EXPLORATIONS

Activity 5

Exploring the Vertex Form of a Quadratic Function

The vertex form of a quadratic function is $f(x) = a(x - h)^2 + k$. By changing the values of a, h and k the shape and position of the graph can be changed. The values of h and k also will determine the maximum or minimum value of the function,called the *vertex.*

In this activity, you will change the parameters of the function and determine the effects on its graph and its vertex.

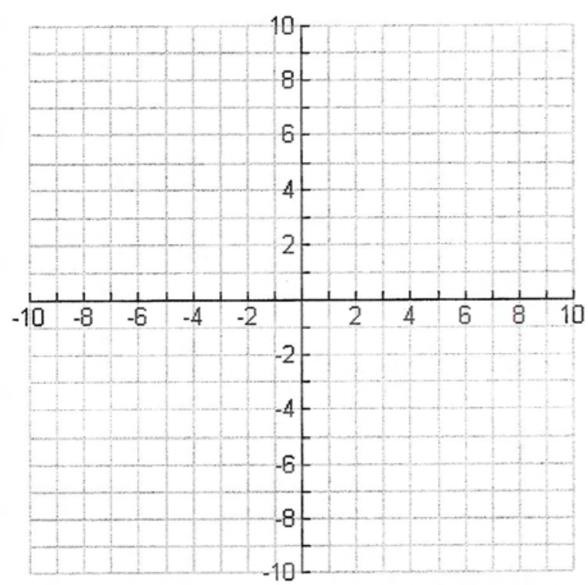
Exploration

1. Open a new TI InterActive! document. Title this document **Vertex Form of a Quadratic Function.** Add your name and the date to this document.

2. Select Math box and define a: = 1. In the next two math boxes, define h: = 0 and k: = 0.

3. Select Graph and define y1(x): = x^2 and y2(x): = a * (x −h) ^{2} + k. Click in the checkbox to the left of each equation to select the equation. Note that the two graphs are the same. Sketch the graph on the grid provided.

   Note: *Use the symbol palette to access the* ^{2}.



4. Click on Save to Document  to paste the graph into your TI InterActive! Document.

Analysis

1. Double-click on k: = 0 and change the value of *k* to each of the values indicated. Click out of the math box and describe the effects on the graph.

   k: = 2 ______________________________

   k: = 3.5 ______________________________

   k: = -3 ______________________________

   k: = -6.5 ______________________________

2. Generalize your findings about the effect of *k* on the graph in Question 3 of the Exploration.

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3. Double-click on k: = -6.5 and change the ⁻6.5 back to 0. Double-click on h: = 0 and change the value of *h* to each of the values indicated. Click out of the math box and describe the effects on the graph.

   h: = 1 ______________________________

   h: = 4.5 ______________________________

   h: = -2 ______________________________

   h: = -7.5 ______________________________

4. Generalize your findings about the effect of *h* on the graph in Question 3 of the Exploration.

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5. Double-click on h: = -7.5 and change the -7.5 back to 0. Double-click on a: = 1 and change the value of *a* to each of the values indicated. Click out of the math box and describe the effects on the graph.

a: = 2 ______________________________

a: = 3 ______________________________

a: = 1/2 ______________________________

a: = 1/4 ______________________________

6. Change the value of *a* to each of the values given below. Then explain the effects on the graph.

a: = -1 ______________________________

a: = -2 ______________________________

a: = -1/2 ______________________________

a: = -1/4 ______________________________

7. Generalize your findings about the effect of *a* on the graph in Question 3 of the Exploration.

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8. Double-click on the graph. Define y2(x): = 1 * (x − 2) ^{2} + 1 and click on the graph. Explain how $y2(x)$ can be obtained from the graph of $y1(x)$.

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9. Click on [Trace] and then on the graph of $y2(x)$. In the Trace Value box, click on the left and right arrows to trace to the lowest point on the graph of $y2(x)$. What are the coordinates of this point? This point is called the *vertex*.

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10. How does this point compare to the values of h and k of $y2(x)$?

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11. Define y2(x): = $-(x + 3)^2 - 4$ and click on the graph. Explain how $y2(x)$ can be obtained from the graph of $y1$.

12. Click on **Trace** and then on the graph of $y2$. Trace to the highest point on the graph of $y2(x)$. What are the coordinates of this point? This point is also called the vertex.

13. How does this point compare to the values of h and k of $y2$?

14. Save this document as **vertex_form.tii.** Print a copy of this document.

Additional Exercises

For each of the following functions, determine the vertex and identify the values of h and k. Explain how the graph can be obtained from the graph of $y = x^2$.

1. $f(x) = 2(x + 5)^2 + 2$

   vertex:________________ h:____________ k:____________

2. $f(x) = 0.5(x - 4)^2 - 4$

   vertex:________________ h:____________ k:____________

3. $f(x) = -2(x + 5)^2 - 6$

   vertex:________________ h:____________ k:____________

4. $f(x) = -0.5(x - 6)^2 + 3$

   vertex:________________ h:____________ k:____________

5. $f(x) = -(x + 4)^2 - 3$

   vertex:________________ h:____________ k:____________

6. How do the vertex and the ordered pair (h, k) compare?

7. How does the value of *a* determine whether the graph has a maximum or a minimum value?

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8. Generalize your findings about the effects of *a*, *h* and *k* on the graph of $f(x): = a(x - h)^2 + k$.

