Activity 23

Objectives

- To investigate the special properties of the midsegment quadrilateral
- To extend the idea of the midsegment triangle to quadrilaterals and other polygons

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Midsegments of Quadrilaterals

Introduction

In Activity 16, Midsegments of a Triangle, you learned about the properties of the midsegments and the midsegment triangle. In this activity, you will extend your understanding of midsegments by investigating the midsegments of a quadrilateral and the midsegment quadrilateral.

This activity makes use of the following definition:

Midsegment of a quadrilateral — a segment connecting the midpoints of any two consecutive sides of the quadrilateral

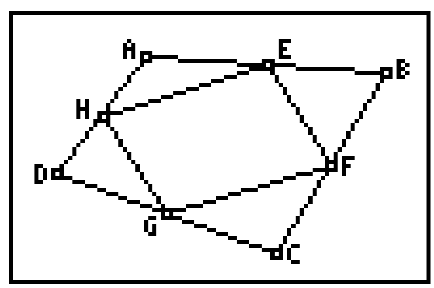
Construction

Construct the midsegment quadrilateral of a general quadrilateral.

Draw a quadrilateral *ABCD* near the center of the screen.

Construct the midpoint of each side of the quadrilateral, and label the midpoints *E*, *F*, *G*, and *H*.

Construct quadrilateral *EFGH.*



Exploration

Use various measurement tools (**Distance and Length**, **Slope**, **Angle**, and **Area**) to investigate the properties of quadrilateral *EFGH*. Drag vertices and/or sides of quadrilateral *ABCD* and observe whether these properties are true for any quadrilateral.

Use various measurement tools to investigate the relationship between quadrilateral *ABCD* and its midsegment quadrilateral *EFGH*. Drag the vertices and/or sides of quadrilateral *ABCD* and observe any relationships that are always true. Be sure to investigate any special properties of quadrilateral *EFGH* when quadrilateral *ABCD* is a special quadrilateral.

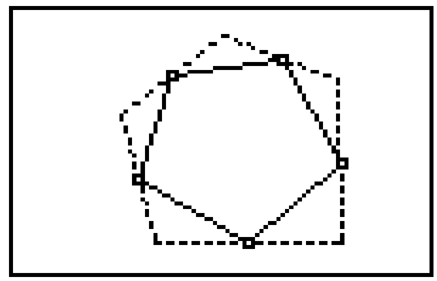
Questions and Conjectures

1. Make a conjecture about the properties of the midsegment quadrilateral of a general quadrilateral *ABCD*. Write a paragraph supporting your conjecture. Hint: Construct a diagonal of quadrilateral *ABCD* to defend your conjecture.

2. Make a conjecture about the relationship between a quadrilateral and its midsegment quadrilateral.

Extension

Make conjectures about the midsegment pentagon and a midsegment hexagon. Explain your reasoning and be prepared to demonstrate.



Teacher Notes

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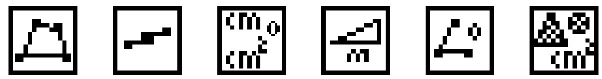
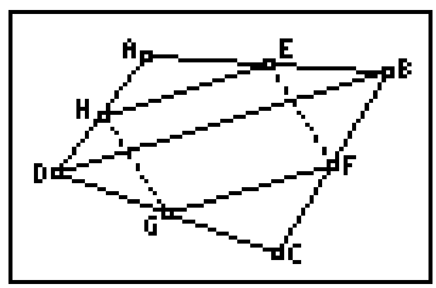


Answers to Questions and Conjectures

1. Make a conjecture about the properties of the midsegment quadrilateral of a general quadrilateral *ABCD*. Write a paragraph supporting your conjecture. Hint: Construct a diagonal of quadrilateral *ABCD* to defend your conjecture.

   The midsegment quadrilateral is a parallelogram. Some students will be able to recognize the properties of a parallelogram such as opposite sides are parallel and congruent, opposite angles are congruent, and the diagonals intersect at their midpoints.

