

## Forces on Point Charges – ID: 9359

Time required  
45 minutes

Topic: Electricity and Magnetism

- Use Coulomb's law to solve problems of force between electric charges.
- Solve problems involving point charges and force using vector addition.

Activity Overview

In this activity, students explore interaction between charged point particles. They first explore graphical vector addition and then use vector addition rules to explore the net forces on charged particles. The preconstructed templates used in this activity include charged particles, represented by points, and forces of interactions between the particles, represented by vectors. When students change magnitudes of charges or distances between the particles, the vectors representing the forces of interaction change correspondingly.

Materials

To complete this activity, each student will require the following:

- TI-Nspire™ technology
- pen or pencil
- copy of student worksheet

TI-Nspire Applications

*Graphs & Geometry, Notes*

Teacher Preparation

Before carrying out this activity, you should review Coulomb's law with students. Students should also be comfortable with the concept of a vector and the rules of vector addition before beginning this activity.

- The screenshots on pages 2–9 demonstrate expected student results. Refer to the screenshots on page 10 for a preview of the student TI-Nspire document (.tns file).
- **To download the student worksheet and .tns file, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter "9359" in the search box.**

Classroom Management

- This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students should record their answers to the questions on notebook paper. If time allows, a whole-class discussion of the activity would be useful for the students.
- The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.
- In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.

The following questions will guide student exploration during this activity:

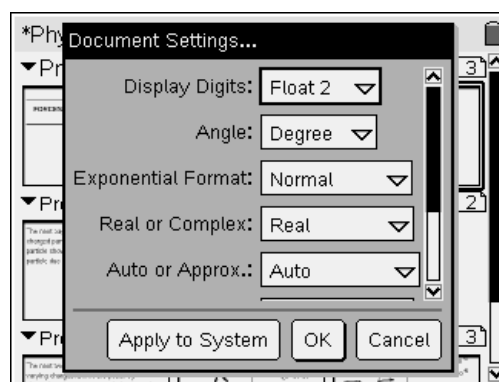
- How do the positions and charges of particles affect the forces acting on the particles?
- How can a preconstructed scenario be used to solve electrostatics problems?

The purpose of this activity is to provide students with an opportunity to explore the interactions between charged particles when three or more particles are present in the system. TI-Nspire technology provides students with a dynamic environment for explorations and tools for graphical and numerical analysis for a variety of situations.

This activity consists of three parts. The preconstructed templates can also be used by the students in and out of class to verify solutions of various problems dealing with point charges and forces between them.

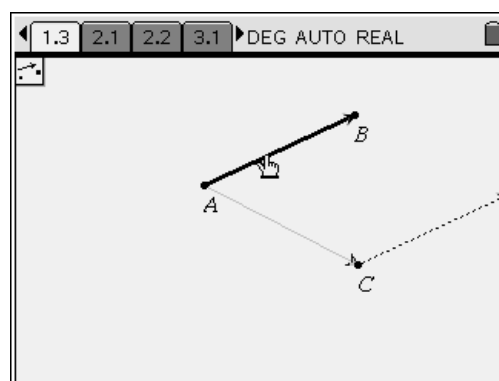
### Problem 1 – Graphical vector addition

**Step 1:** In this part of the activity, students use TI-Nspire features to practice graphical methods of vector addition. First, students should open the file **PhysWeek05\_PointCharges.tns** and read the first two pages. They should then proceed to page 1.3, which contains an empty *Graphs & Geometry* page. Students should change the page to the Plane Geometry view (**Menu > View > Plane Geometry View**) and hide the scale (**Menu > View > Hide Scale**). It is recommended that students change the document settings for the angle measurements to be in degrees for this activity. To change the document settings, students should press **ctrl** to enter the page sorter view, press **menu** to select the menu, and then select **Document Settings**.



**Step 2:** Next, students use the **Vector** tool (**Menu > Points & Lines > Vector**) to draw two coterminal vectors, AB and AC, on page 1.3.

**Step 3:** Next, students should use the **Translation** tool (**Menu > Transformations > Translation**) to translate vector AB to point C. They should label the image of point B as D using the **Text** tool.



**Step 4:** Next, students construct vector AD, the sum of AB and AC. They then find the magnitude and direction of vector AD using the **Length** and **Slope** measurement tools from the **Measurement** menu. They should then answer questions 1 and 2.

