EXPLORATIONS

Activity 1

Divisibility Rules!

Concepts/Skills:

Number theory, divisibility rules, multiples, factors, problem-solving skills

Calculator:

Math Explorer or Explorer Plus

Overview:

Students use the calculator to review and strengthen their knowledge of divisibility rules.

Getting Students Involved

Ask students to verbalize the divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10 and to provide an example and a non-example for each rule. (The divisibility rules are printed on the student activity sheet.)

For example, a number is divisible by 6 if it is divisible by both 2 and 3. Hence, 72 is divisible by 6 because it is divisible by both 2 and 3, but 74 is **not** divisible by 6 because it is **not** divisible by 3. 74 is not divisible by 3 because the sum of its digits (11) is not divisible by 3.

Making Mathematical Connections

Ask whether it is quicker to use a calculator or to use the divisibility rules to determine:

- if 904 is divisible by 2. — *Divisibility rules are quicker.*
- if 375 is divisible by 3. — *Answers may vary. Discussion can help students to recognize, however, the power of the divisibility rule for 3. Those who choose to use the calculator most likely are not yet comfortable with the divisibility rule for 3.*
- if 670 is divisible by 5. — *Divisibility rules are quicker.*
- if 437 is divisible by 9. — *Answers may vary. Students may not have encountered the divisibility rule for 9.*
- if 316 is divisible by 4. — *Answers may vary. Students may not have encountered the divisibility rule for 4.*

♦ if 286 is divisible by 8.	*Probably the calculator is quicker. The divisibility rule for 8 is not particularly helpful in this situation.*
♦ if you are trying to find a number close to 740 that is divisible by 3.	*Answers may vary.* *Some may add the digits and recognize that they are just one short of a multiple of 3 and know that 741 is a number divisible by 3.* *Others may choose to use the constant operation (÷) feature of the calculator (*Calculator Transparency B*) to locate a number that will work. For example, they might enter 7 4 0 ÷ 3 =, recognize that 740 doesn't work, input 7 4 1 =, and have an answer.* *Still other students may use the INT÷ key (*Calculator Transparency J*) to determine a number divisible by 3. For example, they may enter 7 4 0 INT÷ 3 =, and use the answer 246 R2 to determine that they need a number one digit larger so the remainder will be 0.*

Discuss the questions posed above. This will help students learn to make good decisions about when to use calculators and when to use mental math.

This activity provides an excellent example of how a calculator can be used to prove the power of mental mathematics. A student who can instantly answer the questions listed above (before a calculator user can enter the numbers) almost always piques the interest of students who do not know the divisibility rules.

Carrying Out the Investigation

1. Use Activity 1 Transparency 1 to introduce the first Divisibility Rules! problem. As a class, discuss possible strategies for finding the number(s).

For example, we know that the number must be an even number less than 150 but greater than 99. We might start at 100 and input 1 0 0 INT÷ 8 = into the calculator (Calculator Transparency J). The answer 12 R4 informs us that we are four units away from a multiple of 8. Hence, we try 1 0 4 = and get 13 R0. The other answers will occur in intervals of 8, so again we can use the constant operation feature of the calculator by pressing 1 0 4 + 8 = = = = = (Calculator Transparency B) or store a constant operation by pressing + 8 Cons 1 0 4 Cons Cons Cons Cons (Calculator Transparency A).

2. Display Activity 1 Transparency 2. Divide the students into small groups of 2-4 to work on problem 2.

3. Allow one group to provide their solution strategy for the problem. Discuss different strategies used by other groups of students.

4. Assign the next one or two problems. Then debrief the whole group before proceeding.

   The constant feedback sessions enable students to consider additional approaches to incorporate into their processing. We want them to compare strategies, recognize that some are more effective than others, and incorporate the more effective strategies into their own thinking.

Making Sense of What Happened

1. Which set of mystery numbers was the most difficult to discover? Why? Were the numbers hard to find because of the divisibility rule that was involved?

2. When was the calculator most useful? When was it not needed?

3. Which was the easiest set of mystery numbers to discover? Why? How did you approach this problem?

4. How could divisibility rules assist you if you were to reduce 243/396?

Continuing the Investigation

Encourage students to create their own problems and share their problems with other groups.

Solutions

1. Solutions: 104; 112; 120; 128; 136; 144
   Pattern: There is a difference of 8 between solutions.
   Reason: The solutions must be divisible by 8. Numbers divisible by 8 are 8 units apart.

2. Solutions: 42; 84
   Pattern: There is a difference of 42 between solutions.
   Reason: The solutions must be multiples of 21 **and** must be even numbers. When you add 21 to an even number such as 42, you get an odd number (42 + 21 = 63). Hence, the solutions must be 42 (21 + 21) numbers apart.

3. Solutions: 14; 34; 54; 74; 94
   Pattern: There is a difference of 20 between solutions.
   Reason: There is a difference of ten between numbers ending in 4 (14, 24, 34, 44, 54,...). Every other number from this list, however, 4 is divisible by 4 (24, 44, 64,...). Hence, the solutions are 20 numbers apart.

4. Solutions: 36; 54; 72; 90
Pattern: Solutions are 18 numbers apart.
Reason: Numbers divisible by 3, 6, and 9 are multiples of 18. The least common multiple (LCM) of 3, 6, and 9 is 18.

5. Solutions: 60; 120; 180
Pattern: Solutions are 60 numbers apart.
Reason: Numbers divisible by 2, 3, 4, and 5 are multiples of 60. The LCM of 2, 3, 4, and 5 is 60.

