2.1 Land Sections: Writing Addition and Subtraction Sentences

Goals

- Use number sentences to express sums and differences
- Develop strategies for adding and subtracting fractions
- Explore the use of fractions as operators (e.g. $\frac{2}{3}$ of 640 acres)

This problem engages students with an area model for fractions. The students are first asked to name what fraction of a section each person owns. Next students explore combining and removing parts of the section. In addition, each section is equivalent to 640 acres. This allows for questions that show the relative size of parts of the sections in another way—number of acres owned.

In this investigation and the ones that follow, students will be asked to write number (or mathematical) sentences to represent and symbolize situations. During the explore phase of the lesson, pay attention to the students' notation in their work. The summary is also an opportunity to help the students make a connection between their mental computations and number sentences that symbolize what they have done.

Mathematics Background

For background on writing number sentences, see page 4.

Launch 2.1

Before starting Problem 2.1, use the introduction to Investigation 2 in the Student Edition to review what a number sentence is. Make the students aware that there are several names that will be used to indicate that they are to write a number sentence. These might include: mathematical sentence, addition sentence, subtraction sentence, multiplication sentence, and division sentence.

Suggested Questions One way to launch the problem is to have a conversation with your students about naming amounts with fractions. Pose questions like the following:

- *How many sections of land are being discussed in this problem?* (2)
- *How many acres are in a section?* (640)
- *Does anyone own a whole section?* (No. That means everyone owns a fraction that is less than 1.)
- *Who owns the largest piece of a section?* (Maybe Foley or Walker.)
- *About how much does Foley own?* (Between $\frac{1}{4}$ and $\frac{1}{2}$ of a section, maybe $\frac{1}{3}$.)
- *About how much does Burg own?* (A little less than $\frac{1}{4}$ of a section, maybe $\frac{1}{5}$.)
- *What would be a reasonable estimate for Burg + Foley?* (about $\frac{1}{2}$)
- *How much land does Lapp own?* ($\frac{1}{4}$)
- *What do you think it means in Question B when it says to write a number sentence? Can you give an example?* (Students will not know the exact fractional values, but you can use words to write a model and explain that they will replace the words with fractions once they figure out what these values are. For example: Fuentes + Theule = ■.)

Provide students with a copy of Labsheet 2.1. Extra copies of the map are recommended for students who go off in the wrong direction and are too far along to restart on the same sheet. This problem works well in a Think-Pair-Share arrangement. Give the students time to tackle the problem themselves before they share with a partner or a larger group.

Explore 2.1

You may want to check that students have accurately labeled the sections in Question A before they go on to use those values in the rest of the problem. Help students who are struggling to write number sentences. It is often helpful for students to write a number sentence with the names of the landowners, and then substitute numerical values. For example, Bouck + Lapp = Foley is $\frac{1}{16} + \frac{1}{4} = \frac{5}{16}$.

If groups get done early, you can have them show their solution on a transparency of the property map so that it can be shared easily with the class during the summary.

Summarize 2.1

Start the summary by asking the students to discuss how they found the fractions that represent each person's part of land. Any group who did their work on a transparency of the map could show their work and discuss the strategy they used. It is not uncommon to have many different, but equivalent values for each person's share of land.

Suggested Questions Depending on the strategy that students use, several equivalent fractions may be offered for the same landowner. Help students reconcile the different fractions.

- *How did you get $\frac{1}{4}$ for Lapp's part of Section 18?* (If you divide the section into four parts, you can see that Lapp takes up one of the parts.)
- *Did anyone get other fractions for Lapp's part of Section 18?*
- *Lisa said that Lapp owned $\frac{1}{4}$ of Section 18 and Heidi said Lapp owned $\frac{16}{64}$ of Section 18. Who is correct? Why?* (Both are correct. The fractions are equivalent. Students may use diagrams to show this or they may talk about renaming fractions as proof that they are equivalent.)