**Q1.** How does the length of vector AD compare with the lengths of vectors AB and AC when AB and AC are collinear?

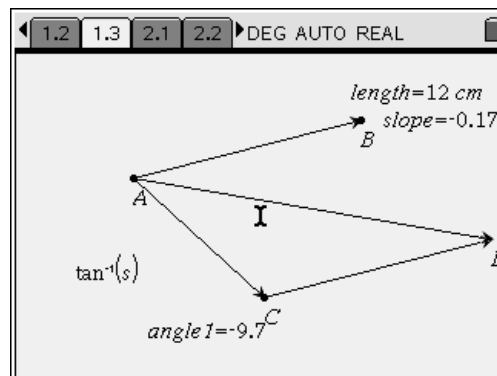
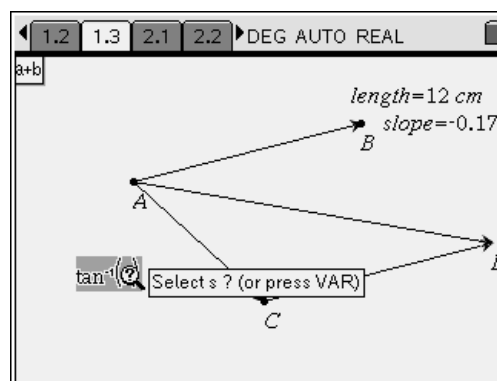
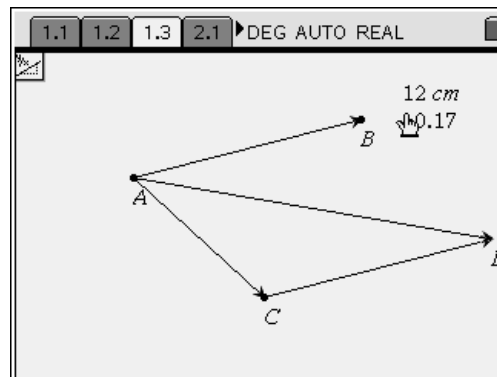
**A.** When AB and AC are parallel, AD is equal to the sum of the lengths of AB and AC.

**Q2.** How does the direction (slope) of vector AD compare with the slopes of vectors AB and AC?

**A.** The slope of AD is partway between the slopes of AB and AC. Students can use the **Slope** measurement tool to observe the direction of the resultant vector, or they can find the angle of the vector to the horizontal. Since the slope of the line is equal to the inverse tangent of the angle, students can use the **Text** and **Calculate** tools to find the angle of the resultant vector. The instructions for using this method are given below:

- Choose the **Text** tool (**Menu > Actions > Text**).
- Click on a blank space anywhere on the screen. The text box will open in the edit mode.
- Enter the expression  $\tan^{-1}(s)$  and press  $\text{enter}$ .
- Choose the **Calculate** tool (**Menu > Actions > Calculate**).
- Click on the expression. When Select s? appears on the screen, move the cursor and click on the value of the slope. Press  $\text{enter}$  and the value of the slope should appear. Drag the expression to the desired location and press  $\text{enter}$  again.

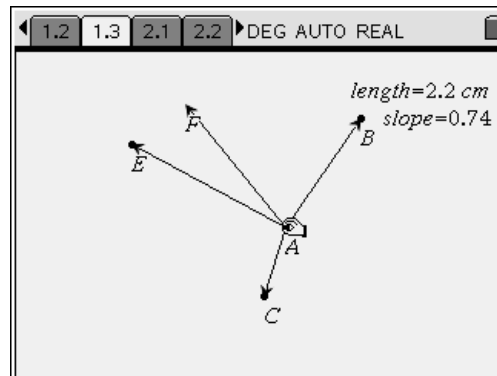
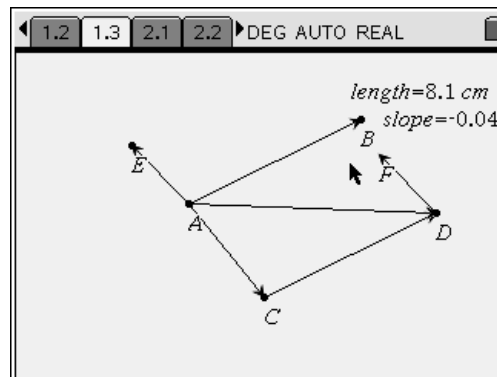
Remind students that, when the angle is in the second or third quadrant, they need to add  $180^\circ$  to the angle calculated using the inverse tangent.



**Step 5:** Next, students explore the addition of three coterminal vectors. They first construct vector AE, which should point in a different direction than vector AB, AC, or AD. Then, they use the **Translation** tool to add vector AE to vector AD. They label the endpoint of this vector *F* and construct vector AF, which is the sum of vectors AB, AC, and AE.

**Step 6:** Next, students explore how vector AF changes as they move points A, B, C, and E around the screen. They should then answer question 3 on the student worksheet.