6. Solutions: 117; 135; 153; 171; 189
Pattern: Solutions are 18 numbers apart.
Reason: The solutions must be multiples of 9 **and** must be odd numbers. Multiples of 9 alternate between odd numbers and even numbers (9, 18, 27, 36, 45, 54,...). Hence, the odd solutions are 18 numbers apart.

7. Solutions: 121; 127; 131; 137; 139; 143; 149
Pattern: No apparent pattern.

8. Solution: 240
Pattern: Not applicable.

9. Solution: 120; 180
Pattern: Solutions are 60 numbers apart.
Reason: The solutions must be multiples of 4, 5, and 6. The LCM of 4, 5, and 6 is 60. Hence, the solutions should be 60 numbers apart.

10. Solutions: 5208; 5220; 5232; 5244; 5256; 5268; 5280; 5292
Pattern: Solutions are 12 numbers apart.
Reason: The solutions must be multiples of 3, 4, and 6. The LCM of 3, 4, and 6 is 12. Hence, the solutions should be 12 numbers apart.

11. Solutions: 301; 329; 343; 399
Pattern: The pattern is rather sporadic.
Reason: The solutions are multiples of 7 that are odd numbers. Odd multiples of 7 in the 300s should be 14 numbers apart: 301, 315, 329, 343, 357, 371, 385, 399. Several of these numbers are deleted because of the other conditions: can't have a 7 in it and can't end in a 5 or 0.

12. Solutions: 629; 647; 683
Pattern: No apparent pattern.

Activity 1, Transparency 1

Divisibility Rules!

Use a calculator (when needed) to discover **all** the possible mystery numbers that satisfy the given conditions, and

If there are multiple solutions, describe a pattern in the answers and explain why that pattern exists.

1. I am a 3-digit number less than 150 that is divisible by 8. Who am I?

Solutions: 104; 112; 120; 128; 136; 144

Pattern: Solutions are 8 numbers apart.

Reason: The solutions must be divisible by 8. Numbers divisible by 8 are 8 units apart.

Activity 1, Transparency 2

Divisibility Rules!

Use a calculator (when needed) to discover **all** the possible mystery numbers that satisfy the given conditions, and

If there are multiple solutions, describe a pattern in the answers and explain why that pattern exists.

2. I am an even 2-digit number that is divisible by 3 and 7. Who am I?

Solutions: 42; 84

Pattern: Solutions are 42 numbers apart.

Reason: The solutions must be multiples of 21 and must be even numbers. When you add 21 to an even number such as 42, you get an odd number (42 + 21 = 63). Hence, the solutions must be 42 numbers apart.

Activity 1

Divisibility Rules!

Name _____

Date _____

For each problem below:

- Use a calculator (when needed) to discover **all** the possible mystery numbers that satisfy the given conditions, and
- If there are multiple solutions, describe a pattern in the answers and explain why that pattern exists.

Reminder Box: Useful Divisibility Rules
A number is divisible by 2 if it is an even number.
A number is divisible by 3 if the sum of its digits are divisible by 3.
A number is divisible by 4 if its last two digits are divisible by 4.
A number is divisible by 5 if its last digit (ones place) is a zero or a five.
A number is divisible by 6 if it is divisible by both 2 and 3.
A number is divisible by 9 if the sum of its digits is divisible by 9.
A number is divisible by 10 if its last digit is a zero.

1. I am a 3-digit number less than 150 that is divisible by 8. Who am I?
   Solution(s): _____
   Pattern: _____
   Reason for Pattern: _____
2. I am an even 2-digit number that is divisible by 3 and 7. Who am I?
   Solution(s): _____
   Pattern: _____
   Reason for Pattern: _____
3. I am a 2-digit number that ends in a 4 but is not divisible by 4. Who am I?
   Solution(s): _____
   Pattern: _____
   Reason for Pattern: _____
4. I am a 2-digit number greater than 20 that is divisible by 3, 6, and 9. Who am I? Solution(s): _____
   Pattern: _____
   Reason for Pattern: _____

5. I am a number that is divisible by 2, 3, 4, and 5 and is less than 200.

   Solution(s): _____

   Pattern: _____

   Reason for Pattern: _____

6. I am an odd number between 100 and 200 that is divisible by 9. Who am I?

   Solution(s): _____

   Pattern: _____

   Reason for Pattern: _____

7. I am an odd number between 120 and 150 that is not divisible by 3, 5, 7, or 9. Who am I? Solution(s): _____

   Pattern: _____

   Reason for Pattern: _____

8. I am a 3-digit number that has a 2 in the hundreds place and is divisible by 2, 3, 4, and 5. Who am I? Solution(s): _____

   Pattern: _____

   Reason for Pattern: _____

9. I am a multiple of 5 between 100 and 200 that is divisible by 4 and 6. Who am I? Solution(s): _____

   Pattern: _____

   Reason for Pattern: _____

10. I am a 4-digit number between 5200 and 5300 that is divisible by 3, 4, and 6. Who am I? Solution(s): _____

    Pattern: _____

    Reason for Pattern: _____

11. I am a 3-digit odd number that has a 3 in the hundreds place, is divisible by 7, is not divisible by 5, and has no sevens in it. Who am I?

    Solution(s): _____

    Pattern: _____

    Reason for Pattern: _____

12. I am a number that is greater than 600 but less than 700. The sum of my digits is 17. I am not divisible by 2, 3, 5, or 7. Who am I?

    Solution(s): _____

    Pattern: _____

    Reason for Pattern: _____