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Teacher Notes

Activity 5

Exploring the Vertex Form of a Quadratic Function

Objective

- Students will investigate the effects of changing the parameters a, h and k of the function $f(x) = a(x - h)^2 + k$
- Students will investigate the vertex of the function $f(x) = a(x - h)^2 + k$.

Applicable TI InterActive! Functions

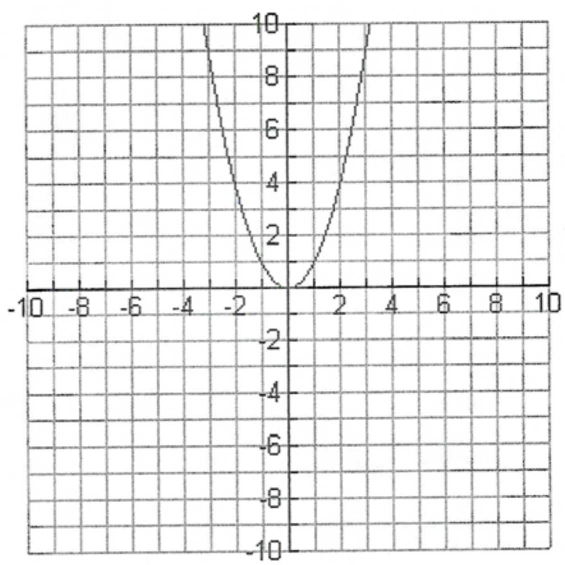
- Define variable:= value
- Graph 
- Trace Trace

Problem

In this activity, students will change the parameters a, h and k of the function $f(x) = a(x - h)^2 + k$ and discuss the effects of changing each on the graph of the parent function $f(x) = x^2$. Additionally, the vertex (h,k) will be determined and the effect of a on the vertex will be observed.

Exploration

3.



Analysis

1. k: = 2 The graph shifts up 2 units.

   k: = 3.5 The graph shifts up 3.5 units.

   k: = -3 The graph shifts down 3 units.

   k: = -6.5 The graph shifts down 6.5 units.

2. If $k > 0$, the graph will be shifted up k units. If $k < 0$, the graph will be shifted down k units.

3. h: = 1 The graph shifts to the right 1 unit.

   h: = 4.5 The graph shifts to the right 4.5 units.

   h: = -2 The graph shifts to the left 2 units.

   h: = -7.5 The graph shifts to the left 7.5 units.

4. If $h > 0$, the graph will be shifted to the right h units. If $h < 0$, the graph will be shifted to the left h units.

5. a: = 2 The graph is stretched vertically by a factor of 2.

   a: = 3 The graph is stretched vertically by a factor of 3.

   a: = 1/2 The graph is compressed vertically by a factor of 1/2.

   a: = 1/4 The graph is compressed vertically by a factor of 1/4.

6. a: = -1 The graph is flipped over the x-axis.

   a: = -2 The graph is flipped over the x-axis and stretched vertically by a factor of 2

   a: = -1/2 The graph is flipped over the x-axis and compressed vertically by a factor of 1/2.

   a: = -1/4 The graph is flipped over the x-axis and compressed vertically by a factor of 1/4.

7. If $a > 0$ the graph will open upwards and for $a < 0$ the graph is flipped over the x-axis and opens downwards. If the absolute value of a is greater than one, then the graph is stretched vertically by a factor of a. If the absolute value of a is less than one, then the graph is compressed vertically by a factor of a.

8. Shift the graph 2 units to the right and up 1 unit.

9. (2, 1)

10. They are the same.

11. Flip the graph over the x-axis, shift the graph 3 units to the left and down 4 units.

12. (-3, -4)

13. They are the same.

Additional Exercises

1. vertex: (-5, 2) *h*: -5 *k*: 2

   Stretch the graph vertically by a factor of 2, shift the graph to the left 5 units and up 2 units.

2. vertex: (4, -4) *h*: 4 *k*: -4

   Compress the graph vertically by a factor of 0.5, shift the graph to the right 4 units and down 4 units.

3. vertex: (-5, -6) *h*: -5 *k*: -6

   Flip the graph over the *x*-axis, stretch the graph vertically by a factor of 2, shift the graph to the left 5 units and down 6 units.

4. vertex: (6, 3) *h*: 6 *k*: 3

   Flip the graph over the *x*-axis, compress the graph vertically by a factor of .5, shift the graph to the right 6 units and up 3 units.

5. vertex: (-4, -3) *h*: -4 *k*: -3

   Flip the graph over the *x*-axis, shift the graph to the left 4 units and down 3 units.

6. The vertex and the ordered pair (h, b) are the same.

7. If $a > 0$, then the graph has a minimum. If $a < 0$, then the graph has a maximum.

8. The value of h shifts the graph left and right and determines the *x*-coordinate of the vertex. The value of k shifts the graph up and down and determines the *y*-coordinate of the vertex. The value of a stretches or compresses the graph horizontally and determines whether the graph has a maximum or a minimum.