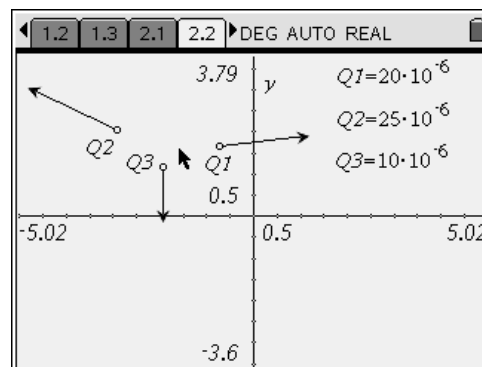
- Q3.** Describe the lengths and positions of vectors AB, AC, and AE that are required to make AF become as small as possible.
- A.** *There are several combinations of vectors AB, AC, and AE that will make vector AF extremely small. Encourage students to compare their results with one another and discuss the different solutions.*



**Problem 2 – Interaction between three similarly charged particles**

Next, students should move to page 2.1 and read the text there. Page 2.2 shows three positively charged particles. The vectors attached to each point show the forces on that particle that are produced by the other two particles. Students can drag each point and thus vary the distances between the particles. The numbers on the right-hand side of the screen give the magnitudes of the charges on the particles. Students can click on these labels and vary the magnitudes of the charges. The charges are given in units of coulombs (C), and distances are in meters.

**Step 1:** Next, students should find the net force on each particle using head-to-tail vector addition. They should then hide the individual force vectors on the particles, so only the net forces are visible, as shown to the right.



**Step 2:** Next, students should vary the locations and charges of points Q1, Q2, and Q3 and observe the results. Then, they should answer questions 4–9 on the student worksheet.

**Q4.** Describe the changes you observed in the net force on each particle when you varied magnitudes and positions of the particles. Did you notice any patterns?

**A.** *Due to the open-ended nature of the question, many different responses are possible. However, students should note that the net forces on the particles decreased as the particles got farther apart and increased as they got closer together. Students should also note that the net forces on the particles are directly related to the charges on the particles (i.e., the larger the charges, the larger the net forces).*

**Q5.** What happens to the net force on a particle when its charge is much larger or smaller than the charges on the other two particles?

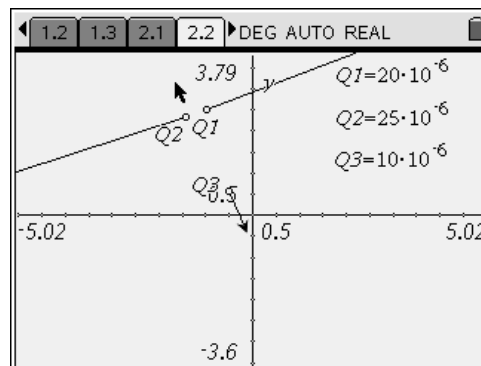
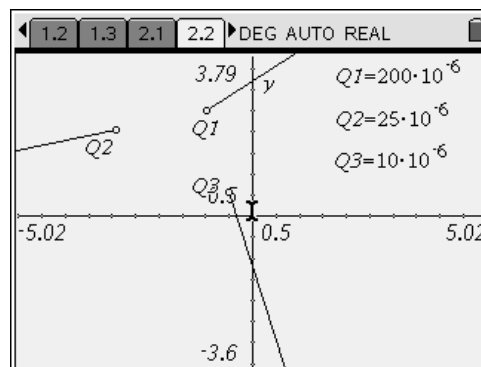
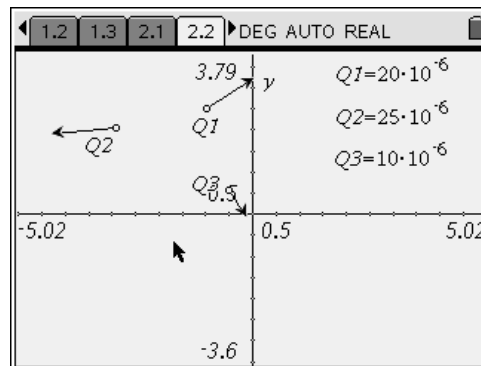
**A.** *When the charge on one particle is much larger than the charges on the other particles, the net forces on all three particles increase significantly. As the charge on one particle approaches zero, the net force on that particle also approaches zero. Note that students may think that the force on a particle can never go to zero if the other two particles have non-zero charge. Guide them to understand that electric forces affect only charged particles, so a particle with a zero charge will experience no net force.*

**Q6.** What happens to the net force when one of the particles moves very close to one of the other particles?

**A.** *The forces on the particles become very large.*

**Q7.** What happens to the net force on a particle when it is moved far away from the other two particles?

**A.** *The forces on the particles become very small.*



**Q8.** What happens to the net force on a particle when it is located exactly between two equally charged particles?

**A.** *The net force on the central particle is zero because the vectors acting on the central particle have equal magnitudes but opposite directions.*

**Q9.** Three particles, each with a charge of  $+11 \mu\text{C}$ , are located at the corners of an equilateral triangle with sides of length 1 m. Calculate the magnitude and direction of the net force on each particle. (Hint: Use the template on page 2.2, adjust the scale if needed, and reproduce the conditions of the problem. Then calculate magnitude and angle for each net force.)