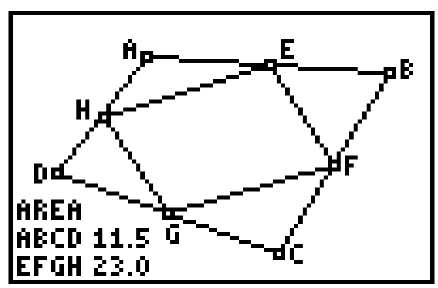
   

   Construct one of the diagonals of quadrilateral *ABCD*. This diagonal separates the quadrilateral into two triangles and the opposite sides of the midsegment quadrilateral are midsegments of the two triangles. Since they are both parallel to the diagonal, they are parallel to each other. Drawing the other diagonal shows the same reasoning for the other pair of parallel sides. This property is true no matter how quadrilateral *ABCD* is dragged and changed including whether it is convex or concave.

2. Make a conjecture about the relationship between a quadrilateral and its midsegment quadrilateral.

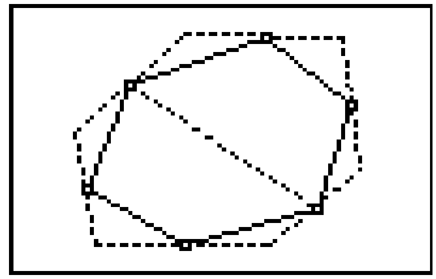
   Some of the conjectures might be:

   - If quadrilateral *ABCD* is a general quadrilateral, parallelogram, or trapezoid, then the midsegment quadrilateral is a parallelogram. (See Question 1.)
   - If quadrilateral *ABCD* is a rhombus or kite, then the midsegment quadrilateral is a rectangle. This is true since the diagonals of a rhombus or kite are perpendicular thus ensuring the sides of the midsegment quadrilateral are perpendicular.
   - If quadrilateral *ABCD* is a rectangle or an isosceles trapezoid, then the midsegment quadrilateral is a rhombus. This is true since the diagonals of a rectangle or an isosceles trapezoid are congruent thus ensuring that all four sides of the midsegment quadrilateral are congruent.
   - If quadrilateral *ABCD* is a square, then the midsegment quadrilateral is a square. This is true since the diagonals of a square are both perpendicular and congruent thus ensuring the sides of the midsegment quadrilateral are perpendicular and congruent.
   - The perimeter of the midsegment quadrilateral is equal to the sum of the lengths of the diagonals. This is true since the length of each midsegment is equal to one-half the length of the diagonal it is parallel to. (See Activity 16, Midsegments of a Triangle.)
   - The area of the midsegment quadrilateral is one-half the area of quadrilateral *ABCD*. It can be shown that the combined area of the four triangles formed by the midsegments (for example, ΔBEF) is equal to one-fourth the sum of the areas of the four triangles formed by the diagonals (for example, ΔBAC). This is true since each midsegment triangle's area is one-fourth the area of the original triangle (see Activity 16, Midsegments of a Triangle). Since the sum of the areas of the triangles formed by the diagonals is twice the area of quadrilateral *ABCD*, the sum of the area of the triangles formed by the midsegments is one-half the area of quadrilateral *ABCD*. Therefore, since the area of quadrilateral *EFGH* is equal to the area of quadrilateral *ABCD* minus the areas of the triangles formed by the midsegments, this relationship holds true for all midsegment quadrilaterals.



Answers to Extension

Make conjectures about the midsegment pentagon and a midsegment hexagon. Explain your reasoning and be prepared to demonstrate.



The Cabri® Jr. application has no tool that will draw a pentagon or a hexagon as a single object. Use the **Segment** tool to connect five (or six) consecutive segments together into a closed figure. To measure the area of the figure, overlay triangles and/or quadrilaterals onto the figure, compute the individual sub-areas, and then sum the sub-areas to find the total area of the figure. The perimeter of a figure can be found by adding the individual side lengths two values at a time.

Another way to investigate this relationship would be to use Cabri Geometry II™ Plus software.

Unlike midsegment triangles and quadrilaterals, polygons with greater than four sides have no general relationships regarding midsegments. Even though students may conjecture that the pattern continues, it is important that they verify their conjectures to note that the patterns fails. The verification process points out the importance of counterexamples in mathematical reasoning and proof.