The summary should include an exploration of the strategies that students use to combine quantities. Have students talk about why their number sentences make sense. Continue to emphasize the role of equivalence when discussing the number sentences that students wrote. This provides another chance to help students focus on the power of equivalent representation of fractions and on the area model for fractions. The reasoning that is required is important to understand the computations.

Here is a conversation that took part in a classroom when talking about Question C.

Classroom Dialogue Model

Teacher *Would one group begin by sharing a number sentence they wrote for Question C and explain why it makes sense?*

Cody *We did Section 19 + Lapp + Gardella + Bouck = $1\frac{1}{2}$. For the number sentence, we wrote $1 + \frac{1}{4} + \frac{3}{16} + \frac{1}{16} = 1\frac{1}{2}$.*

Teacher *How do you know those equal $1\frac{1}{2}$?*

Cody *If you look at the map, you can fit Bouck's land into the upper right corner of Gardella's section. This will fill up half of Section 18.*

Trista *We used the same people but wrote the number sentence $1 + \frac{4}{16} + \frac{3}{16} + \frac{1}{16} = 1\frac{8}{16} = 1\frac{1}{2}$. If you rename Lapp's $\frac{1}{4}$ as $\frac{4}{16}$, it is easy to add the 16ths and get $\frac{8}{16}$ or $\frac{1}{2}$.*

Teacher *How do you know that $\frac{1}{4}$ equals $\frac{4}{16}$?*

Charlie *Because we know that if you multiply or divide the numerator and denominator by the same number you get an equivalent fraction. For $\frac{1}{4}$, we multiplied the numerator and denominator by 4 to get $\frac{4}{16}$.*

Cody *Why did you do that?*

Anne *Well, the denominators tell us the sizes of the pieces, and because the denominators are the same, we know that for these two fractions, the sizes of the pieces are the same. But we have different amounts of pieces, because the numerators are different, and it is the numerators that tell us how many pieces we have. So, with all the pieces in 16ths, we can easily add the number of pieces and see how many 16ths there are.*

Teacher *So your strategy was to use equivalent fractions to rename the pieces with the same denominators so the pieces were the same size and you could easily tell how many there were all together?*

Anne *Yes, it is easy to add up each person's land when the fractions have the same denominator.*

Suggested Questions After a group has presented their strategy, ask the class questions like the following:

- *What do others think about this group's strategy? Does it seem reasonable?*
- *Did anyone have a different answer or use a different strategy?*

As you move into the additional parts of the problem, be sure to have students share their number sentences.

Have a few students show their strategies for solving Question E. Here we are foreshadowing multiplication with fractions by having students find a fraction of a number other than 1. This is in contrast to their work in Question A where they found a fractional part of a whole (1).

Question F is a practice problem and can be assigned as homework if time is short. Note that this is an important problem to discuss. This problem uses situations where subtraction may be a useful strategy. For students who do use subtraction, ask them how they would write a number sentence to show their solution.

Suggested Questions

- *How do you decide if addition or subtraction will help solve a problem?*
- *What strategies help you add or subtract fractions?*

2.1 Writing Addition and Subtraction Sentences

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Use number sentences to express sums and differences
- Develop strategies for adding and subtracting fractions
- Explore the use of fractions as operators (e.g., $\frac{2}{3}$ of 640 acres)

Launch

Use the introduction to Investigation 2 in the Student Edition to review number sentences. Read Problem 2.1.

- *How many sections of land are being discussed in this problem?*
- *Does anyone own a whole section?*
- *Who owns the largest piece of a section?*
- *What do you think it means in Question B when it says to write a number sentence? Can you give an example?*

Provide a copy of Labsheet 2.1. Use a Think-Pair-Share grouping arrangement.

Materials

- Transparency 2.1
- Labsheet 2.1 (1 per student but have extra copies for students who need to start again)

Vocabulary

- number sentence

Explore

You may want to check that students have accurately labeled the sections in Question A before they go on to use those values in the rest of Problem 2.1. It is helpful for students to write number sentences with the names of the landowners, and then substitute numerical values. You might have groups put their solutions on a transparency of the property map to share in the summary.