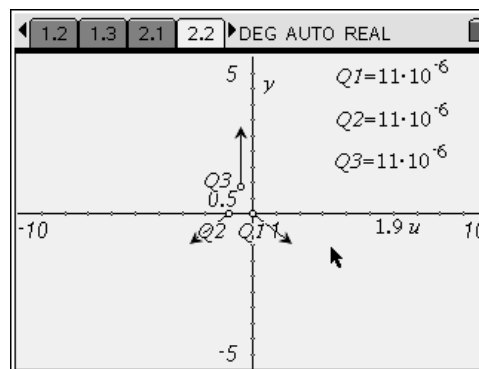
**A.** *Students can use the template on page 2.2 to solve this problem graphically. They can place the three charges, for example, on the points*

*$(-1, 0)$ ,  $(0, 0)$ , and  $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ . They can then*

*display the coordinates of points Q1, Q2, and Q3 using the **Coordinates and Equations** tool (**Menu > Actions > Coordinates and Equations**). They can use the **Length and Slope** measurement tools to determine the magnitudes and directions of the resulting forces. They should find that the magnitude of the force on each particle is 1.9 N and the directions are as follows:*

*For  $(0, 0)$ ,  $\theta = -30^\circ$ ; for  $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ ,  $\theta = 90^\circ$ ; and*

*for  $(-1, 0)$ ,  $\theta = 210^\circ$ . Students will probably need to adjust the scale of the graph (**Menu > Window > Window Settings**) to see the net forces clearly. Students should also calculate the magnitudes and directions analytically, using Coulomb's law and vector addition rules. Coulomb's law yields the net force on each particle, as shown on the next page (make sure students convert  $\mu\text{C}$  to C):*



$$F_c = 2k \frac{q^2}{r^2} \cos(30^\circ)$$

$$F_c = (2)(9 \times 10^9 \frac{N \cdot m^2}{C^2}) \left( \frac{11 \times 10^{-6} C}{1 m} \right)^2 (0.866)$$

$$F_c = 1.9 N$$

Students can use the law of cosines to determine the direction of each vector.

**Problem 3 – Interaction between particles of opposite charge**

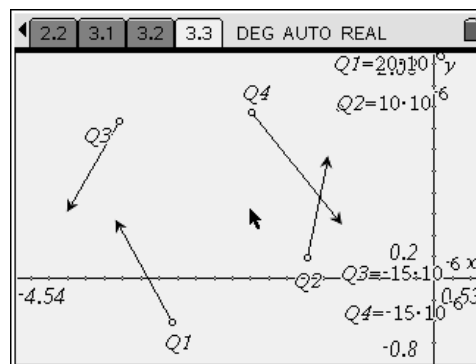
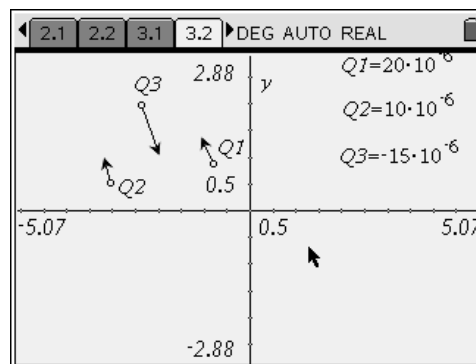
In this part of the activity, students explore the interactions between positively and negatively charged particles. Note that in each template, the sign of the charge cannot be changed. Students should read page 3.1 before moving on to the simulations. (Note: To save time, you may construct the net force vectors for this problem ahead of time in the .tns file, before transferring the file to the students' handhelds or computers.)

**Step 1:** Students should first move to page 3.2, which shows two positively charged particles and one negatively charged particle. They should use head-to-tail vector addition to find the net force on each particle, and then hide the individual force vectors on each particle so only the net forces are visible. They should change the positions and charges of the particles and observe the results.

**Step 2:** Next, students should move to page 3.3, which shows two positively charged particles and two negatively charged particles. They should again find the net force on each particle and hide the component forces. After exploring the effects of position and charge on these forces, students should answer questions 10–14 on the student worksheet.

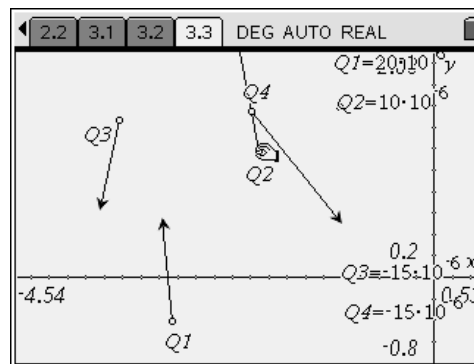
**Q10.** What happens to the net forces when oppositely charged particles move very close to one another?

