F8 and select only the perpendicular line.
Press F2 and choose **1:Hide**. This will hide the perpendicular line. Press F8 and choose the **straightedge** tool.
Using the **segment** tool, construct the perpendicular segment DE. Your screens should be similar to the ones
below.
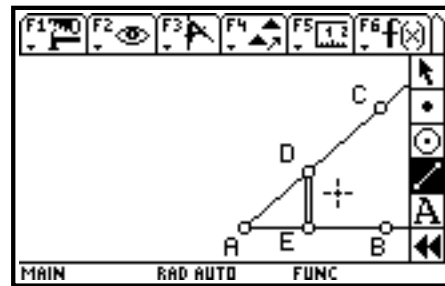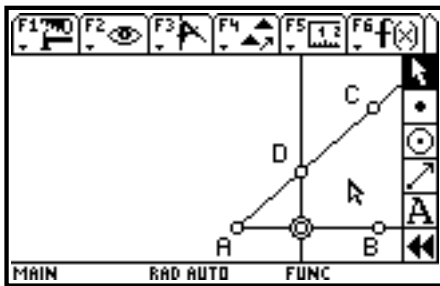
Right triangle ADE has now been constructed. As point D is dragged on ray AC, the triangle will remain a right
triangle. Also observe that if point D is dragged on ray AC, the various versions of triangle ADE are all similar.
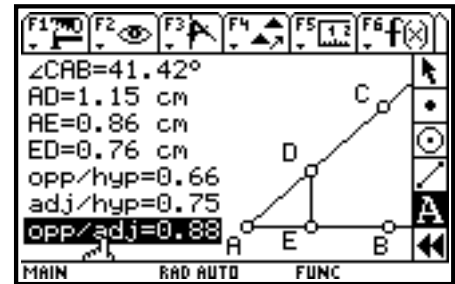
Press F8 and choose the **pointer** tool. Select point C, point A, and then point B. Press F5 and choose
5:Angle. The measure of angle CAB will be displayed. Press F8 and choose the **pointer** tool. Select points A
and D, remembering to deselect the previous measure before selecting the endpoints of the segment.   Press F5
and choose **2:Distance**. The length of segment AD will be displayed. In a similar manner, calculate the lengths
of segment AE and segment DE. Your screens should be similar to the ones below.
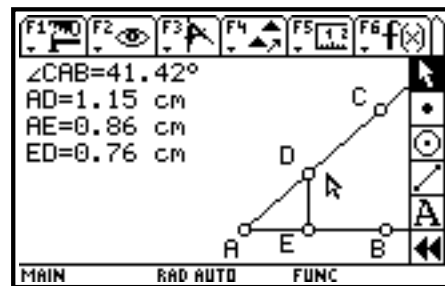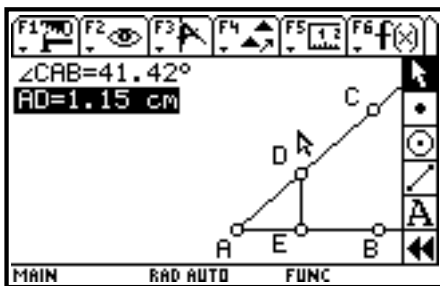
Press F5 and choose **C:Calculate**. Press F1 to display the measurements. Calculate the ratio of DE to AD. In regard to their relative positions in the triangle, this is the ratio of the length of the side opposite angle CAB to the length of the hypotenuse. Label this ratio as **opp/hyp=**. Your screens should be similar to the ones below.
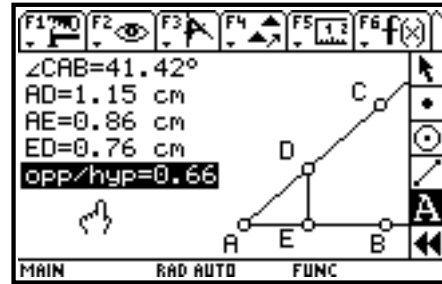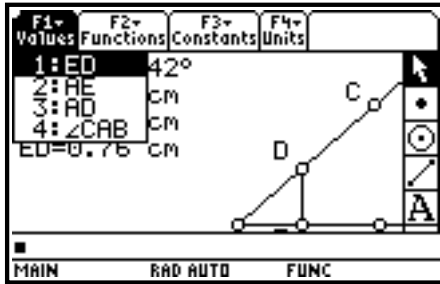
In a similar manner, calculate the ratio of AE to AD. This is the ratio of the length of the side adjacent to angle CAB to the length of the hypotenuse. Label this ratio **adj/hyp=**. Finally, calculate the ratio of DE to AE . This is the ratio of the length of the side opposite angle CAB to the length of the side adjacent to this angle. Label this ratio **opp/adj=**.

Press F8 and choose the **pointer** tool. Drag point D on ray AC and observe the "changes" in the measures and ratios. Make a conjecture about the results and justify your conjecture.

Teacher Note: Dragging point D only produces similar triangles; the ratio of the sides will always remain the same, since corresponding sides are proportional in similar triangles.
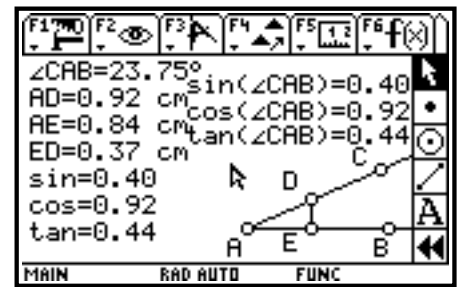
Press F8 and choose the **pointer** tool. Drag point C and thus ray AC to a different location on the screen. This will change the measure of angle CAB . Drag point D on ray AC a second time and observe the "changes" in the measures and ratios. Make a conjecture about the results and justify your conjecture.

Teacher Note: Dragging point D only produces similar triangles. Although the measures of the segments change, the ratios remain the same. The ratios are determined by the angle CAB.

The three ratios determined by the given angle can be renamed. The ratio of the side opposite the angle to the hypotenuse will be called the **sine** ratio, abbreviated **sin**. The ratio of the side adjacent to the angle to the hypotenuse will be called the **cosine** ratio, abbreviated **cos**. The ratio of the side opposite the angle to the side adjacent to the angle will be called the **tangent** ratio, abbreviated **tan**. Press F8 and choose the **label** tool. Change the labels of these ratios to the appropriate names.
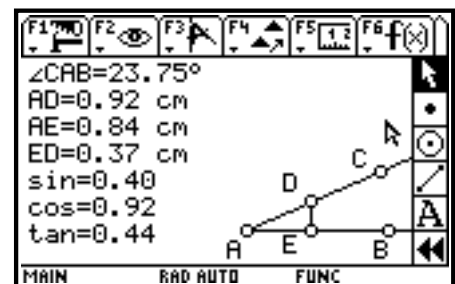
It is important to know that these ratios are determined by the angle. To confirm these values, press F5 and choose **C:Calculate**. Press F2 and choose **1: sin**. Then press F1 and choose the angle measure, remembering to type in the closing parenthesis. In a similar manner, calculate the cosine of the angle and the tangent of the angle. Compare these function values with the previously calculated ratios.

Press F8 and choose the **pointer** tool. Drag point C, thus changing the measure of angle CAB. Compare the function values with the calculated ratios that are displayed.

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