Summarize

As students share their solutions for Questions A–D, ask questions like the following:

- *How did you get $\frac{1}{4}$ for Lapp?*
- *Did anyone get other fractions for Lapp's part of Section 18?*
- *Lisa said that Lapp owned $\frac{1}{4}$ of Section 18, and Heidi said that Lapp owned $\frac{16}{64}$ of Section 18. Who is correct? Why?*

As students share strategies, have them talk about why their number sentences make sense. Continue to emphasize the role of equivalence when adding and subtracting fractions.

- *How do you know that those values together equal $1\frac{1}{2}$?*
- *What do others think about this group's strategy?*

Materials

- Student notebooks

continued on next page

Summarize

continued

- *Did anyone have a different answer or use a different strategy?*

Question E foreshadows multiplication with fractions by having students find a fraction of a number other than 1.

Question F is practice and can be assigned as homework. Note that this is an important problem to discuss.

- *How do you decide if a problem is addition or subtraction?*
- *What strategies help you add or subtract fractions?*

ACE Assignment Guide for Problem 2.1

Differentiated Instruction
Solutions for All Learners

Core 2, 29

Other *Applications* 1, *Connections* 28

Labsheet 2ACE Exercise 1 is provided if Exercise 1 is assigned.

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 2.1

A. Lapp: $\frac{1}{4}$; Bouck: $\frac{1}{16}$; Wong: $\frac{3}{32}$; Stewart: $\frac{5}{32}$; Krebs: $\frac{1}{32}$; Fitz: $\frac{5}{32}$; Gardella: $\frac{3}{16}$; Fuentes: $\frac{1}{16}$; Foley: $\frac{5}{16}$; Theule: $\frac{3}{16}$; Burg: $\frac{3}{16}$; Walker: $\frac{5}{16}$.

B. Fuentes + Theule = $\frac{4}{16}$; $\frac{1}{16} + \frac{3}{16} = \frac{4}{16}$

C. 1. Possible answers: All of section 19 + Lapp + Gardella + Bouck, or

$1 + \frac{4}{16} + \frac{3}{16} + \frac{1}{16} = 1\frac{1}{2}$;

All of section 18 + Foley + Theule, or

$1 + \frac{5}{16} + \frac{3}{16} = 1\frac{1}{2}$.

2. See part (1).

D. 1. $\frac{1}{16} + \frac{1}{4} = \frac{5}{16}$. To show this is true, you can rewrite Lapp's section as $\frac{4}{16}$.

$\frac{1}{16} + \frac{4}{16} = \frac{5}{16}$.

2. Possible answer: Bouck + Wong = Stewart or $\frac{1}{16} + \frac{3}{32} = \frac{5}{32}$.

3. Possible answer: Burg + Fuentes + Bouck = Walker or $\frac{3}{16} + \frac{1}{16} + \frac{1}{16} = \frac{5}{16}$.

E. Lapp: 160 acres; Bouck: 40 acres; Wong: 60 acres; Stewart: 100 acres; Krebs: 20 acres; Fitz: 100 acres; Gardella: 120 acres; Fuentes: 40 acres; Foley: 200 acres; Theule: 120 acres; Burg: 120 acres; Walker: 200 acres

Possible explanation: Each section can be broken into 32 parts, so each part is 20 acres. Then Lapp has 8 of the 32 parts, or 160 acres; Bouck has 2 parts, or 40 acres; Wong has 3 parts, or 60 acres; and so on.

F. 1. $\frac{21}{32}$; Lapp + Gardella + Fuentes + Fitz = $\frac{1}{4} + \frac{3}{16} + \frac{1}{16} + \frac{5}{32} = \frac{21}{32}$

2. $\frac{11}{32}$; Wong + Stewart + Krebs + Bouck = $\frac{3}{32} + \frac{5}{32} + \frac{1}{32} + \frac{1}{16} = \frac{11}{32}$, or $1 - \frac{21}{32} = \frac{11}{32}$

3. Lapp by $\frac{10}{32}$; $\frac{21}{32} - \frac{11}{32} = \frac{10}{32}$