- A. *The magnitudes of forces on the oppositely charged particles increase significantly and approach infinity. Their directions become opposite. At the same time, the net force on the third particle approaches zero.*



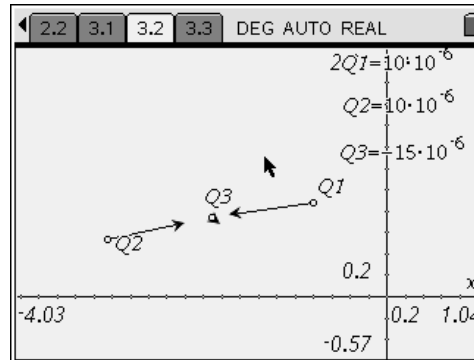
**Q11.** What happens to the net force on a negatively charged particle when it is located exactly between two particles of equal positive charge?

- A. *When a negatively charged particle is located exactly between two equally positively charged particles, the net force on the negatively charged particle is zero because the two force vectors acting on the central particle have equal magnitudes but opposite directions.*



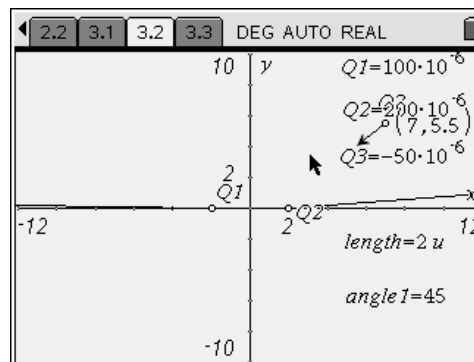
**Q12.** Explore other symmetrical arrangements of the particles, and describe any patterns that you observe.

- A. *Encourage students to explore different symmetrical arrangements of particles and discuss their results with the class. Encourage them to predict the net forces resulting from various symmetrical arrangements, and then use the templates on pages 3.2 and 3.3 to test their predictions.*



**Q13.** A particle of charge  $+100 \mu\text{C}$  is located at  $(-2, 0)$ , and a particle of charge  $+200 \mu\text{C}$  is located at  $(2, 0)$ . Where should a negatively charged particle with charge  $-50 \mu\text{C}$  be placed so that the net force on this particle has a magnitude of  $2 \text{ N}$  and is directed at  $-135^\circ$ ? (Hint: You can see the coordinates of a point by choosing **Menu > Actions > Coordinates and Equations** and then clicking on the point.)

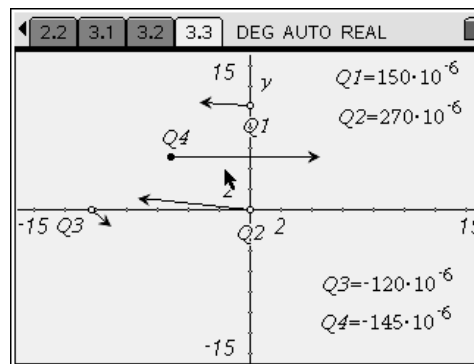
- A. *Two possible coordinates are  $(7, 5.6)$  and  $(-0.385, 0.082)$ . This particular problem is too advanced for analytical solution. It is sufficient if students verify their graphical solution by calculations. In order to do that, they will need to measure the magnitudes and angles of all forces and show that the net force in the system is zero. Note: If students struggle with this problem, you may first ask them to explore a simpler problem, such as determining the magnitude and direction of the net forces on the particles when they are placed at specific locations.*





**Q14.** A right isosceles triangle is formed by the charges  $Q_1 = +150 \mu\text{C}$ ,  $Q_2 > 0$ , and  $Q_3 = -120 \mu\text{C}$  located at points  $(0,10)$ ,  $(0, 0)$ , and  $(-10, 0)$ , respectively. The fourth charge,  $Q_4 = -145 \mu\text{C}$ , is located at the midpoint of the hypotenuse. The net force on charge  $Q_4$  is 10 N in the positive  $x$ -direction. What is the charge on  $Q_2$ ?

- A.** *The charge on particle  $Q_2$  should be approximately  $+270 \mu\text{C}$ . Once again, students should verify that the net force in the system is zero after they find their solution. If you wish, you may assign additional problems or explorations to student pairs or groups for homework or independent projects.*



# Force on Point Charges – ID: 9359

(Student)TI-Nspire File: *PhysWeek05\_PointCharges.tns*

<p><b>FORCES ON POINT CHARGES</b></p> <p><b>Physics</b></p> <p>Electrostatics</p>	<p>Forces, like all vectors, have magnitudes and directions. Operations with vectors have their own rules. In this part of the activity, you will learn how to use different tools of TI-Nspire technology to add vectors and to find their magnitudes and directions. Refer to your worksheet for directions for this part of the activity.</p>	
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<p>The next page shows three positively charged particles. The vectors on each particle show the repulsive forces on that particle due to the other particles.</p>		<p>The next two pages show particles with varying charges. Some are positively charged, and some are negatively charged. The vectors on each particle show the attractive or repulsive forces on that particle due to the other particles.</p>
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# Forces on Point Charges

ID: 9359

Name \_\_\_\_\_

Class \_\_\_\_\_

*In this activity, you will*

- explore the effect of charge and distance on interaction of three or more charged particles
- solve problems involving finding net forces of interaction between three or more charged particles

Open the file **PhysWeek05\_PointCharges.tns** on your handheld or computer, and follow along with your teacher for the first two pages. Move to page 1.2 and wait for further instructions from your teacher.

