The Rule of 72

By John F. Mahoney

Authors of personal finance books tout the "Rule of 72" as a way of determining how quickly an investment will double at a certain interest rate. For example, to find out approximately how long it will take an investment to double at a compounded growth rate of 5%, divide 72 by 5 to get 14.4 years.

How good is this rule? We can use our graphing calculator to find out. Pre-calculus and calculus students know that the formula for computing the balance after *t* years of an initial investment of *$P* compounded continuously at an annual rate of r can be calculated using the formula: $B = Pe^{rt}$
Since we are interested in doubling the funds, we can let B = 2P and calculate *t* in terms of *r* :

$$2P = Pe^{rt} \quad \leftrightarrow \quad 2 = e^{rt} \quad \leftrightarrow \quad 1n2 = rt \quad \leftrightarrow \quad \frac{1n2}{r} = t$$

In 2 is approximately *.693147* which implies that it would be more accurate to reconfigure the rule as the "Rule of 69." Why, then, are people advised to use the "Rule of 72?" I suspect that the reason is that *72* has a lot more factors than *69* does and therefore the division is easier using *72* rather than *69*. Students with access to the TI-89 or TI-92 can use their calculator to get the above results shown below:

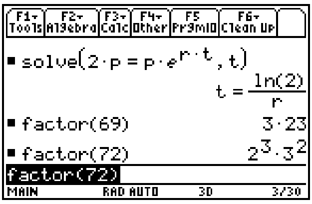

Figure 1

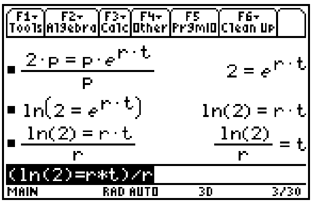

Figure 2

From a teaching point of view, however, it is more useful to use the calculator to explicitly show each step of the derivation. In Figure 2 we first divide both sides of the equation by *p*. Then we take the natural log of both sides. Finally we divide both sides by *r* in order to solve for *t*.

How good then, is the "Rule of 72?" I made a table of values on my TI-83. In L_1 I used the command seq*(x, x, 2, 15, 1)* to get some typical interest rates. In L_2 I used the command "ln(2)*100/L1" to compute the exact doubling times as shown in Figure 3. I graphed the L_2 vs. L_1 in Figure 4.

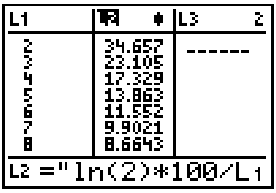

Figure 3

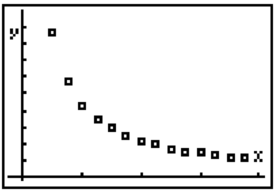

Figure 4

I then graphed (Figure 5) the equation $Y_2 = 72/x$ to see how close it came to the values of the scatter plot. We can see from the graph that while the "Rule of 72" does not really accurately model the data, it is nonetheless pretty good. We can quantify how good the model is by setting L_3 ="$Y_2(L_1) - L_2$" (see Figure 6) and observe that the error in using the Rule of 72 is less than a year if the interest rate is at least 3%.

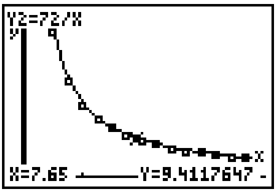

Figure 5

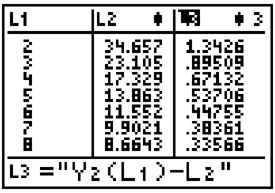

Figure 6