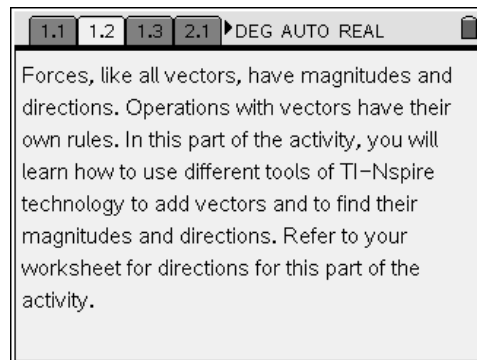
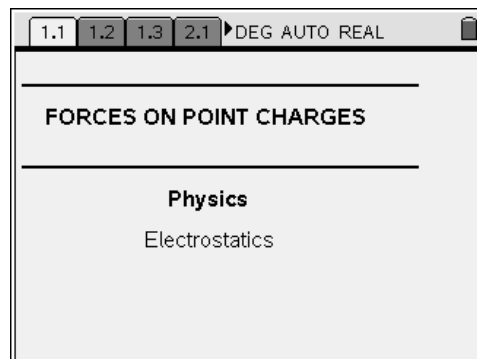
Remember that Coulomb's law describes the force between two point charges, as follows:

$$F = k \frac{qQ}{r^2}$$

In this equation, *k* is defined as Coulomb's constant

( $k = 9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ ), *q* and *Q* are the magnitudes of the charged particles, and *r* is the distance between the particles.

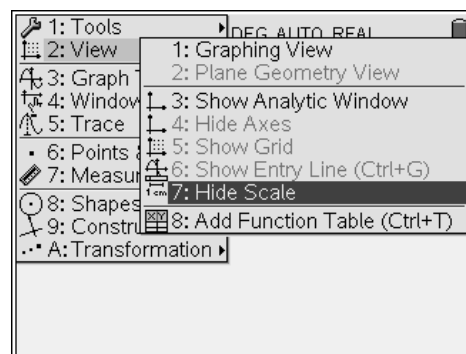
In a system of three or more charged particles, each pair of particles interacts according to Coulomb's law. When two or more forces act on a particle, all charges and distances affect the strength and direction of the force of interaction. In order to explore this effect, we need to be able to determine this net force of interaction on a given particle due to all charged particles. Since forces are vectors, you will need to use vector addition rules.



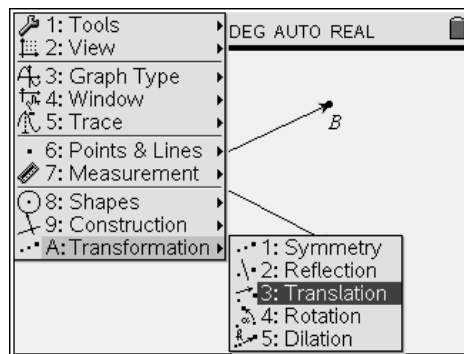
## Problem 1 – Graphical vector addition

**Step 1:** Page 1.3 contains a blank *Graphs & Geometry* page. Change the view of this page to Plane Geometry view (**Menu > View > Plane Geometry View**) and hide the scale (**Menu > View > Hide Scale**).

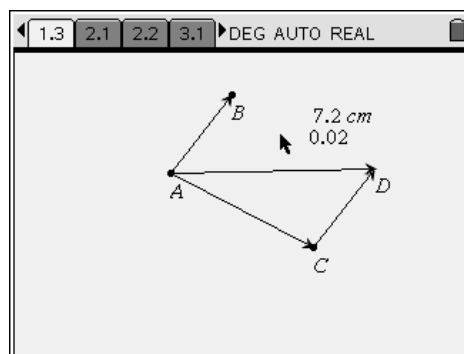
**Step 2:** Draw two vectors, AB and AC, using the **Vector** tool (**Menu > Points & Lines > Vector**). To draw vector AB, first select the **Vector** tool. Then, click once on any point on the page. Press  $\text{CAPS} \text{ } \text{A}$  to label the starting point of the vector A. Then, drag the cursor to another point, single-click again, and label the ending point of the vector B. To draw vector AC, click again on point A and create another vector that points in another direction. Label the endpoint of this vector C.



**Step 3:** Next, you will explore the head-to-tail rule of vector addition. According to this rule, you can add two vectors by placing the head of one vector at the tail of the second. The sum of the vectors is then the vector from the tail of the first vector to the head of the second vector. To explore this rule using vectors AB and AC, you must first translate vector AB so that its tail is aligned with the head of vector AC (point C). You can use the **Translation** tool to copy vector AB so that point A is aligned with point C. To do this, choose the **Translation** tool (**Menu > Transformation > Translation**). Click on vector AC and then on vector AB and press  $\text{enter}$ . Vector AB will be translated to point C. Draw a point (**Menu > Points & Lines > Point**) at the end of this new vector and label it *D*.

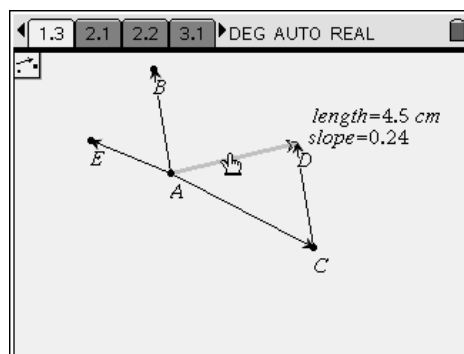


**Step 4:** Use the **Vector** tool to construct vector AD. Vector AD is the sum of vectors AB and AC. Drag point B around the screen to observe how vector AD changes as the magnitude and direction of vector AB changes. Then, use the **Length** and **Slope** tools (**Menu > Measurement > Length** and **Menu > Measurement > Slope**) to measure and label the lengths and slopes of vectors AB, AC, and AD. To use the measurement tools, first select the appropriate tool from the menu. Then, click once on the vector you wish to measure. The measurement will appear on the screen. Drag the measurement to where you want it to be, and then click again to place the measurement there. To label a measurement, double-click on the number, and then type the label next to the number.



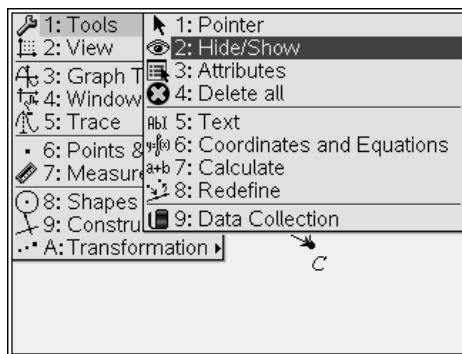
- Q15.** How does the length of vector AD compare with the lengths of vectors AB and AC when AB and AC are collinear?
- Q16.** How does the direction (slope) of vector AD compare with the slopes of vectors AB and AC? (Hint: The angle of the vector is equal to the inverse tangent of the slope. You can use the **Text** and **Calculate** tools to calculate the angle from the slope measurement you have made.)

**Step 5:** Next, you will explore the addition of three vectors. To add three vectors, you must first add two of the vectors, and then add the third vector to the sum of the other two. Use the **Vector** tool to construct vector AE pointing in a different direction than vectors AB, AC, and AD. Adding vector AE to vectors AB and AC is equivalent to adding vector AE to vector AD. To add these vectors, open the **Translation** tool. Click on vector AD and then on vector AE. Vector AE will then be translated to point D. Add a point at the end of this vector, and label it *F*. You can then construct vector AF, the sum of vectors AB, AC, and AD.



**Step 6:** Move points A, B, C, and E around the screen, and observe how vector AF changes. If you wish, you may hide vectors CD, AD, and DF by choosing **Menu > Tools > Hide/Show** and clicking on each vector in turn. This will make it easier to see how vector AF is related to vectors AB, AC, and AE.

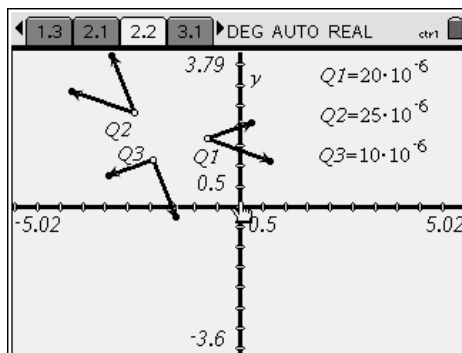
**Q17.** Describe the lengths and positions of vectors AB, AC, and AE that are required to make AF become as small as possible.



**Problem 2 – Interaction between three similarly charged particles**

Page 2.2 shows three charged particles. Because all three particles are positively charged, the forces between them are all repulsive. The vectors attached to each point show the forces on that particle that are produced by the other two particles. You can drag each point and thus vary the distances between the particles. The numbers on the right-hand side of the screen give the magnitudes of the charges on the particles. You can click on these labels and vary the magnitudes of the charges. The charges are given in units of coulombs (C), and distances are in meters.

**Step 1:** Find the net force on each particle using head-to-tail vector addition. Then, hide the individual forces on each particle (the original vectors present on the page).



**Step 2:** Vary the locations and magnitudes of the charges. Observe the effects of charge magnitudes and distances between the charges on the magnitude and direction of the net force on each particle.

**Q18.** Describe the changes you observed in the net force on each particle when you varied magnitudes and positions of the particles. Did you notice any patterns?

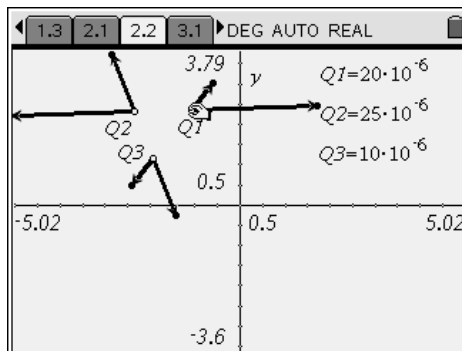
**Q19.** What happens to the net force on a particle when its charge is much larger or smaller than the charges on the other two particles?

**Q20.** What happens to the net force when one of the particles moves very close to one of the other particles?

**Q21.** What happens to the net force on a particle when it is moved far away from the other two particles?

**Q22.** What happens to the net force on a particle when it is located exactly between two equally charged particles?

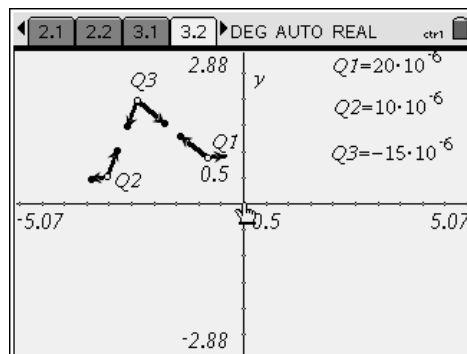
**Q23.** Three particles, each with a charge of  $+11 \mu\text{C}$ , are located at the corners of an equilateral triangle with sides of length 1 m. Calculate the magnitude and direction of the net force on each particle. (Hint: Use the template on page 2.2, adjust the scale if needed, and reproduce the conditions of the problem. Then calculate magnitude and angle for each net force.)



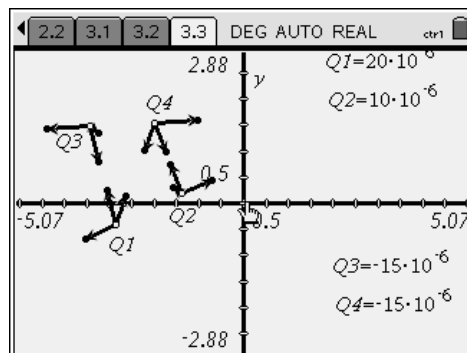
**Problem 3 – Interaction between particles of opposite charge**

In this problem, you will explore how interactions between oppositely charged particles differ from the interactions between particles with the same charge.

**Step 1:** Move to page 3.2, which shows two positively charged particles and one negatively charged particle. Find the net force on each particle using head-to-tail vector addition. Then, hide the individual forces on each particle (the original vectors present on the page). Move the particles around and observe the effects on the resultant forces.



**Step 2:** Next, move to page 3.3, which shows two positively charged particles and two negatively charged particles. Find the net force on each particle using head-to-tail vector addition. Then, hide the individual forces on each particle (the original vectors present on the page). Move the particles around and observe the effects on the resultant forces.



- Q24.** What happens to the net forces when oppositely charged particles move very close to one another?
- Q25.** What happens to the net force on a negatively charged particle when it is located exactly between two particles of equal positive charge?
- Q26.** Explore other symmetrical arrangements of the particles, and describe any patterns that you observe.
- Q27.** A particle of charge  $+100 \mu\text{C}$  is located at  $(-2, 0)$ , and a particle of charge  $+200 \mu\text{C}$  is located at  $(2, 0)$ . Where should a negatively charged particle with charge  $-50 \mu\text{C}$  be placed so that the net force on this particle has a magnitude of  $2 \text{ N}$  and is directed at  $-135^\circ$ ? (Hint: You can see the coordinates of a point by choosing **Menu > Actions > Coordinates and Equations** and then clicking on the point.)
- Q28.** A right isosceles triangle is formed by the charges  $Q_1 = +150 \mu\text{C}$ ,  $Q_2 > 0$ , and  $Q_3 = -120 \mu\text{C}$  located at points  $(0, 10)$ ,  $(0, 0)$ , and  $(-10, 0)$ , respectively. The fourth charge,  $Q_4 = -145 \mu\text{C}$ , is located at the midpoint of the hypotenuse. The net force on charge  $Q_4$  is  $10 \text{ N}$  in the positive  $x$ -direction. What is the charge on  $Q_2$ ?